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THE
RHIND MATHEMATICAL PAPYRUS

BRITISH MUSEUM 10057 AND 10058

INTRODUCTION, TRANSCRIPTION, TRANSLATION AND COMMENTARY

BY

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PREFACE

It is now nearly fifty years since Eisenlohr published his translation of and commentary on the Rhind Mathematical Papyrus, and the mathematician and the Egyptologist are still almost entirely dependent on this edition for their knowledge of the subject. Yet during these years our knowledge of the Egyptian language has been doubled and trebled, and Eisenlohr's translations, guided though they were to some extent by mathematical necessity, must be very seriously revised and modified. What is more, new documents have come to light: the New York fragments, the Kahun fragments, the Moscow papyrus, the Berlin fragments, not to mention later demotic, Coptic and Greek documents, must all be taken into account if a correct view is to be obtained of the mathematical abilities of the Egyptian.

Every attempt has been made to render the book intelligible to the mathematician who has no knowledge whatsoever of the Egyptian language. On the other hand, the Egyptologist with little knowledge of mathematics may enter on it without fear: Egyptian mathematics was a simple affair, and the author has tried throughout to deal with it in its own simple terms without clothing it in a modern dress which is totally foreign to it.

The work was begun in 1911 and was well advanced in August 1914. From that time it lay untouched until 1920, since when it has been practically rewritten, the changes being mainly in the direction of greater simplicity of treatment. It was not until early in the present year that I received the photographs of the Moscow Papyrus. These were sent to me on the natural understanding that I should not publish their contents. I will only say of them that in spite of their very high interest they have led me to modify practically nothing in my volume, though I should have been very sorry to have had to publish it in ignorance of them.

As usual my work contains many an admirable suggestion from Dr. Alan H. Gardiner, unacknowledged, at his own request, and I owe some valuable points to Mr. Battiscombe Gunn. I have also to thank Professor R. C. Archibald, of Brown University, Providence, R.I., for the liberality with which he put at my disposal his admirable bibliography of the Rhind Papyrus. As this bibliography is to be published in full in America in the near future, I have thought it unnecessary to append to my volume what could at best be no more than a selection from it. That I was able to obtain for study photographs of the Moscow Papyrus is mainly due to Dr. Fritjof Nansen, whose sympathies in the work were kindly enlisted by Professor D'Arcy Thompson, of St. Andrews. Professor Strouwé, of Petrograd, who is to publish the papyrus, very unselfishly agreed to my having the photographs, and the Director of the Museum of Fine Arts in Moscow, Dr. W. Ghiatzintoff, was kind enough to have them made for me.

T. ERIC PEET

LIVERPOOL,
June 23rd, 1923.

LIST OF ABBREVIATIONS.

<i>A.Z.</i>	<i>Zeitschrift für ägyptische Sprache.</i>
<i>B.M.Facs.</i>	<i>Facsimile of the Rhind Mathematical Papyrus in the British Museum, London, 1898.</i>
CANTOR	CANTOR, M., <i>Vorlesungen über Geschichte der Mathematik</i> , 2nd ed., Vol. I, Leipzig, 1894.
EISENLOHR	EISENLOHR, <i>Ein mathematisches Handbuch der alten Ägypter, übersetzt und erklärt</i> , Leipzig, 1877. (A second edition without plates, 1891.)
GRIFFITH, K.P.	GRIFFITH, F. LL., <i>Hieratic Papyri from Kahun and Gurob</i> , London, 1898.
HEATH	HEATH, SIR THOMAS, <i>A History of Greek Mathematics</i> , Oxford, 1921.
<i>J.E.A.</i>	<i>Journal of Egyptian Archaeology.</i>
L., D.	LEPSIUS, RICHARD, <i>Denkmaeler aus Aegypten und Aethiopien</i> , Berlin, n.d.
<i>P.S.B.A.</i>	<i>Proceedings of the Society of Biblical Archaeology.</i>
SETHE, V.Z.Z.	SETHE, KURT, <i>Von Zahlen und Zahlworten bei den alten Ägyptern</i> , Strassburg, 1916.
Urk.	<i>Urkunden des ägyptischen Altertums</i> , ed. GEORG STEINDORFF, Leipzig, various dates.

CONVENTIONAL SIGNS USED IN THE TRANSLATION.

Square brackets [] enclose restorations of damaged or lost passages in the papyrus. They are never used in this volume as a mathematical symbol.

Pointed brackets < > enclose words or passages which never stood in the papyrus, but whose omission there is due to an error on the part of the scribe.

THE RHIND MATHEMATICAL PAPYRUS

INTRODUCTORY

PREVIOUS WORK ON THE PAPYRUS.

THE first scholar to study the papyrus would appear to have been Lenormant, who published a note on it as early as 1867.¹ He gave some account of its contents, naturally not altogether accurate—he speaks of the determination of the volume of a pyramid—and dated the document to the XIIth Dynasty. In the following year Dr. Birch described the papyrus with a few quotations in hieroglyphs.² The next student was Brugsch, who, in 1874, published a short article giving some of the more important technical terms used in the document.³

Little of importance appeared henceforward until in 1877 Eisenlohr published his volume *Ein mathematisches Handbuch der alten Aegypter*. This was accompanied by a hieratic text which was practically a reproduction of facsimile plates made by the Trustees of the British Museum in 1869 and delayed in publication. A set of these plates had been lent to Eisenlohr during his stay in London in 1872, a courtesy which he seems to have repaid by publishing a tracing of them without authority. The British Museum publication of the admirable and almost perfect facsimile did not take place until 1898.⁴

After the appearance of Eisenlohr's edition no work was done on the papyrus until 1891, when a series of brilliant articles from the pen of Griffith began to appear in the *Proceedings of the Society of Biblical Archaeology*.⁵ These dealt not only with the Rhind Papyrus itself, but with the subject of Egyptian weights and measures in general.

Since that time numerous articles treating of specific problems in the papyrus have appeared, mainly by Borchardt and Schack-Schackenburg. These are dealt with in those sections of the papyrus to which they refer. The only other work which calls for special mention here is F. Hultsch's *Die Elemente der ägyptischen Teilungsrechnung in Abhandlungen der Kgl. Sächs. Gesellschaft der Wissenschaften, phil.-hist. Klasse*, vol. 17, no. 1, Leipzig, 1895. This is an exhaustive survey covering most of the ground of Egyptian fractional arithmetic as exemplified in the Rhind Papyrus.

DESCRIPTION OF THE PAPYRUS.

The Rhind Papyrus as it now lies in the British Museum consists of two pieces separately mounted between sheets of plate glass and numbered 10057 and 10058 respectively. These two pieces once formed a single roll, and were probably separated in modern times by an unskilful unroller. There is now a gap between them, and the Historical Society of New York possesses a number of fragments which come from this gap. A glance at Plate E,

¹ *Note relative à un papyrus égyptien contenant un fragment d'un traité de géométrie appliquée à l'arpentage, Comptes rendus de l'Acad. des Sciences*, Paris, vol. 65, p. 903.

² *Ä.Z.*, 1868, 108–10.

³ *Ä.Z.*, 1874, 147–49.

⁴ *Facsimile of the Rhind Mathematical Papyrus*, London, 1898.

⁵ Vols. XIII, 328 ff.; XIV, 26 ff.; XVI, 164 ff., 201 ff., 230 ff.

in which these fragments are arranged so far as possible in their proper places, will show that it would have been impossible to cut the papyrus in two vertically at any point in this gap without mutilating a problem. This tells strongly against Griffith's suggestion¹ that the papyrus was cut in two by its owner in ancient times for reasons of convenience, and it is far more likely that the damage was done after the finding in modern times, probably in an ill-advised attempt at unrolling.

There is no evidence of value as to how the fragments came to be separated from the larger sheet or sheets. The two portions in the British Museum were bought by Mr. A. H. Rhind in Luxor in 1858, and said to have been found with others in a chamber in the ruins of one of the small buildings near the Ramesseum.² The entry in the Catalogue of the Egyptian Collection of the New York Historical Society is as follows:—"265. Fragments of two or more papyri, containing numbers and quantities. Found with fragments of the Medical Papyrus and No. 262." The Medical Papyrus in question is the now well-known Edwin Smith Papyrus, and No. 262 is a tiny fragment of hieratic writing containing the name of Tuthmosis I. The mathematical fragments came into the possession of the Society in 1907 as part of the Edwin Smith Collection. It seems likely that the dates March 17/62 and Dec. 10/63, written on the paper mounts by Edwin Smith, represent the time when he acquired the fragments; at least they were in his hands by those dates.³

If these two last are indeed the dates of acquisition, and it is not easy to see what else they could be, it would seem that the native finders of the roll attempted to open it in or before 1858, when they sold the main portion to Mr. Rhind, but that they kept back the fragments and sold them to Mr. Smith in two instalments on the dates given.

What is of still greater interest, if it be true, is the statement that the fragments were found with fragments of the Medical Papyrus and with a dated scrap of the reign of Tuthmosis I. If we could trust this statement we should infer that the Arabs found a cache of scientific documents dating, like the Rhind and the Edwin Smith, from the Hyksos Period, stored away not earlier than the reign of Tuthmosis I. No one, however, who knows the habits of the native finder and dealer will be unwise enough to make any deduction at all.

The two sheets in the British Museum, Fig. 1, may be described as follows:—

Papyrus 10058.

Present length 206 cm., height 33 cm.

Recto. The recto consists of five pages of papyrus, each about 395 mm. broad, except the first, and the last, which is incomplete. The gumming is admirably done. At the right-hand end there is a blank space of about 10 cm., after which the title begins in vertical columns. This is followed by a double vertical black line, and from this point leftwards the sheet is ruled in black into six horizontal registers or bands throughout. The whole of this recto is devoted, with the exception of the title already referred to, to the division of 2 by the odd numbers from 3 to 101.

Verso. This face is very heavily patched, not only at the blank left-hand end, but also on the right: this patching was clearly done in ancient times, and where signs had disappeared on a lost fragment they have been written in in very black ink on the patches by a later hand. At 18 cm. from the right-hand end is a double vertical ruling in black, as on the recto. Left of this the papyrus is ruled out into six horizontal registers. The writing begins at the same end as on the other face. On the right, outside the double line, is the carelessly

¹ *P.S.B.A.*, XVI, 164-5. Griffith had not seen the fragments.

² *B.M.Facs.*, Preface.

³ I owe this information to Mrs. C. Ransom Williams. For the very interesting details of Mr. Smith's stay in Luxor see Breasted in *Recueil d'Études Égyptologiques Champollion*, Paris, 1922, 385 ff.

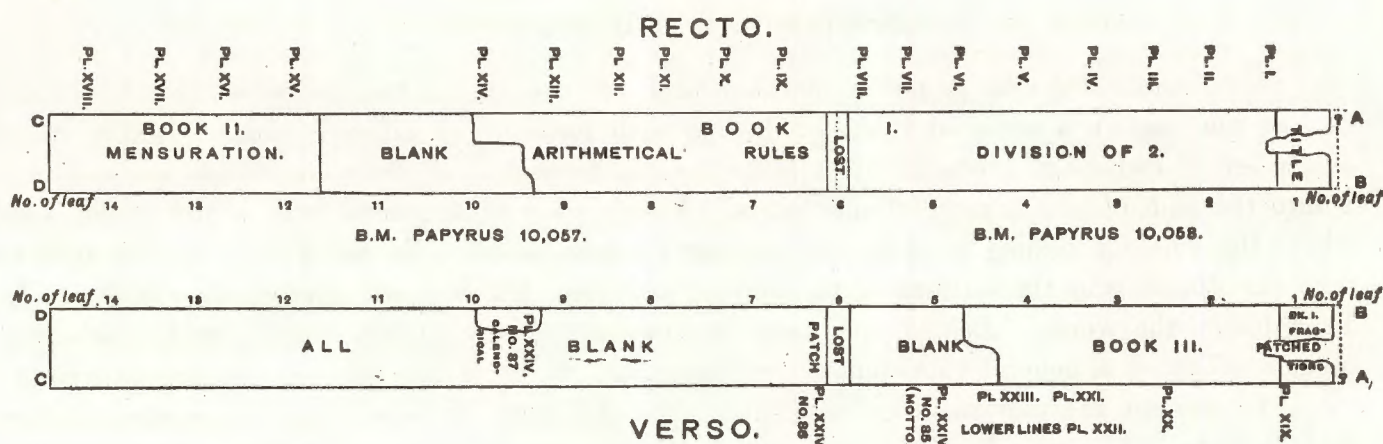


Fig. 1 (after GRIFFITH, *P.S.B.A.*, XVI, Pl. 1.) The plate numbers refer to the *B.M.Facs.*

written No. 61. It is followed by Nos. 62 to 84, and to the left of this last the papyrus is blank to its end (57 cm. away), except for the curious No. 85, written upside down near the bottom and about half-way along the blank space.

Papyrus 10057.

Present length 319 cm., height 33 cm. or just over, the edge being actually under the framing.

Recto. The pages are from 39 to 40 cm. broad, and the gumming is very accurate. The left-hand end is fragmentary. The whole face is ruled out in six horizontal registers. Problem No. 1 begins the recto and we run without a break up to No. 40, after which there is a blank of about 55 cm.: No. 41 begins a fresh page, and the problems continue up to No. 60, which ends the recto.

Verso. This face is quite blank except for the calendrical entry No. 87, which is written near the top about half-way along. At the right-hand end the papyrus has been patched with a piece of another papyrus bearing the fragment of accounts numbered No. 86.

The whole papyrus is of a good bright colour. Two kinds of ink were used, a fairly dark black and a bright red. This last is employed for headings, and also in order to bring into prominence certain figures in the problems. The length of the papyrus before it was cut in two was about 543 cm., and its recto showed 14 sheets, each from 383 to 400 mm. in width, except the first and last which were narrower, unless this is due to damage. The pages of writing are very variable in breadth.

DATE OF THE PAPYRUS.

The Rhind Papyrus is dated in the 33rd year of the Hyksos King Auserre^c Apophis, who must have ruled at some time between 1788 and 1580 B.C. There is no reason to doubt the scribe's own statement¹ that it was a copy of an older document written in the time of King Nemare^c, Amenemmes III of the XIIth Dynasty, who was on the throne from about 1849 to 1801 B.C. It is thus the latest of our hieratic mathematical papyri, the Moscow Papyrus and the Kahun and Berlin fragments all being definitely XIIth Dynasty documents: its prototype clearly belonged to the same *milieu* as these.

¹ Griffith, *P.S.B.A.*, XIV, 436 note, doubted the truth of this in its entirety, on the ground that the double- and quadruple-*hekat* found in some examples in Rhind were not in use in the XIIth Dynasty. He would hardly maintain this now, for his own decipherment of the Kahun fragments has shown that the double-*hekat* was regularly used in the early Middle Kingdom, and in the case of the quadruple-*hekat* we ought not to argue *ex silentio* that it was not in use as early as Nemare^c.

CONTENTS OF THE PAPYRUS.

The Rhind Papyrus is not a mathematical treatise in the modern sense, that is to say it does not contain a series of rules for dealing with problems of different kinds. It consists of a number of examples, preceded by a table for the resolution of fractions whose numerator is 2 into the sum of two or more aliquot parts. A suggestion of a general rule occurs in No. 61B, where the rule for finding $\frac{2}{3}$ of an aliquot part is formulated. To get $\frac{2}{3}$ of $\frac{1}{5}$ we are told to take the double and the 6-times of the aliquot part (*sic*), which would give as answer $\frac{1}{10} + \frac{1}{30}$. Then follow the words "Behold one does likewise in the case of any aliquot part which may occur." This is a general rule, but, if we except No. 66, it is the only one in the papyrus.

In content the papyrus does not stand alone, for none of those which we possess contain general rules, but merely series of tables and of examples worked out by their aid, and we are justified in doubting whether such things as theoretical general treatises existed in Egypt. It would be fully in keeping with what we know of the concrete nature of Egyptian thought had their mathematicians failed to formulate general principles and confined themselves to the working out of concrete instances.

The papyrus begins with the long table of resolution of fractions whose numerator is 2, as mentioned above, preceded only by a very short title and by the name of the maker of the copy and the date of its making. These together occupied the whole of the recto of Papyrus 10058, and the table extended over a portion of the gap between the two papyri from which come the New York fragments.

This table is followed by a number of examples to which Eisenlohr has given the serial numbers 1-84. Nos. 1-40 are purely arithmetical, and are in the main examples of the multiplication and division of fractions. They are followed by a considerable blank in the papyrus (see above, p. 2), after which follow a number of problems in the measurement of areas, volumes and angles of slope, Nos. 41-60. These bring us to the end of the recto of 10057.

If we now turn the whole papyrus over on its longer axis we find at the beginning, *i.e.* at the right-hand end of 10058, No. 61, which deals with a purely arithmetical matter, the multiplication of fractions, and seems out of place here. It is in fact written outside the ruling of the sheet (see above, p. 3), and was clearly not part of the scribe's original scheme for this portion of the papyrus. Possibly he added it afterwards in this very accessible blank space because it contained a table of fractions to which he frequently needed to refer. It is followed by a group of miscellaneous problems, all purely arithmetical in type and couched in concrete terms, numbered 62-84. These complete the mathematical portion of the papyrus, but do not carry us even as far as the end of the verso of 10058, the rest of which is blank except for the two curious columns of signs numbered 85, which are written upside down at the bottom of the page about half-way along the blank space.

Passing across to the verso of 10057 we find that its right-hand end is patched with a piece of papyrus (No. 86) bearing some accounts. The whole verso is blank except for the so-called calendrical entries, No. 87, written at the top about half-way along.

The following is a synopsis of the contents:—

Table of resolution of fractions with numerator 2.

Book I. Arithmetic.

Nos. 1-6. Division of various numbers of loaves equally between 10 men.

Nos. 7-20. First group of completion-calculations (*sekem*) involving multiplication of fractions.

Nos. 21-23. Second group of completion-calculations, involving simple addition of fractions.

Nos. 24-34. Arithmetical solution by trial of equations of the first degree. (Eisenlohr's *hau*-calculations.)

Nos. 35-38. Similar equations involving the bushel or *hekat*.

Nos. 39 and 40. Division of loaves between men in unequal proportions.

Book II. Mensuration.

Part I. Volumes and cubic content in corn.

Nos. 41-43. Cylindrical containers.

Nos. 44-46. Rectangular parallelipedal containers.

No. 47. Expression in correct form¹ of $\frac{1}{10}$, $\frac{1}{20}$, up to $\frac{1}{100}$ of a *hekat*, disguised as a sum in cubic content.

Part II. Areas.

No. 48. Area of square and circle compared.

No. 49. Rectangle.

No. 50. Circle.

No. 51. Triangle.

No. 52. Truncated triangle.

No. 53. Trapezoid (?).

Nos. 54 and 55. Division of given area of land into equal-sized fields.

Part III. Batter, or angle of slope.

Nos. 56-59. Batter of pyramid.

No. 60. Slope of a cone (?).

Book III. Miscellaneous problems in arithmetic.

No. 61. Multiplication of fractions (probably out of place, see above).

No. 62. Proportionate values of precious metals.

No. 63. Division of loaves in unequal proportions.

No. 64. Division of barley into shares in arithmetical progression.

No. 65. Division of loaves in unequal proportions.

No. 66. Daily portion of a yearly ration of fat.

No. 67. Reckoning of livestock.

No. 68. Division of 100 *hekat* of corn in unequal proportions.

Nos. 69-78. So-called *pefsu*-reckonings. Conversion of grain into bread and beer, and the barter of these last.

No. 79. Geometrical progression.

No. 80-1. Conversion of fractions of the *hekat* ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, etc.) into *henu*.

No. 82-3. Food estimate for a poultry yard.

No. 84. Estimate of food of an ox-stall.

Additions.

No. 85. Unintelligible group of signs.

No. 86. Fragment of accounts.

No. 87. Calendrical entries.

¹ See p. 25.

DOCUMENTS AVAILABLE FOR THE STUDY OF EGYPTIAN MATHEMATICS.

A.—EARLY DOCUMENTS.

The ancient Egyptian documents which deal with mathematics as such are not numerous. They comprise the following papyri and tablets, all dating from the Middle Kingdom, with the exception of the Rhind, which, while actually written down in its present form in Hyksos times, is according to its own statement a copy of a Middle Kingdom original.

1. The Rhind Mathematical Papyrus, now in the British Museum, except for some fragments in the possession of the Historical Society of New York.

2. The Moscow Mathematical Papyrus, in the Museum of Fine Arts of Moscow. This document, which dates from the XIIth Dynasty, has lain unpublished at Moscow for many years, and though by the kindness of the Director of the Museum of Fine Arts of Moscow and Professor Strouvé, who is to publish the papyrus, I have been allowed to have photographs of it, I should be betraying my trust did I add anything to the short note in *Ancient Egypt*¹ by the late Professor Turaiev. In this note was published one problem from the papyrus, which seems to give the correct determination of the volume of a truncated pyramid on a square base (p. 93). The writer also stated that the papyrus contained "19 problems, some of which give us new types of calculation unknown till now, and therefore somewhat difficult to comprehend. Four of these problems are geometrical ones. The first shows how to define the length of the sides of a quadrilateral, when the relation of the sides and the area of the quadrilateral are known. The two next give a method of calculating the area of a triangle: a method already known to us." I will only add to this that though the papyrus is of the highest interest owing to its early date and admirable state of preservation (in part at least) it contains nothing, with the exception of the problem of the truncated pyramid, which will greatly modify the conception of Egyptian mathematics given to us by the already published papyri and fragments.

3. The Kahun fragments. These were found at Kahun in 1889 by Professor Flinders Petrie and published by F. Ll. Griffith in the volume called *Hieratic Papyri from Kahun and Gurob* (London, 1898), pp. 15–18 of the text volume and Plate VIII. The mathematical contents of the fragments are as follows:—

(a) Table of resolutions of all fractions whose denominator is odd and whose numerator is 2, from $\frac{2}{3}$ to $\frac{2}{27}$, into the sum of two or more fractions with numerator unity. Pl. VIII, lines 1–10.

(b) Multiplication of $\frac{1}{3} + \frac{1}{12}$ by 9 (line 11), perhaps a fragment of some longer problem.

(c) The number 110 is divided by 8 and the result stated to be $13\frac{2}{3} + \frac{1}{12}$. From this $\frac{2}{3} + \frac{1}{6}$ is continuously subtracted nine times. (Line 12.)

(d) To find the content in *khar* of a cylinder of diameter 12 and height 8 cubits (lines 13–14; see *Ä.Z.*, 35, 150–2, and 37, 78–9).

(e) A list of eight very large numbers. The fragment is full of lacunae and the numbers seem to have no connection one with another. Perhaps it was an addition. (Lines 15–22.)

(f) Problem which can be expressed algebraically as follows: If $\frac{1}{2}x - \frac{1}{4}x = 5$, find x . (Lines 23–28.)

(g) A difficult problem, with beginning lost, dealing with the content in *henu* of a container or containers, parallelopipedal in form, the sides of whose bases are to one another in a fixed ratio (lines 30–42). The solution involves the use of square root.

(h) Accounts of a poultry yard. (Lines 43–62.)

4. Berlin Papyrus 6619, published by Schack-Schackenburg in *Ä.Z.*, 38, 135 ff. with Tafel IV: provenance not stated. The papyrus consists of four fragments reproduced on Tafel IV under the numbers 1–4.

¹ 1917, 100–102.

No. 1 is a problem, To divide 100 square cubits into two squares whose sides are in the proportion $1:\frac{3}{4}$. It involves the use of square root.

No. 2. Fragment of a problem in exchanges of various kinds of grain. Too much is lost to allow the real nature of the sum to be discerned.

No. 3. A problem in numbers involving the correct determination of the square root of $6\frac{1}{4}$ as $2\frac{1}{2}$.

No. 4. Too fragmentary for diagnosis.

5. Two wooden tablets in the Cairo Museum. These bear the catalogue numbers 25367 and 25368. They are writing tablets of the usual stuccoed type and were found at Akhmîm. One bears on one side a letter and a list of servants, and on the other certain mathematical calculations. The other tablet bears on one side a list of servants and a mathematical calculation, while the reverse is entirely occupied by calculations. The former tablet is dated in year 28 of an unnamed king. The style of the script and the names of the persons point to the Middle Kingdom.

The calculations, which are five in number, were first published, but badly misunderstood, by Daressy.¹ Möller next referred to them,² stating that in them certain fractions in later times used only to denote parts of the *hekat* or bushel were actually multiplied together, and must therefore have been at this period pure fractions. Sethe has pointed out that the major premise of this syllogism is false.³

In reality these calculations⁴ are nothing more or less than the finding of a third, a seventh, a tenth, an eleventh and a thirteenth respectively of a *hekat* or bushel in terms of the parts of the bushel used in ordinary everyday transactions, namely the $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, $\frac{1}{64}$, for each of which there existed a special sign (see p. 25), and the $\frac{1}{320}$ th part, called the *ro*. We ourselves are not accustomed to think in sevenths or thirteenths of a ton but reduce these to hundredweights, quarters, pounds and ounces, weights of which we have a fairly clear conception: so the Egyptian reduced these unwieldy fractions of the bushel to certain fixed parts, and his system was superior to ours in that each part was exactly half of the next larger.

A couple of examples will enable the reader to grasp the bearing of these tables. Thus one seventh of a bushel is shown to be equivalent to

$$(\frac{1}{8} + \frac{1}{64}) \text{ bushel} + (\frac{1}{2} + \frac{1}{7} + \frac{1}{14}) \text{ ro},$$

while a third of a bushel works out to

$$(\frac{1}{4} + \frac{1}{16} + \frac{1}{64}) \text{ bushel} + 1\frac{2}{3} \text{ ro}.$$

In addition to the above documents the various account papyri are of importance for the study of mathematics and especially of weights and measures. The most important of these so far published are Papyrus Bulaq 18, see *Ä.Z.*, 57, 51-68, and those dealt with by Spiegelberg in his *Rechnungen aus der Zeit Setis I.*

B.—SOME LATER DOCUMENTS FROM EGYPT.

There are three later documents from Egypt which call for notice here, though in considering them it must be remembered that owing to their late date we must make great allowances for the possibility of contamination from Greek mathematics, and must not use them to prove anything with regard to the state of the science in the earlier periods in Egypt. These are the Demotic Papyrus, the Byzantine and Coptic tables of fractions, and the Akhmîm Papyrus.

¹ *Recueil de Travaux*, XXVIII, 62 ff.

³ *V.Z.Z.*, 74, note 2.

² *Ä.Z.*, XLVIII, 99.

⁴ See my article in *J.E.A.*, IX, 91 ff.

1. The Demotic Papyrus.¹ This is a document stated to be in the Library of the Egypt Exploration Society, and to have been dated by Griffith to the Roman period. It contains tables giving resolutions into aliquot parts² of fractions with various denominators, which originally probably ran from 3 (Hultsch suggests $\frac{2}{3}$!) to beyond 15. An example will make this clear. The table dealing with the denominator 7 is as follows (partly restored):—

$$\begin{aligned} 1 \div 7 &= \frac{1}{7} \\ 2 \div 7 &= \frac{1}{4} + \frac{1}{28} \\ 3 \div 7 &= \frac{1}{3} + \frac{1}{14} + \frac{1}{42} \\ 4 \div 7 &= \frac{1}{2} + \frac{1}{14} \\ 5 \div 7 &= \frac{2}{3} + \frac{1}{21} \\ 6 \div 7 &= \frac{1}{2} + \frac{1}{4} + \frac{1}{14} + \frac{1}{28} \\ 7 \div 7 &= 1 \end{aligned}$$

In the same way in the case of any denominator n the numerators run from 1 to $n - 1$, omitting such as can be reduced at once to a single aliquot part, e.g. $\frac{2}{8} = \frac{1}{4}$ or $\frac{3}{9} = \frac{1}{3}$.

2. Several tables of fractions of Byzantine date from Egypt are known. The best of them are those published by Sir Herbert Thompson in *Ancient Egypt*, 1914, 52 ff. These are written on wood and are, as is clear from their numbering, the two last of a series of 16 such tables; they deal with the division of whole numbers by 15 and 16 respectively, the results being expressed in aliquot parts. Thus,

$$\begin{aligned} \text{The 15th part of 1} &= \frac{1}{15} \\ \text{„ 2} &= \frac{1}{10} + \frac{1}{30} \\ \text{„ 3} &= \frac{1}{5} \\ \text{„ 4} &= \frac{1}{4} + \frac{1}{60} \\ \text{„ 5} &= \frac{1}{3} \\ \text{„ 6} &= \frac{1}{3} + \frac{1}{15} \quad \text{and so on.} \end{aligned}$$

The whole numbers divided in the 15-table run from 1 to 15, in the 16-table from 1 to 16. The preceding tables, which are lost, doubtless dealt with division by the lower digits 2 to 14; the first table perhaps dealt with multiplication by $\frac{2}{3}$.³

3. A Coptic ostrakon published by Crum⁴ contains a similar table, the full meaning of which was first recognized by Sethe.⁵ The divisor is here 31, and the numbers to be divided run from 1 to 31 with the addition of $\frac{1}{2}$ and $\frac{1}{3}$.

4. The Mathematical Papyrus of Akhmîm⁶ was discovered by natives in the cemeteries of that town. It is in the form of a bound book with leather cover, and is in fairly good preservation. There is no direct indication of date, but arguments from the nature of the burials in the cemetery and the style of the writing would indicate a date between the sixth and the ninth centuries A.D. The contents of the papyrus consist of a series of tables of fractions followed by a number of problems partly illustrating these.

The tables give $\frac{2}{3}$ and the various aliquot parts of each of a long series of whole numbers. The aliquot parts are $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and so on down to $\frac{1}{20}$. As far as the one-tenth table the whole numbers run by units from 1 to 10, then by tens up to 100, by hundreds up to 1,000, and by thousands up to 10,000. In the remaining tables the whole numbers only

¹ RÉVILLOUT, E., *Mélanges sur la métrologie, l'économie politique et l'histoire de l'ancienne Égypte*, Paris, 1895; HULTSCH, F., *Neue Beiträge zur ägyptischen Teilungsrechnung*, in *Bibliotheca Mathematica*, ser. 3, vol. II, 1901, 177-84.

² $\frac{2}{3}$ is, as usual, reckoned as an aliquot part.

³ See SETHE, *V.Z.Z.*, 70-71.

⁴ *Coptic Ostraca*, No. 480.

⁵ *V.Z.Z.*, 71-72.

⁶ BAILLET, J., *Le papyrus mathématique d'Akhmîm* (*Mémoires de la Mission Archéol. Franç. au Caire*, vol. 9, fasc. 1), Paris, 1892. See further LORIA, G., *Un nuovo documento relativo alla logistica greco-egiziana* in *Bibliotheca Mathematica*, ser. 2, vol. VII, 1893, 79 ff.

go as high as the denominator of the aliquot part, *e.g.*, in the table of $\frac{1}{11}$ they run only from 1 to 11, as in the Byzantine and Coptic tables described above.

The problems are exactly 50 in number and range over various subjects, the volume of various containers, division of earnings between several workmen, questions of interest on money and so on. Baillet rightly remarks that as against the Rhind the Greek papyrus shows less interest in the actual solving of its problems and more in the detail of their solution. In the treatment of fractions it shows much that is new.¹ Yet here still, as in the Rhind, the only fractions dealt with are aliquot parts (with the old exception of $\frac{2}{3}$). There is, however, considerably more skill shown in their handling, and the calculator has undoubtedly acquired greater power over them. Thus it is possible from the working to show that the resolution of proper fractions into the sum of two or more aliquot parts was carried out according to certain fixed formulae: it will be seen later that all attempts to show that this was the case in the Rhind table of resolutions have failed.

The main result of the documents above described is to show that in Egypt, as elsewhere in the east, the advanced mathematics of the Greeks had not even in this period succeeded in replacing the cruder methods of earlier civilizations. Here at Akhmîm we have a system in which all fractions except aliquot parts must be eschewed, at a time when the equivalent of the modern notation of proper fractions had been known to the Greeks for some centuries.

DATE OF ORIGIN OF EGYPTIAN MATHEMATICS.

Our information on this point is sadly defective. The Rhind Papyrus dates from the Hyksos Period, though it claims to be a copy of a document prepared in the XIIth Dynasty, in the reign of Amenemhet III. This may well be, since both the Moscow Papyrus and the Kahun fragments date from that Dynasty. But how much earlier must we go to find the beginnings? Surely the complicated fabric of Egyptian mathematics can hardly have been built up in a century or even two, and it is tempting to suppose that the main discoveries of mathematics should be dated to the Old Kingdom. There is a very definite tendency among Egyptologists to put this period down as the Golden Age of Egyptian knowledge and wisdom. There can be little doubt that some of the literary papyri have their roots in this era, as for example the Proverbs of Ptahhotep, and the antiquated constructions of the medical papyri make it possible that the science of medicine, such as it was, had its spring in the Old Kingdom.

Of definite evidence for this early date there is none. All we know is that by the beginning of the First Dynasty the system of notation was complete up to the sign for 1,000,000 (see p. 11, note 1). In the IVth Dynasty we find in the tomb of Methen that the land measures of the Rhind Papyrus are already in full development in a form which involves correct determination of the area of the rectangle, but not of necessity of the triangle or circle. There appears to be no early evidence with regard to measures of capacity,² though one may almost take it for granted that with the measurement of the field on which the corn was grown went that of the containers in which it was stored and sold. That measurement by weighing was practised can hardly be denied in view of various objects of Old Kingdom date which can scarcely be anything but weights, as for example the stone weight of Khufu,³ formerly in the Hilton Price collection, though the attempts to establish a standard from these objects have been far from satisfactory.

¹ *E.g.*, the resolution of an aliquot part into the sum of two or more smaller ones, $\frac{1}{4} = \frac{1}{8} + \frac{1}{8}$. Cf. however the Rhind resolution of $\frac{1}{101}$, p. 47.

² The hieratic signs for the dimidiated portions of the *hekat* occur in a VIth Dynasty papyrus (Berlin, P 10500, unpublished). See *A.Z.*, 48, 100.

³ *P.S.B.A.*, XIV, 435-6, 442.

From these feeble indications we pass straight to the fully developed mathematical system of the XIIth Dynasty, the early stages in the building up of which are entirely concealed from us.

GENERAL CHARACTER OF EGYPTIAN MATHEMATICS.

The outstanding feature of Egyptian mathematics is its intensely practical character. This is not peculiar to mathematics, for it is typical of all the sciences in Egypt. As Plato alone of the Greeks seems to have realized,¹ the Egyptians were essentially a "nation of shop-keepers," and interest in or speculation concerning a subject for its own sake was totally foreign to their minds.

To realize this we have only to take a glance through the problems of the Rhind Papyrus. Here everything is expressed in concrete terms. The Egyptian does not speak or think of 8 as an abstract number, he thinks of 8 loaves or 8 sheep. He does not work out the slope of the sides of a pyramid because it interests him to know it, but because he needs a practical working rule to give to the mason who is to dress the stones (see under No. 56). If he resolves $\frac{2}{13}$ into $\frac{1}{8} + \frac{1}{52} + \frac{1}{104}$ it is not because this fact in itself appeals in any way to his curiosity, but simply because sooner or later he will come across the fraction $\frac{2}{13}$ in a sum, and since he has no machinery for dealing with fractions whose numerators are greater than unity he will then urgently need the resolution above stated.

Perhaps it is in keeping with this attitude that there is in our papyrus practically no instance of the use of a general formula,² each case being worked out on its own merits, and cases which to us seem analogous being sometimes dealt with by totally different methods.

In these facts we may see the cause why Egyptian mathematics stagnated, as they undoubtedly did. By the XIIth Dynasty the mathematician was already able to work out any problem which he was liable to meet in ordinary life. He could measure a field or a granary, and divide wages or booty in fixed proportions, and after all what more was needed?³ For further progress one of two conditions must be realized. Either altered conditions of civilization must present fresh problems for solution, or a genius must arise thirsting for knowledge for its own sake. Egypt could never produce such a genius, and it remained for Greece to do it.

Plutarch says of Thales, comparing him with other "philosophers" whose study was practical politics, that "he carried his speculations beyond things of common utility." Heath⁴ is perhaps not too bold when he illustrates this with a passage from Proclus' Summary:—"Thales discovered many propositions himself, and instructed his successors in the principles underlying many others, his method of attack being in some cases more general, in others more empirical." Here undoubtedly lies the main difference between Greek and Egyptian mathematics. When first the School of Pythagoras, somewhere round about 500 B.C., began to evolve the Theory of Numbers it had already taken a step which put Greek mathematics on a different plane from Egyptian: for the Egyptian there could be no theory of numbers, only a practice. Still less could an Egyptian have appreciated the metaphysical subtleties of Plato's treatment of mathematics in the sixth and seventh books of the Republic.

¹ *Republic*, iv, 436.

² Nos. 61B and 66 are perhaps the only exceptions.

³ The reasons for the stagnation of medicine, however, are quite different. It could hardly be said that the medical knowledge of the XIIth Dynasty was sufficient to meet the normal need. What prevented development was the fact that the science was permeated with magic, from which it never seemed able to break away.

⁴ I, 128.

1. SYSTEM OF NOTATION.

The Egyptian system as we find it at the beginning of the Dynastic Period, and as it continued throughout history, was decimal.¹ A unit was represented by a vertical stroke |, two by two strokes, and so on up to 9. Ten was represented by ∩, 20 by two such signs, and so on up to 90. For 100 a new unit @ appears, and this repeated the requisite number of times served from 200 up to 900. For 1,000 ⚓ was used, for 10,000 ⌋, for 100,000 ⚡, and for 1,000,000 ⚡.³ Thus 143,257 would be written ⚡ ⌋⌋⌋ ⚓⚓⚓ @ @ ∩∩∩ ∩∩∩.

In the cursive ink-written script known as hieratic many of these numbers took on ligatured and contracted forms, the four strokes, for example, being shortened into a horizontal line. Hieratic forms for the numerals already existed as early as the First Dynasty,⁴ and ran through Egyptian history until replaced by the demotic in the Persian period.

Despite the fact that in historical times the system is definitely decimal it contains faint traces of having originally been quinary. The evidence for this is too intricate to be discussed here, but the main points of the latest pronouncement on the subject, that of Jéquier, are as follows.⁵ The numbers from 1 to 5 have names resembling the African (Hamitic) names, and are part of the African inheritance of the Egyptians. The numbers from 6-10 have names offering some analogies with the Semitic names and are a later acquisition. The tens from 10 to 40 have special names which correspond neither to those of the Egyptian 1-5 nor to those of the tens in either Hamitic or Semitic languages. The tens from 50 to 90 are formed from the numbers 5-9, of which they are perhaps plural forms. These results must not be regarded as final, and will doubtless meet with considerable criticism. What would appear almost certain, however, is that there are remnants in the Egyptian system of a primitive quinary system based on finger numbering (the number 5 was represented by the figure of a hand), complicated by a later extension to a decimal system formed by the addition of the second hand. Exactly what portions of this system are due to African and Semitic origins respectively is still a matter of almost complete conjecture.

The defects of this system are obvious. In the first place it was cumbersome, for in order to write such a number as 879 no fewer than 24 signs had to be made. This was to a certain extent neutralized in hieratic, where almost every unit, ten, hundred, and thousand developed a contracted form. The other defect of the system was the absence of anything in the nature of value by position, a disadvantage which it shared with the Greek notation and which was only circumvented by the Arab mathematicians, who are said to have derived positional notation from the Hindus and who passed it on to us. See, however, p. 28.

As against these defects the system had one virtue which the Greek could not claim: it lent itself admirably to multiplication and division by 10, for in order to multiply 98 by 10 it was only necessary to turn the 8 units into ten-signs and the 9 ten-signs into hundred-signs. The result of this was that multiplication and division by 10 played a large role in the elementary processes of Egyptian reckoning.

2. THE SIMPLE ARITHMETICAL PROCESSES.

There is only one truly fundamental process in arithmetic, that of counting. What we are accustomed to call the four simple rules are not fundamental, but are results of counting committed to memory. Thus, when I say 8 and 7 make 15 I am not performing a basic process, I am merely repeating a fact which I know from memory, and the child who says

¹ In the reign of Narmer, 1st Dynasty or just before, we find the notation in full use up to 1,000,000; QUIBELL, *Hieraconpolis* I, Pl. XXVI. B.

² See, however, Gunn in *J.E.A.*, III, 280.

³ For the ring-sign in later times see *ibidem*.

⁴ PETRIE, *Royal Tombs of the First Dynasty*, I, Pl. XIX, 11.

⁵ *Recueil d'Études Égyptologiques Champollion*, 1922, 467 ff. Cf. SETHE, *V.Z.Z.*, 24-26.

8 and 7 are 14 is not making an error of calculation, but merely one of memory. If I wish to prove that 8 and 7 really do make 15 I must count out 8 objects, then 7 more. I must then count both lots together and I shall get 15. Just as addition—and therefore also subtraction—is a pure act of memory, except perhaps in simple cases such as 1 and 1 is 2, where we may almost be said to count, so, too, multiplication and division are mere acts of memory. When we say 9 multiplied by 6 is 54 we do not count, we merely repeat a fact learnt by heart. To prove that it is so we must make 9 rows of 6 objects each and count right through from 1 to 54.

The result is that the ability of a nation or an individual to make rapid arithmetical calculations depends in a great measure on memory equipment. If I have all the multiplication tables from 2 times to 19 times in my head I shall in general perform arithmetical calculations with more speed and comfort than one whose equipment does not reach beyond 12 times 12.

How did the Egyptian stand in this respect? At the outset he possessed one slight advantage over us in the matter of addition, for the very nature of his hieroglyphic notation enabled him, if he so wished, to dispense with most of the memory work so familiar to us. To write down 8 he had to make 8 strokes, and to write down 7 he had to make 7 strokes. The consequence was that when he had to add 8 and 7 the 15 strokes were all actually there before his eyes, and all he had to do was to count them. Similarly 80 and 70 could be added by mere counting, and 800 and 700 and so on. In the same way subtraction was a mere matter of counting. This must have been extremely pleasant for the Egyptian schoolboy, but it must have reacted disastrously on the development of the arithmetician, for it is clear that where there is little incentive to memorize little memorizing will be done. At the same time it is certain that the power of adding numbers by memory was developed among the professional mathematicians and accountants, otherwise they would never have evolved the hieratic numerals, in which the separate strokes etc. are no longer to be discerned. We may therefore credit the Egyptian with a certain facility for addition of simple numbers by memory.

The technical phrases for addition are mainly derived from the common use of the preposition *hr* in the sense of "in addition to," doubtless a very early derivative from the literal meaning of "on." Thus in No. 26 we read, (A number) $\frac{1}{4}f$ *hr.f*, "whose fourth part is added to it": here we have a simple nominal sentence. More often, however, a verb is added, either *wsh* or *dit*, both meaning "to put" or "place." In No. 72 we find *wsh-hr-k* 100 *hr-s*, "You are to add 100 to it"; and in No. 22, $\frac{1}{5} + \frac{1}{10}$ *m wsh hr.f*, " $\frac{1}{5} + \frac{1}{10}$ is what is added to it," where *wsh* is presumably a Neuter Passive Participle.¹ The use of *wsh* in this connection is doubtless very primitive, the meaning being almost literal, "to place upon."² In Nos. 41 and 42 *dit* is used in place of *wsh*.

The verb *dmd*, "to unite," can also be used of adding: in No. 52 we have *dmd-hr-k A hr B*, "You are to unite A and B." The invariable noun for "total" is, as in the account papyri, *dmd*.

The usual verb for to subtract is *hbi*,³ determined by the cross sticks which Grapow⁴ has shown to characterize verbs whose primitive meaning is akin to "breaking." In this papyrus it is used either absolutely, e.g. No. 41, *hb-hr-k* $\frac{1}{9}$ *n* 9, "You are to subtract one-ninth of 9" (sc. from 9), or followed by the preposition *hnt*,⁵ No. 43, *hb-hr-k* 1 *hnt* 9, "You are to subtract 1 from 9." The word used for "remainder" is the familiar *d:t* or *wd:t* of the account papyri.⁶

In Nos. 21-23 a process amounting to simple subtraction is indicated by the verb *skm*,

¹ Or an abstract noun. Cf. Pap. Bulaq 18; *Ä.Z.*, 57, 55.

² Or is it short for *wsh tp* "count," for which see below.

³ For a different use of *hbi* see Nos. 54 and 55.

⁴ *Ä.Z.*, 49, 116 ff.

⁵ *hbt hft* $\frac{1}{10}$ in No. 82 means "to subtract at the rate of $\frac{1}{10}$," not "from $\frac{1}{10}$."

⁶ For a discussion of this word see SPIEGELBERG, *Rechnungen aus der Zeit Setis I*, Text, 40-41 and 49.

"to complete."¹ That this is not specifically a word for subtraction is clear from the examples Nos. 7-20, where it is used of making one quantity up to another by the addition of aliquot parts of itself.²

When we pass on to multiplication and its complement, division, the nakedness of the land is indeed apparent. The Egyptian never multiplied directly, *i.e.* by sheer act of memory, by any numbers except 2 and 10. The latter indeed did not even involve an act of memory, for it is clear that in a decimal system with a notation of the Egyptian type all that has to be done to multiply by ten is to turn unit-signs into ten-signs, ten-signs into hundred-signs, and so on. Thus 45, $\text{nnnn} \overset{\text{lll}}{\text{ll}}$, multiplied by 10 is $\text{e} \text{e} \text{e} \text{e} \overset{\text{nnn}}{\text{nn}}$, *i.e.* 450. By reversing this process division by 10 could clearly be accomplished in a manner which was purely mechanical, and which, it will easily be seen, was hardly made the less so by the development of the hieratic numerals referred to above.

Multiplication by 2, however, was a true memory process with the Egyptian. He not only had tables giving him its results, but he almost certainly knew them by heart. Further than this, however, he did not go. He never multiplied by any higher number, except of course 10. The result was that in order to perform multiplication by other numbers he had to make what shift he could with 2 and 10. Thus 12 times a number was obtained by adding 2 times to 10 times. To multiply 15 by 13 the following process was gone through:—

$$\begin{array}{rcl} \text{—} 1 \times 15 & = & 15 \\ 2 \times 15 & = & 30 \\ \text{—} 4 \times 15 & = & 60 \\ \text{—} 8 \times 15 & = & 120 \\ \text{Total } 13 \times 15 & = & 195 \end{array}$$

Here the Egyptian merely kept on doubling; he noticed that the multipliers 1, 4 and 8 added up to 13, and that therefore the products corresponding to these must amount to 13 times 15. To simplify his work he ticked off the multipliers in question and then ran down the right column adding together the products opposite the ticks. It will be seen that in this way, using no multiplier other than 2, any required multiplier could be arrived at. The process could in many cases be shortened by the use of the multiplier 10, which by doubling would give 20 and so on, but in general it would seem that the mathematician of Rhind preferred to work solely in powers of 2.

Division was accomplished by reversing this process. Thus, to divide 77 by 7 we do as follows:—

$$\begin{array}{rcl} \text{—} 1 \times 7 & = & 7 \\ \text{—} 2 \times 7 & = & 14 \\ 4 \times 7 & = & 28 \\ \text{—} 8 \times 7 & = & 56 \\ \text{Total } 11 \times 7 & = & 77 \end{array}$$

We note that the three products 7, 14 and 56 add up precisely to the required 77. We therefore tick off the lines containing those products and add up the corresponding multipliers, 1, 2 and 8, giving the answer 11.

This process of multiplication or division was known to the Egyptians as *wꜥh tp*. In Old and Middle³ Egyptian *wꜥh tp* is "to incline the head," and there can be little doubt that

¹ For a somewhat similar but perhaps intransitive use of *km* see No. 37 and notes thereto.

² In No. 28 addition and subtraction are indicated by the sign of the human legs facing to the right and the left respectively. For the reading of these signs see the notes there.

³ *E.g.* Pap. Bulaq 18 (MARIETTE), Pl. XXVII, 2, 20. Cf. *Ä.Z.*, 57, 61.

"nodding the head" was a primitive operation in the process of counting, and so perhaps became the phrase for "to count," the head being nodded, not necessarily at every digit, but possibly at every five or ten counted off on the fingers. At any rate, *w:h tp* (often *irt¹ w:h-tp*) *m 4 r² spw 5* means literally "count (make a counting) with 4 5 times," or "multiply 4 by 5." Similarly "divide 77 by 7" was rendered *w:h tp m 7 r gmt 77*, "count with 7 to find 77." This general sense "to count with" is well illustrated by such examples as No. 43, where *w:h tp m 8* "count with 8," or even more generally "operate on 8," is followed by "you are to add to it one third of it; it becomes $10\frac{2}{3}$." Here the specific translation "multiply" or "divide" is impossible.

Division, as we have just seen, is usually expressed by *w:h tp*. There is, however, another technical term for the process, namely *nš A hnt³ B*, "divide A by B."⁴ This can be used whether A is greater or less than B, e.g. No. 66, *nš 3200 hnt 365*, "divide 3200 by 365: result $8\frac{2}{3} + \frac{1}{10} + \frac{1}{2190}$," and No. 35, *nš 1 hnt 3 $\frac{1}{3}$* . The verb *nš* means to "call" or "summon," and the preposition *hnt* must here have its usual sense of "out of" or "from among." Possibly the original picture is that of one counting up to A and calling out at every Bth number, or possibly the term is purely a metaphor. If the former be the correct explanation the use in cases where B is greater than A must be a later extension. It is worthy of notice that this term is used in the statement of the so-called resolutions of fractions whose numerator is 2. Where we say "resolve $\frac{2}{x}$ " the Egyptian merely said *nš 2 hnt X*, "divide 2 by X" (see below).

Two other cases of *nš* must be noted here. In No. 44 we read *tp n nš šš ifd*, "example of reckoning a cubical container," i.e. of finding its content in corn, and in No. 56 we have *tp n nš mr*, "example of working out a pyramid," the problem being to find the slope of its sides, given its base and its vertical height. It is difficult to see how these more complex uses of the term could be derived from its technical sense of divide, and they are more probably metaphorical uses of its literal meaning of "call" or "summon."

The papyrus contains a few rare examples of multiplication or division direct by numbers greater than 2. Thus in No. 37 we find one third of 30 given as 10, with no working or explanation, and also one third of 90 as 30. The first of these is merely the reverse of a division by 10, and as for the second it will be seen below that division by 3 was well within the powers of the reckoner, though it usually necessitated two steps. Facts of the type $\frac{1}{x} \times x = 1$ also form a frequent exception to the general rule, apparent only, since they do not in reality involve division at all.

The result of the process of multiplication or division, and indeed the result of any mathematical process, is expressed by the verb *hpr* "to become," generally followed by the preposition *m*. The most usual method is to use the *šdm:f* or *šdm-hr:f* form of the verb with the Neuter (Masculine in form) 3rd Singular suffix pronoun. Thus, "Multiply 4 by 5," *hpr:f m 20*, or *hpr-hr:f m 20*, "it becomes 20." Sometimes the resulting figure is used as subject: *hpr-hr 4*, "4 results" (No. 5). In Berlin Pap. 6619 we once find *hpr-hr m 1 $\frac{1}{4}$* without apparent subject, but this may be an error, though such an omission of subject is not at all unusual in Egyptian.⁵

A more elaborate phrase for expressing result is *hprt im pw 1200* (Nos. 76 and 78; cf. Pap. Kah., Pl. VIII, *passim*), a regular nominal clause in which *hprt* is the Neuter Active Participle, "1200 is what results therefrom."

¹ In No. 30 *irt* alone without *w:h tp* is used, in the form *ir-hr-k*, of division. That this is not an accident is clear from a second use in the passive in the same sum, and from Pap. Kah., Pl. VIII, 27, 37 and 39.

² *r* is often omitted.

³ In No. 57 we have *nš hft*, perhaps merely in error for *hnt*.

⁴ The Egyptian for "take a quarter of it" is *irt $\frac{1}{4}$ f*, literally "make its quarter." Cf. Nos. 26 and 44.

⁵ Similarly, perhaps, in No. 62, though the following Relative Form *did-k* may here have been regarded as grammatical subject to *hpr* despite the intervention of *m 4*. See, however, p. 105.

3. FRACTIONS.

The Egyptian fractional notation was a very simple one. With the sole exception of $\frac{2}{3}$ no fraction was ever written which had a numerator greater than unity,¹ or in other words, with this same exception, all Egyptian fractions on paper were aliquot parts, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, etc. If in the course of a problem, owing to a multiplication by 2 (the only digit except 10 by which the Egyptians multiplied directly) a fraction arose or threatened to arise whose numerator, as we should say, was 2, it was immediately resolved into the sum of two or more whose numerators were unity. In other words, the Egyptian never wrote the fraction $\frac{2}{11}$; if he multiplied $\frac{1}{11}$ by 2 the result was $\frac{1}{6} + \frac{1}{66}$.

How far this was a consequence and how far a cause of the very restricted notation of fractions it would be difficult to say. To write one-thirteenth the Egyptian simply wrote the numeral 13 underneath the sign $\ominus$ (reduced in hieratic to a dot). This sign, as Sethe has shown,² must here mean "a part." The only fractions not expressed in this way were $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{2}{3}$, for which special hieratic signs existed. Just as doubling formed the basis of all multiplication, so halving lay at the root of most division. Two of the most important measures in Egyptian daily life, the acre ($\dot{s}t;t$) and the bushel ($hk;t$) were divided up into halves, quarters, eighths, etc. These divisions may go back to a stage in reckoning even more primitive than that of simple aliquot parts, a stage when only the $\frac{1}{2}$ and its powers $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$ and $\frac{1}{32}$ were used.

In this connection the parts of the acre are of special interest, for $\frac{1}{2}$ -acre is written with the sign $\curvearrowright$, which reads *rmn* "arm," sometimes "side," and which may be an earlier word for $\frac{1}{2}$ in the general sense than the better known $\longleftarrow$, *gs*. Still more important is the sign for $\frac{1}{4}$ -acre, which is a cross, $\times$, or to be more exact a pair of sticks crossed. We know that in late times this $\frac{1}{4}$ -acre was called *hsp*, and Sethe³ points out that this is the same as the earlier *h'sb*, a word meaning "to break." Combining this fact with the pictogram of the crossed sticks, he argues that these last are in reality two fragments of one broken stick, and that this old word for $\frac{1}{4}$ was the fraction or "breaking" par excellence, a conjecture which is borne out by the fact that *h'sb* was in historical times the word for "to count" or "reckon." In hieratic the crossed sticks remained throughout the sign for $\frac{1}{4}$, but in hieroglyphic they were replaced at an early stage by the normal $\overline{\text{||||}}$ except in the special senses of $\frac{1}{4}$ -acre and $\frac{1}{4}$ -bushel.

Side by side with these early dimidiated fractions there must have existed in quite early times another set based on division into three, since in this way we can best explain the unique position of $\frac{2}{3}$ among Egyptian fractions and the existence at all periods of a special hieratic sign both for this and for $\frac{1}{3}$. Originally $\frac{2}{3}$ was written in hieroglyphic thus $\overline{\text{|||}}$, with the old word *r* meaning "a part" and two equal strokes attached to it under its left-hand end. Later the two strokes worked their way to the centre and one became longer than the other, so that the sign came to be written $\overline{\text{|||}}$ or $\overline{\text{|||}}$. The original form of the sign leaves little doubt that the group was originally called by the Egyptians, as by most nations, "two-parts." That it was a very primitive and fundamental conception in the Egyptian mind is clear from the fact that in mathematics one-third of a number or quantity was invariably found by first obtaining two-thirds and then halving it. The curious writing of $\frac{1}{3}$ in hieratic, which cannot possibly be brought into relation with the normal hieroglyphic $\overline{\text{|||}}$, is undoubtedly to be traced back to a primitive system of division into three, indeed Möller and Sethe have suggested that the earliest form of the hieratic sign may well be derived from the word "part," $\ominus$, with a single vertical stroke attached to it under its left-hand end,⁴ meaning "one part," division into three being assumed.

The precise nature of the Egyptian fractional system and its methods of working has

¹ There are very rare examples of $\frac{3}{4}$, written as "three parts" analogously to $\frac{2}{3}$, "two parts" (see below).

² V.Z.Z., 85-87.

³ V.Z.Z., 75-78.

⁴ V.Z.Z., 82.

been analysed by Hultsch in a long study of considerable complexity, entitled *Die Elemente der ägyptischen Theilungsrechnung*.¹ He rightly begins by pointing out that the result of any divisional process in Egyptian must be arranged in whole numbers followed by a series of aliquot parts in order of magnitude. Thus the result of dividing 2 by 13 was written $\frac{1}{8} + \frac{1}{52} + \frac{1}{104}$. This is in fact a question of notation. Just as there is a notation for whole numbers in tens, hundreds, etc., so there is a fixed notation for quantities less than unity, namely the series $\frac{2}{3}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, etc.; after all this is not so unlike the modern decimal notation, where the quantities lower than unity are expressed in terms of $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, etc.

Hultsch now goes on to say (p. 9), "Was nach ägyptischer Anschauung Vielheitstheilungen oder noch nicht zu Ende geführte Divisionen waren, das sind für uns Brüche mit Zählern, die grösser als 1 sind." "What the Egyptian looked upon as divisions of numbers greater than unity or unaccomplished divisional processes are to us fractions with numerators greater than 1." If this means that the Egyptian had no *conception* of a fraction whose numerator was greater than unity, and that he would have regarded our fraction $\frac{5}{8}$ only as 5 divided by 8, and never as 5 eighth-parts of unity, it is too sweeping a statement. It is true that he had no notation for such quantities, but the argument from what he was capable of expressing in symbols to what he was capable of conceiving is a *non sequitur*, and the suggestion that his notation must surely have kept pace with his conception will fall on deaf ears in the case of those acquainted with the amazing conservatism of the Egyptian mind in every branch of life.

What is more, there are at least certain cases in which it is obvious that a fraction with numerator greater than 1 was conceived and that very clearly. Thus Sethe's researches have shown from the earliest writings of the symbol for $\frac{2}{3}$ that this fraction was originally written "the 2 parts," i.e. the 2 parts of a unit conceived as divided into three, just as we now speak of "three parts," meaning $\frac{3}{4}$. Similarly the Egyptian reckoner, when doubling fractional quantities, is wont without any discussion to replace twice $\frac{1}{14}$ by $\frac{1}{7}$: in these cases, where the unit in his mind is indisputably 1, it seems idle to deny that he reasoned through $\frac{2}{14}$ (though he could not write it) to $\frac{1}{7}$, or to assert that his mind-process was "the unaccomplished division 2 by 14 is the same as 1 by 7." The same is true of his method of adding $\frac{1}{14}$ and $\frac{1}{14}$. His mental picture was not "1 divided by 14 added to 1 divided by 14 amounts to 2 divided by 14, which is the same thing as 1 divided by 7," but simply "If a unit is divided into 14 parts and 2 of them are taken the result is one-seventh part of the unit"; this is clear from his conception of aliquot parts and from his ability to double them. If we are to suppose that $\frac{2}{3}$ stood merely for the division of 2 units by 3 and not for 2 third-parts of a unit, how are we to explain the Egyptian's ability to double $\frac{2}{3}$ and obtain $1\frac{1}{3}$, a direct process which occurs over and over again in the papyrus? If Hultsch's theory were correct the result of doubling 2 divided by 3 could only be 4 divided by 3, whereas what the Egyptian actually gets is a unit plus its third part.

It would thus appear that the Egyptian had a perfectly clear conception of fractions of the type $\frac{2}{n}$ in the sense of two *n*th-parts of a unit, and not merely in the sense of 2 divided by *n*. There are not wanting suggestions which take us even farther. In No. 18 the fractions $\frac{1}{6} + \frac{1}{9} + \frac{1}{18}$ are to be added, and the answer is set down without any working as $\frac{1}{3}$. How was this done, and why is the method of common denominator² or *bloc extractif* not shown here as in the companion examples? Probably because $\frac{1}{6} + \frac{1}{18}$ was, from the table at the opening of the papyrus, seen to be equivalent to two-ninths and the addition of the other $\frac{1}{9}$ gave three-ninths or $\frac{1}{3}$. The same process is employed in No. 17.

Indications of this kind are not to be neglected,³ and we cannot therefore follow Hultsch in his sweeping assertion. The *conception* of fractions with numerators greater than unity

¹ *Abhandl. der Kgl. Sächs. Gesellsch. der Wiss., phil.-hist. Classe*, Band XVII. Leipzig, 1895.

² See below, p. 17-18.

³ Surely the process $\frac{1}{6} \times \frac{1}{4} = \frac{1}{24}$, common in the papyrus, involves the conception of $\frac{1}{6}$ as four twenty-fourth parts of the unit.

seems to be inherent in some of the processes of Egyptian mathematics, but the *notation* did not keep pace with it.

In order to appreciate the powers and limitations of the Egyptian mathematician in dealing with fractions it is essential to bear this notation in mind throughout. What enabled him to keep such a simple apparatus undeveloped throughout ages was the fact that he never learned to multiply directly by any other number than 2. The result was that in his processes he was rarely likely to be threatened with worse fractions than doubled aliquot parts, e.g. "twice the thirteenth part." He had discovered that all such quantities could be resolved into the sum of two or more aliquot parts, and had actually worked out tables for such resolutions, running from twice a fifth-part to twice a 101st-part. This simple apparatus, a copy of which begins our Rhind Papyrus, saved him the trouble of evolving a more complicated fractional notation.

It is apparent from the Egyptian notation and general conception of fractions that their addition and subtraction could only play a limited part in mathematics. Since none but aliquot parts were used an answer in the form $1\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$ was in no way repugnant to the Egyptian mind, and even if it had been the notation offered no remedy. There were, however, occasions when additions had to be made, generally in sums where some fractional quantity had been multiplied by some other quantity, whole or fractional, and it was required to show that the result was equal to some whole number or to some quantity involving only very simple fractions. Thus in the proof of No. 32 we have to show that $(1\frac{1}{6} + \frac{1}{12} + \frac{1}{14} + \frac{1}{28}) \times (1\frac{1}{3} + \frac{1}{4}) = 2$. The working is as follows:—

1	$1\frac{1}{6}$	$+ \frac{1}{12} + \frac{1}{14} + \frac{1}{28}$
$\frac{1}{3}$	$\frac{1}{3}$	$+ \frac{1}{18} + \frac{1}{36} + \frac{1}{42} + \frac{1}{84}$
$\frac{1}{4}$	$\frac{1}{4}$	$+ \frac{1}{24} + \frac{1}{48} + \frac{1}{56} + \frac{1}{112}$
Total	$1\frac{1}{2} + \frac{1}{4}$	
Remainder	$\frac{1}{4}$	

$\frac{1}{12} + \frac{1}{14} + \frac{1}{28} + \frac{1}{18} + \frac{1}{36} + \frac{1}{42} + \frac{1}{84} + \frac{1}{24} + \frac{1}{48} + \frac{1}{56} + \frac{1}{112}$
76 8 4 50 $\frac{2}{3}$ 25 $\frac{1}{3}$ 2 $\frac{2}{3}$ 1 $\frac{1}{3}$ 38 19 2 1
Total 228 i.e. $\frac{1}{4}$

In the first line the multiplicand is set out with the multiplier 1 before it. In the second line it is multiplied by $\frac{1}{3}$, and in the third by $\frac{1}{4}$. These three products have now to be added. The simpler quantities $1\frac{1}{6} + \frac{1}{3} + \frac{1}{4}$ clearly give $1\frac{1}{2} + \frac{1}{4}$, since $\frac{1}{6} + \frac{1}{3}$ is $\frac{1}{2}$, a combination well known to the Egyptian and frequently used. We have now only to show that the sum of the more complicated fractions to the right of the vertical line amounts to the remaining $\frac{1}{4}$. Here the Egyptian employs a method which at first sight appears to be that of a common denominator. All the fractions or aliquot parts seem to be reduced to terms of the highest aliquot part,¹ namely the 912th part: under each fraction is placed in red (here represented by italics) the number of 912ths contained in that fraction, a number which, it will be observed, is not in all cases a whole number. This step must involve a certain amount of rough working, which is always omitted in the papyrus. The red figures are now added and seen to come to 228, which is $\frac{1}{4}$ of 912. Therefore the sum of all these fractions is the required $\frac{1}{4}$, and adding on the already obtained $1\frac{1}{2} + \frac{1}{4}$ we get 2 for the product of the two original quantities.

This method obviously differs in small details from the modern method of common denominator. For instance, we always choose as our denominator the smallest number into which all the separate denominators will divide integrally. The Egyptian, unfamiliar with the principle of factors, often used a denominator which was smaller than the L.C.M., and

¹ In No. 33 the number chosen is not the highest aliquot part.

consequently had fractional quotients (rarely involving smaller fractions than $\frac{1}{8}$). It is hardly necessary to mention the further difference that the numerators of the fractions to be added were all unity.

These, however, are distinctions of mere detail, and despite them the general principle involved might be the same. This is denied by both Hultsch¹ and Rodet. The former would describe the process given above as follows.² The unit employed was originally 1, and this unit (*Stammeinheit*) was kept so long as addition in terms of it proved feasible, i.e. up to the point where the simpler fractions had been added up to $1\frac{1}{2} + \frac{1}{4}$. Then, according to Hultsch, to facilitate the addition of the complicated fractions to the right of the vertical line a new unit is chosen (*Hülfeinheit*), namely $\frac{1}{912} = 1$. The aliquot parts to be added are now multiplied each by 912, that is to say, they "are transformed into multiples or parts of the *Hülfeinheit*." The results are added and come to 228, which being $\frac{1}{4}$ of 912 enables us to "return to the *Stammeinheit*," which in this case is unity.

But here again the question is surely one merely of notation. What is done is in essence precisely what happens in our own method of addition by common denominator. We may clothe this in whatever words or signs we like; the facts remain the same. We may say that the Egyptian uses a new unit, namely $\frac{1}{912}$, but what else do we do when we choose a common denominator 912? We have only to look at Hultsch's attempt to word the problem in order to see this. For instance, on p. 112 he describes the following addition:—

$$\begin{array}{cccccccc} \frac{1}{32} & \frac{1}{16} & \frac{1}{12} & \frac{1}{6} & \frac{1}{6} & \frac{1}{288} & \frac{1}{6} & \frac{1}{288} \\ 9 & 18 & 24 & 3 & 8 & 1 & 8 & 1 \end{array}$$

His concluding words are "Hierauf folgt im Texte die Summe mit den Worten 'zusammen $\frac{1}{4}$ 72.' Das soll bedeuten 'zusammen 72 Hülfeinheiten (deren jede = $\frac{1}{288}$ ist), das ist $\frac{1}{4}$ der *Stammeinheit*.'" It is clear that if the numbers which are added to give 72 are all Hülfeinheiten, i.e. 288th parts of unity, then the process is precisely the modern method of common denominator, and the Egyptian is simply replacing $\frac{1}{32}$ by nine-288ths, a conception which, incidentally, Hultsch himself has denied to him (above, p. 16).

Rodet³ adopts an entirely different view. He says, "il est bien certain qu'Aahmesu ne réduisait pas ses fractions à un dénominateur commun, mais que, comme on l'a fait après lui pendant vingt-six et trente siècles encore, il choisissait un nombre, bloc extractif, fonds commun ou comme on voudra l'appeler, d'où il puisse tirer toutes ces fractions, soit, comme ses successeurs, à l'état d'entiers, soit, comme il s'en contentait, à l'état d'à peu près entiers, mais, dans ce cas, avec une fraction d'expression simple; et c'est sur les substitués ainsi obtenus pour ses fractions qu'il opérait." Rodet supports this view by references to eastern mathematicians of the Middle Ages and later, by whom the method which he describes appears to have been practised. The number, called by Rodet *bloc extractif*, seems to have been called *môré* by Aben-ezra (12th century A.D.) and *mokhrag* by Maḥmūd of Herat. The *mokhrag* of a group of fractions is an integer which by multiplication will turn each of the fractions into an integer. It is in fact a common denominator viewed from a slightly different angle. Thus if we wish to add $\frac{1}{4}$ and $\frac{1}{5}$ we reduce them to the common denominator 20: we say that $\frac{1}{4}$ is 5-twentieths and that $\frac{1}{5}$ is 4-twentieths, total 9-twentieths. The method of *mokhrag* is according to Rodet different. Here, unable to add $\frac{1}{4}$ and $\frac{1}{5}$ in terms of the unit 1, the reckoner takes the *mokhrag* 20 as a unit, and adds $\frac{1}{4}$ and $\frac{1}{5}$ of that, result 9; answer 9-twentieths. But here again the difference is purely one of notation. The Arab mathematician avoids actually saying that $\frac{1}{4}$ is equivalent to 5-twentieths, but he is bound to admit it tacitly, for in the end his *mokhrag* must become a denominator and he confesses it when he uses the word "twentieths."

¹ *Op. cit.*, 9-10.

² *Op. cit.*, 112.

³ *Journal Asiatique*, 1881, 196-215.

The fact is that both Hultsch and Rodet have been deceived by notation. There is and can be only one way of adding fractions, though there may be several ways of writing down the process. The fractions $\frac{1}{4}$ and $\frac{1}{5}$ are quite irreconcilable as they stand, and we can only combine them by reducing them to some smaller part of unity of which they are both multiples. We may do this in the modern way by means of the common denominator 20, or we may do it in the Arab way by means of the *mokhrag* 20. Avoid the notation $\frac{5}{20}$ as we may, we cannot in the end escape the fact that the 20 really stands for the twentieth part of some unit, and that the 5 is 5-twentieths of that unit. The process as seen in the Rhind papyrus is particularly deceptive since all the complicated additions there used are in the nature of proofs, i.e. the result is known to be some very simple aliquot part, e.g. $\frac{1}{4}$ or $\frac{1}{8}$, and though our denominator or *mokhrag* may be 960, the fact that we are really working in 960ths is apt to be overlooked or forgotten when the addition comes to 240 or 120, and the 960 drops out of sight, leaving only a simple $\frac{1}{4}$ or $\frac{1}{8}$.

The ease and accuracy with which the Egyptian dealt with these very complicated-looking fractions often compel our admiration. At the same time, in matters of everyday occurrence they were probably not very frequent. As will be seen below, the measures of capacity and of area were largely based on the principle of halving, with the result that the fractions involved were very easily added. Thus $\frac{1}{32} + \frac{1}{32}$ at once gives $\frac{1}{16}$, which can be taken up with another $\frac{1}{16}$ giving $\frac{1}{8}$, which will combine with another $\frac{1}{8}$ to give $\frac{1}{4}$, and so on.

The papyrus contains no definite example of the subtraction of fractions except such simple cases as 2 less $1\frac{1}{2} + \frac{1}{4} = \frac{1}{4}$. See, however, p. 58.

As in the case of whole numbers, so in the case of fractions the only multipliers in general use were 10 and 2, though, as will shortly be seen, the anomalous $\frac{2}{3}$ opened up possibilities of division by 3 which were not neglected. Since all the fractions were merely aliquot parts, it is obvious that a fraction could at once be divided by 2 or by 10 by simply multiplying the denominator, as we should call it, by 2 or 10.¹ Thus the Egyptian had no difficulty in seeing that $\frac{1}{5} \times \frac{1}{10}$ was $\frac{1}{50}$, or, in other words, that if you divided a tenth aliquot part of a whole into 5 the result would be 50th aliquot parts. Multiplication, however, was a much more difficult matter, for there was literally no "numerator" to multiply, owing to the primitive notation adopted. The Egyptian could not write or deal with such quantities as $\frac{3}{20}$ or $\frac{19}{47}$, and fractional reckoning was only possible because he had discovered the possibility of expressing twice any aliquot part of a quantity, say two-sevenths, as the sum of two or more different aliquot parts of the same quantity, namely one-fourth and one twenty-eighth. In modern terms he had found that every fraction with numerator 2 could be resolved into the sum of two or more fractions whose numerator was 1. He had even drawn up a table of such resolutions from $\frac{2}{3}$, $\frac{2}{5}$, $\frac{2}{7}$ up to $\frac{2}{99}$ and $\frac{2}{101}$, the fractions with even denominators being of course omitted, since he saw that they reduced at once to aliquot parts ($\frac{2}{62} = \frac{1}{31}$). Since he never multiplied by a number greater than 2,² it is clear that this table sufficed for any fractional operations arising in the course of multiplication.

Thus to multiply ($1\frac{1}{2} + \frac{1}{7}$) by $2\frac{1}{4}$ we proceed as follows:—

$$\begin{array}{rcl} & 1 & 1\frac{1}{2} + \frac{1}{7} \\ \text{—} 2 & & 3\frac{1}{4} + \frac{1}{28} \text{ (}\frac{2}{7} \text{ being } \frac{1}{4} + \frac{1}{28}\text{)} \\ \text{—} \frac{1}{4} & & \frac{1}{4} + \frac{1}{8} + \frac{1}{28} \end{array}$$

The second line is got by doubling the first and resolving the twice $\frac{1}{7}$ by table into $\frac{1}{4} + \frac{1}{28}$. The third line is $\frac{1}{4}$ of the first. We now tick off the required multipliers and add the corresponding products. The two $\frac{1}{4}$ s give $\frac{1}{2}$ and the two $\frac{1}{28}$ s give $\frac{1}{14}$. Result $3\frac{1}{2} + \frac{1}{8} + \frac{1}{14}$.

¹ A fifth was obtained by doubling a tenth.

² Fractions are practically never multiplied by 10 except when their denominator is a multiple of 10. For an exception see No. 4, where the simple $\frac{1}{2} + \frac{1}{3}$ is multiplied by 10 and gives 7. In No. 56 $\frac{1}{10}$ of ($1\frac{1}{3} + 1\frac{1}{5}$) is stated, without proof, to be $\frac{1}{10} + \frac{1}{25}$.

Division, as in the case of whole numbers, is simply the reversal of this process, trial multipliers being taken until in the products-column products can be seen which add up to the required dividend.

It is clear that by the introduction of fractions the bounds of Egyptian multiplication and division have been greatly widened. They were still further widened by the ingenious use made of the anomalous fraction two-thirds, "the two parts." Strange as it may seem to us, the Egyptian was accustomed to take two-thirds of a number by a single process. No doubt he used tables for the purpose, but the mere fact that tables existed is one more testimony to the fundamental nature of the concept of "the two parts" in the Egyptian mind. Most of us when asked to give two-thirds of 5 would do it by taking one third and doubling it. So far was the Egyptian from doing this that his sole means of finding one-third of a quantity was to take two-thirds and then halve it.¹ In the case of fractions he made use of the equation $\frac{2}{3} = \frac{1}{2} + \frac{1}{6}$, see No. 61B, so that all he had to do was to multiply the "denominator" of the fraction by 2 and then by 6, thus:—

$$\frac{2}{3} \text{ of } \frac{1}{5} = \frac{1}{10} + \frac{1}{30}$$

This ability to take two-thirds opened up a new series of divisions, for halving it gave one-third, which in its turn could be halved to give one sixth, one twelfth, and so on; in other words, it facilitated division by 3, 6, 12, 24, etc.: No. 32 is a good example of this.

As a general rule the operations performed in the papyrus are accomplished before our eyes by the methods detailed above. There are a few exceptions, however. Thus in No. 37 $\frac{1}{3}$ of $\frac{1}{3}$ is stated to be $\frac{1}{9}$, and $\frac{1}{3}$ of $\frac{1}{4} + \frac{1}{32}$ is given as $\frac{1}{16} + \frac{1}{32}$, the necessary two-thirds line being omitted. Other cases would seem to involve a conception which comes near that of the improper fraction. Thus $\frac{1}{10}$ of $3\frac{1}{3}$ is stated to be $\frac{1}{3}$ in No. 35, and in No. 38 $\frac{1}{2}$ of $3\frac{1}{7}$ is said to be $\frac{1}{7}$, but the daring nature of this statement is felt to demand an apologetic explanation, which is solemnly given.

4. OTHER MATHEMATICAL PROCESSES KNOWN TO THE EGYPTIANS.

a. Square and square root.

That the conception of squaring was familiar is known to us from the problem of the truncated pyramid from the Moscow Papyrus.² Here the phrase used for "square 4" is *ir-kr-k 4 pn m Δ*,³ "You are to make this 4 in square." No conjecture can be hazarded as to the reading of the sign here used for "square": from the point of view of mathematical clarity it is unfortunate that the same sign should be used in Rhind No. 28 for addition.

No example of square root occurs in Rhind, but Pap. Berlin 6619, Pap. Kahun Pl. VIII, l. 40, and Pap. Moscow (unpublished) show that the idea of square root existed and that the technical term for it was *knbt*, literally "corner" or "angle," the idea presumably being that the original number, say 16, represented the area of a square, while the length of each of the two sides containing any corner of it was its square root, 4. The Egyptians were even capable of taking the root of quantities involving simple fractions, the quantities whose root is taken in Berlin 6619 being $6\frac{1}{4}$ and $1\frac{1}{2} + \frac{1}{16}$.

b. Solution of equations.

It will be seen below that the problems Nos. 24–38 involve the solution of equations of the first degree with one unknown by means of a simple method of trial.

Equations of the second degree where there is virtually only one unknown were also understood. In the Berlin Papyrus 6619 (see above, p. 6–7) we have to divide 100 square

¹ There is an apparent exception in No. 42, where $\frac{1}{18}$ th of a quantity is got direct from $\frac{1}{6}$ th of it instead of through $\frac{1}{3}$ th (i.e. $\frac{2}{3}$ of $\frac{1}{6}$ th).

² *Ancient Egypt*, 1917, 100–102.

³ Here facing as in the hieratic.

cubits into two squares whose sides are to one another in the ratio 1 to $\frac{3}{4}$. The difficult problem *g* in the Kahun Papyrus (see above, p. 6) involves a similar equation.¹

c. Progression.

No. 64 contains the solution of an arithmetical progression, given the sum and the common difference. In No. 40 we are given the number of terms (5), the sum, and the fact that the sum of the two lowest terms is one-seventh of the sum of the three highest, and we are asked to find the common difference. The word here used for common difference is *twmw*, and at first sight it would appear to be a technical term used of arithmetical progression. This, however, can hardly be the case, for it is used in a somewhat different sense in No. 39, where there is no progression, and we must therefore suppose that the use in No. 40 is a specialized employment of a more general term.

No. 79 contains a geometric progression whose first term is unity. The solution (*q.v.*) shows that the Egyptian had a very clear grasp of the nature of a series of this particular type, which he sums neatly and accurately, but does not tell us whether he was equally at home with a series in which the first term was not unity.

5. GEOMETRY.

The areas of the square and rectangle were correctly estimated; that of the circle was found by squaring $\frac{8}{9}$ of its diameter, a very fair approximation. For the triangle see pp. 51 ff. The volumes of cube and rectangular parallelopiped were known, while the Moscow Papyrus gives the correct solution for a frustrum of a regular pyramid on a square base, p. 93. The volume of a cylinder was got by multiplying the area of its base obtained as above by the height. For the determination of an angle by its cotangent see p. 98.

METHOD OF SETTING OUT THE SUMS.

Each sum consists of: 1. Title; 2. Statement of problem; 3. Working out; and 4. Proof.

1. The title begins with the words *tp n* "Example² of." Thus in No. 52 the title reads "Example of calculating a trapezoid of land." Very typical of these titles is the use of the verb *irt* "to make" or "to do" in the sense of "calculating" or "dealing with." An extreme case is seen in Nos. 2-6, where *irt t; X* replaces the "Example of dividing (*psš*) *X* loaves" of No. 1.

2. The statement of the problem introduces the data and should begin with the words *mī dd(w) nk*, "If they say to you," i.e. "If you are asked." In No. 52 this section runs, "If you are asked: a trapezoid of land, 10 *khet* in its *mryt*, 6 in its base and 4 in its cut side, what is its land-content?" In some cases, e.g. No. 68, we have *ir dd nk sš*, "If the scribe says to you," in place of *mī dd(w) nk*. This is followed by *šdm:f*, "Let him hear . . ."

3. Of the working out little need be said here except that the directions given are generally in the verbal form *šdm-hr-k*,³ which we may perhaps render in English by the slightly antiquated form, "You are to do so and so."

¹ See further SIMON, *Geschichte der Mathematik im Altertum*, 41-2.

² This meaning seems to be established by the last words of No. 66, "You shall do likewise in the case of anything asked of you similar to this example," *mī tp pn*.

³ The use of this form is typical of all the mathematical papyri as of the medical. It has two distinct uses, firstly to express a polite command or direction, "You are to . . ." and secondly to express the result of an operation, *hpr-hr-f m X*, "It becomes X." In No. 55 it is preceded in this last sense by the particle *hr*. The use of this unusual and archaic verbal form is doubtless due to the fact that the diction of the sciences of mathematics and medicine was probably fixed in the Old Kingdom, and continued almost unaltered into the Middle Empire.

working" in Nos. 41-3, *tp n ššmt*, "section(?) of working" in No. 44, and *ššmt* simply in Nos. 46, 58, and 60. In all these cases the meaning is the same, the word being applied not to the whole working but only to what we should call the "rough working." In other words, the whole working is divided into two parts: the first, with no heading, merely states the actual operations performed and gives their results; the second part, headed *ššmt*, *kī n ššmt* or *tp n ššmt*, gives the laborious detail of the actual multiplications and divisions used.

Whether the confining of the *ššmt* to this latter part only of the working is merely due to the mis-handling of the original arrangement of the sums by our scribe, owing to his ignorance and the exigencies of his narrow registers, it is impossible to say. It may be so, and the word may in reality be the title of the whole working out. We can, however, only go by what we see before our eyes, and in the papyrus as we have it *ššmt* is a word for the "working out," either the whole or the rougher part of it.

More difficult is the remaining phrase *irt mī hpr*. This we must approach first from the philological side. *irt* can only be the Infinitive or the Impersonal Passive, i.e. it can only mean "the doing" or "one does (it)." *hpr* must be either an Impersonal *šdm.f*, "it happens," or a Neuter Participle Active with Masculine form,¹ "that which happens" or "has happened."² The fact that a title is likely to be a noun in form is in favour of taking *irt* as an Infinitive, and in this case the grammatical construction of *hpr* will scarcely affect the meaning, which will be "The doing as it happens" or "The doing according to that which happens." In other words, the phrase should denote some portion of the sum in which an application of some number or quantity to the actual facts of a case takes place.

With this in mind we must now examine the uses of the phrase in the papyrus. It occurs 26 times, generally in groups of sums of the same type. Its use appears to fall under four separate heads:—

1. As title of a proof in which the answer found is subjected to the conditions laid down in the problem and shown to satisfy them and thus to be correct: Nos. 1-6, 24-25, 62-64, and 75-77. In Nos. 1-6 it should be noted that though the phrase covers all the working given it must not be rendered "working" as opposed to "proof," for in these sums the answer is guessed and the only "working" supplied is in reality a proof of this answer.

2. As title of detailed rough work (multiplications and divisions) added after a general outline of the working has been given: Nos. 50, 52, 66. Here it corresponds exactly to the more common use of *ššmt*.

3. In a perfectly literal sense similar to that of 1 above, but in application to a step in the working, not to a proof. The best instance is the first step of No. 40, where we may translate freely, "What would actually happen supposing that the difference of share were $5\frac{1}{2}$." The other case, No. 35, is less obvious, the step consisting in the application of the number 1 to the actual facts of the problem as set. It is necessary to remark, however, that in both these cases the step headed *irt mī hpr* is the first, and in consequence it is possible that the phrase applied to the whole working and not to this step alone, despite the fact that the present arrangement of the papyrus gives the latter impression.

4. As title of the whole working. Here it stands immediately after the setting out of the problem, but the instances of it are somewhat unsatisfactory. Thus we have it in 43, while in the precisely similar 41 and 42 it is omitted. Similarly 51 has it, but the parallel 50 and 52 omit it. It occurs in 49, a very inaccurate and unreliable piece of copying. In 67 it precedes the whole of the work, which, however, is all of the nature of rough work.

¹ Contrast, however, the Feminine *hprt im pw* of Nos. 76 and 78.

² For *mī hpr* meaning "as it actually happens" or "happened" see Pap. Ebers, 67, 5, BRUGSCH, *Wörterbuch* 1341, line 3, and *Urk.*, IV, 121, 14. In the last case *irt* precedes, but the sense is difficult to seize.

³ Baillet (*Pap. Math. Akhmim*, 60) is certainly wrong when he says "la formule *irt mī hpr* se retrouve dans le οὐτω ποίει" ("proceed as follows") of the Akhmim papyrus. The Egyptian cannot mean this.

$$\begin{aligned}\frac{1}{8} \text{ setat} &= s(?) \text{, } \text{𐩰} \triangleright \text{ or } \text{𐩱} \triangleright \text{ (late writings)}^1 \\ \frac{1}{16} \text{ setat} &= sw, \text{ } \text{𐩲} \triangleright \text{ (late)} \\ \frac{1}{32} \text{ setat} &= r m, \text{ } \text{𐩳} \triangleright \text{ (late)}\end{aligned}$$

Of these parts only the $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{8}$ occur in our papyrus, Nos. 53 and 54. The whole of this system of division of the *setat* is referred by Sethe to a very early stage in Egyptian civilization.²

For practical purposes in land-measuring there was a tendency for a unit called the "cubit-of-land" and a 1000-fold multiple of it called the "thousand-of-land" to be used in preference to the *setat*. A cubit-of-land is a narrow strip of land 100 cubits long and 1 cubit broad and is expressed in Rhind by — (the ordinary cubit sign, see No. 55): this unit is clearly one-hundredth part of a *setat*. The thousand-of-land was written in early times $\text{𐩴} \text{𐩵} \text{𐩶}$: in Rhind, however, it is regarded as a unit and indicated simply by a vertical stroke. Thus in No. 53 we find 7 *setat* doubled and the result given as 𐩷 , i.e. 1 thousand-of-land and 4 *setat*. Fractions of the *setat* are expressed in the dimidiated fractions described above, so far as possible, and the small remainders in cubits-of-land (i.e. hundredths of the *setat*). In No. 54 we have, quite exceptionally, a special hieratic sign for 10 cubits-of-land resembling the numeral 30.

3. MEASURES OF CAPACITY.

The unit of capacity is the *hekat* or bushel, containing 10 *henu*, each *henu* being about 29.2 cubic inches according to the examples which have survived. It would seem, however, that despite this numerical relation the *henu* and the *hekat* may have independent origins, for the *henu* was not used in actual reckoning as a part of the *hekat* but as a totally distinct unit. The *hekat* was divided into $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$ and $\frac{1}{64}$, each of which had a separate sign and doubtless a separate name. The *hekat* was further divided into 320 *ro* (—), the word had here probably its early sense of "part"), so that 1 *henu* contained 32 *ro*. Fractions of a *hekat* other than those enumerated above ($\frac{1}{2}$ to $\frac{1}{64}$) were not tolerated, but were reduced to terms of these and of the *ro*. Thus $\frac{1}{3} \text{ hekat} = (\frac{1}{4} + \frac{1}{16} + \frac{1}{64}) \text{ hekat} + 1\frac{2}{3} \text{ ro}$. See above, p. 7.

The notation used in hieroglyphic for these dimidiated parts of the *hekat* was as follows, written from left to right:—

$\frac{1}{2} \text{ hekat}$	𐩸	$\frac{1}{16} \text{ hekat}$	𐩹
$\frac{1}{4}$	𐩺	$\frac{1}{32}$	𐩻
$\frac{1}{8}$	𐩼	$\frac{1}{64}$	𐩽

Möller was the first scholar to perceive that these signs can all be arranged together to form the figure of the *wadit*, or sacred eye, Fig. 2. It must not be inferred, however, that



Fig. 2.

¹ The short hieratic form for 𐩰 is used in Rhind.

² SETHE, *V.Z.Z.*, 74 ff.

³ *E.g.* L., D., II, 7.

⁴ Both signs appear to me to be reversed in MÖLLER, *Paläographie*, Nos. 708, 711. Contrast *Ä.Z.*, 48, 101, line 10, and bear in mind that all the signs are there written in the opposite direction.

this was their origin: these measures were in constant use in Egyptian medicine, and it may have been an ingenious discovery of some scribe that they could be arranged in such a way as to form the sacred eye so closely connected in Egyptian eyes with healing power.¹

For multiples of the *hekat* we find a special notation in the papyri of this period. A tall stroke standing in front of the *hekat* sign thus $1 \overline{\text{hekat}}$ stands for 100 *hekat*, two strokes for 200 and so on. 50 *hekat* is expressed by the ordinary sign for $\frac{1}{2}$ placed after the *hekat* sign, $\overline{\text{hekat}} \frac{1}{2}$, 25 by the sign for $\frac{1}{4}$ $\overline{\text{hekat}} \frac{1}{4}$.² The two signs together would indicate 75 *hekat*. Ten *hekat* are shown by a tall stroke after the *hekat* sign, and 5, 6, 7 and 8 by special hieratic signs to which we do not know the hieroglyphic equivalents.³ Single *hekat* up to 4 are indicated by dots, after the *hekat* sign, and fractions of a *hekat* follow in the usual Horus-eye notation. Thus $11 \overline{\text{hekat}} \frac{1}{10} \times 100$ stands for 287 *hekat*.

In No. 82 we are introduced to a double-*hekat* unit, written $\overline{\overline{\text{hekat}}}$. This made its appearance, so far as we know, in the Middle Kingdom, for it is first met with in the Kahun Papyri. It is equipped with its dimidiated parts just as is the single *hekat*, each being presumably double the corresponding part of the single *hekat*. This double-*hekat*, written $\overline{\overline{\text{hekat}}}$, also continued in use into the XVIIIth Dynasty, for it is used in *pfšw* reckonings on a stela from Tell el-'Amarneh.⁴

Nos. 41 to 47 make use of a still larger unit, the quadruple-*hekat* or "great quadruple-*hekat*" (for the various writings see the problems referred to). This too is divided by halves after the manner of the single *hekat*, and continued to be used, especially for measuring grain, until well on into the New Kingdom.

The only other measure of capacity used in Rhind is the *khar* (*h:r*) $\overline{\overline{\overline{\text{hekat}}}}$, which, as is clear from Nos. 41 ff., contained 5 quadruple-*hekat*. It is also the capacity of two-thirds of a cubic cubit. This same measure is written $\overline{\overline{\overline{\text{hekat}}}}$ in the Westcar Papyrus (12, 4), but seems to die out in the New Kingdom, when it is replaced for bulky substances by a $\overline{\overline{\overline{\text{hekat}}}}$ consisting of 4 quadruple-*hekat*. The reading of this new measure is unknown, since it is nowhere written out, but it is not impossible that it continued to be read *h:r*, though its value had fallen from 5 quadruple-*hekat* to 4.

The $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{3}{4}$ of this measure, which correspond to 1, 2, and 3 quadruple-*hekat* respectively, are expressed by dots, one, two or three as the case may be. The notation described by SPIEGELBERG, *Rechnungen aus der Zeit Setis I*, Text, p. 49, is probably incorrect. Louvre Pap. 3226 is perfectly clear on the point. Some later papyri at Turin (unpublished) are, however, less clear, and need fresh collation and working out.

4. WEIGHTS.

The complicated system of weights used in Egypt plays but little part in this papyrus. Suffice it to say that in No. 62 we have the *deben* (*dbn*, formerly read *uten*) of which the *kite* (*kdt*) is the tenth part. For the "ring" as a definite weight see below under No. 62. The weight established for the *deben* by actual weighing of New Kingdom specimens is between 1400 and 1500 grains.⁵

¹ The hieratic forms, which show but slight resemblance to the parts of the eye, are doubtless older than the hieroglyphic, none of which have yet been found earlier than the XVIIIth Dynasty, some not before the XXth. It was perhaps in establishing the hieroglyphic forms from the hieratic that the sacred eye myth was brought into play.

² In No. 82 there is a unique instance of the fraction $\frac{1}{2}$ preceded by $\overline{\text{hekat}}$ standing for $33\frac{1}{2}$ *hekat*, a clumsy expedient since it leaves us with $\frac{1}{2}$ *hekat* which must be resolved into correct Horus-eye notation.

³ See MÖLLER, *Paläographie*, I, Nos. 699-702.

⁴ P.S.B.A., XV, 306. Cf. too DÜMICHEN, *Kalender-Inschriften*, Pl. XXXIX.

⁵ See Petrie's article *Egyptian Weights and Measures* in *Encyclopedia Britannica*; also HULTSCH, *Die Gewichte des Altertums*, EISENLOHR, *Altägyptische Maasse* in *Recueil de Travaux*, XVIII, 29 ff., and DECOURDEMANCHE, *Notes sur les poids égyptiens*, in *Annales du Service*, XIII, 125-160.

COMPARISON OF EGYPTIAN MATHEMATICS WITH BABYLONIAN.

As early as 3000 B.C. it would seem that the world was in possession of two separate and highly developed systems of mathematics, the Egyptian and the Babylonian. Archaeology may never take us far enough back into the roots of civilization to enable us to decide whether these two systems had entirely independent origins, or, if not, what was their common source, and how much each owed to it and to the other. We can only take them as they stand when they first come under our observation and compare them.

For the modern the mathematics of Babylonia¹ possess an interest which those of Egypt cannot rival, for in them lies, it would seem, the origin of the modern division of the circle into 360 degrees and of the day into 24 hours. The term Babylonian is however here a complex one, for we must distinguish between two elements in Babylonia, the non-Semitic Sumerians and the Semitic Akkadians with their later Semitic followers. These Semites of Babylon appear to have used a system which was purely decimal, like the Egyptian, but which very soon became contaminated by the sexagesimal system of the Sumerians.² From the extent to which Babylonian mathematics is dominated by the sexagesimal notation it is clear that most of it owed its origin to the Sumerians, and we are therefore justified in going back to the earliest times in the enquiry which follows.

1. CONTRAST BETWEEN EGYPTIAN AND SUMERIAN MATHEMATICAL RECORDS.

There is an essential difference between the mathematical material which has come down to us from the two lands. From Egypt we have two complete papyri of problems and considerable fragments of others, in other words we have mathematics in use. From Sumeria we have little except the means employed, consisting of numerous tablets containing tables of multiplication and division, of squares, of square roots and of cube roots. Except for what can be gathered from astronomical³ reckonings and from calculations in weights and measures, mostly very unfruitful, we have no pictures of these tables in action. This wide difference in the nature of the material preserved makes it difficult to institute comparisons of any value between Egypt and Sumeria.

2. NOTATION.

The Egyptian notation is decimal, its units being 1, 10, 100 and so on. The Sumerian is essentially sexagesimal, that is to say its main units are 1, 60, 3600, etc. At the same time this is not entirely primitive, and there clearly must have been a time when the Sumerians could not count up to 60. Such a time is marked by the survival in the sexagesimal system of a subsidiary unit 10. We may go back even farther than this, for the names of the numbers from 6 to 10 appear to be secondary formations from those for 1 to 5, which points to a stage where the fingers of one hand were used to count up to five (for a similar possibility in Egyptian see above, p. 11).

In the Sumerian sexagesimal system this unit 10 occupied a subordinate place, which can be best illustrated by the fact that 100, 1000, etc. were not units in the system⁴ and had

¹ The latest works on the subject are THUREAU-DANGIN, *Numération et métrologie sumériennes*, in *Revue d'Assyriologie*, XVIII, 123 ff.; HILPRECHT, *Mathematical, metrological and chronological tablets from the temple library of Nippur* (The Babylonian Expedition of the University of Pennsylvania, Vol. XX, Part 1); THUREAU-DANGIN, *L'u, le qa et la mine, leur mesure et leur rapport*, in *Journal Asiatique*, 1909, 79-112.

² See THUREAU-DANGIN, *Orientalische Literaturzeitung*, 1909, 383, n. 2.

³ Perhaps astrological would be a more correct term, see HILPRECHT, *op. cit.*, 34, and quotation there given from Bezold.

⁴ They were used by the Akkadians, however, and even survived the Sumerianizing of the Akkadian arithmetic.

no special signs: here we have an essential difference from Egyptian. The number 10 was however used to form new units, for just as the unit 1 could be multiplied by 10 to give a new unit with special sign, so also the sexagesimal units 60, 3600, etc. could each be multiplied by 10 to give 600, 36,000, and 2,160,000, a series of fresh units each with a special name.

The highness of the unit 60 had for its consequence a tendency to lead to very high numbers, for 60 cubed was already 216,000. This may be compared with the fully developed system of high numbers in the Egyptian system, reached by quite a different route.

In one important point the Sumerian system completely surpassed that of Egypt. In the purely mathematical tablets we find a system, imperfect it must be admitted, of positional notation. Supposing that in our decimal notation we write 365. We read this (3×10^2) plus (6×10) plus (5×1). Our unit being 10 we can represent each portion of it by a single digit, 3, 6 and 5. In a system with a main unit 60 and a subsidiary unit 10 we should have to represent each portion by two digits, thus 32.12.43 would stand for (32×60^2) plus (12×60) plus (43×1), or 115,963. This is precisely what the Sumerians did, and it was here that their secondary unit 10 served them in such good stead. In the mathematical tablets two signs alone are used in this notation, the sign for 1 and the sign for 10. The number above referred to would be as follows:—three tens and two units; a ten and two units; four tens and three units: the fact that the first group is to be multiplied by 60^2 , the second by 60, and the last by 1 is taken for granted, just as the multiplication of 3, 6 and 5 by 100, 10 and 1 respectively is in our decimal notation for 365.

The system was still further perfected by the use of a sign for zero in cases where a middle term was missing, *e.g.* in 12.0.33, which stood for (12×60^2) plus (0×60) plus (33×1). Here we have all the elements of positional notation with one exception: there was nothing to correspond to our decimal point. In other words, if I write in modern terms 365 I know that the 5 is units, the 6 tens, and the 3 hundreds; and if I write 36.5 I know that the 5 is tenths, the 6 units, and the 3 tens. When, however, the Sumerian wrote 12.25.33 he had no means of showing whether the lowest unit, that to be multiplied by 33 was 3600, 60, 1, $\frac{1}{60}$, or some other sexagesimal unit. All that was fixed was that whatever the lowest of these units (to be multiplied by 33) was, the next (to be multiplied by 25) was 60 times greater, and the highest (to be multiplied by 12) 3600 times as great. An interesting corollary of this will be noticed below in connection with the tables of division and multiplication.

3. FRACTIONS.

In the true sexagesimal system the highest fractional unit was naturally $\frac{1}{60}$ and the next $\frac{1}{3600}$. The use of the secondary unit 10, however, gave also $\frac{1}{6}$ ($\frac{1}{60} \times 10$) and $\frac{1}{360}$ ($\frac{1}{3600} \times 10$). In practical everyday reckonings the fraction $\frac{1}{6}$ assumed a paramount position both in the Sumerian system and in the later Babylonian systems derived from it, and from a comparatively early period we find a notation for $\frac{1}{6}$, $\frac{1}{3}$, $\frac{1}{2}$, and $\frac{2}{3}$. As early as the First Dynasty of Babylon we find a notation for $\frac{2}{3}$ as well as for $\frac{1}{4}$ and $\frac{1}{5}$.

Thus in Babylonia we have up to the present no instances of the complicated fractional reckonings to which the Egyptian mathematician seems to have been perfectly accustomed. This may be the merest accident, and probably is, for in a system whose first fractional units are $\frac{1}{60}$ and $\frac{1}{3600}$ calculations involving fractions with high denominators could hardly have been avoided. The use of the fraction $\frac{2}{3}$ is paralleled in Egypt, but the existence of $\frac{2}{3}$ shows us that the Babylonians had in some cases at least passed beyond the Egyptian convention which demanded that all fractions except $\frac{2}{3}$ should be aliquot parts.¹

¹ SETHE, *V.Z.Z.*, 103, is not inclined to ascribe to Babylonian influence the appearance of a sign for $\frac{2}{3}$ in Egypt in Ptolemaic times.

4. TABLES OF MULTIPLICATION AND DIVISION.

In early Egypt nothing has yet been discovered in the way of tables except those for dividing 2 by the various odd numbers. It would be rash to say that multiplication tables did not exist, but at the same time there is a reason why they should never have needed a very high development. The Egyptian system of multiplication by doubling was a slow one, but it was sure; it is indeed astonishing how simply and even how quickly the most complicated multipliers can be constructed by this method, especially when we remember that the purely mechanical process of multiplication by 10 can be introduced as an aid at any moment. The fact is that the Egyptian's apparatus for multiplication was so powerful that he could well dispense with tables, and we must be prepared for the possibility that he actually did, though excavation may prove this to be wrong.

The Sumerian, on the other hand, was a lover of tables. Unfortunately we have no idea whatsoever how his multiplications were accomplished. One might, however, hazard the guess that at quite an early period he had acquired considerable power over the process, otherwise he could hardly have faced the multiplicative difficulties involved in a notational system based on so high a figure as 60. The third unit in his system was no less than 3600!

The multiplication tables known to us come mostly from the library at Nippur. Each table multiplies by a certain multiplier the numbers from 1 to 20 inclusive, then 30, 40 and 50; it will be observed that in this way any multiplicand from 1 to 59 can be obtained either directly or by a simple addition. The reason why these multiplicands stop at 59 is obvious. Having regard to the positional system of notation multiplication by 60 did not alter the figures in the multiplicand, but merely the units. Thus to multiply by 60 was as easy for the Sumerian as it is for us to multiply by 10. We merely move the decimal point or insert a cypher, leaving the digits of the multiplier as they are. He too left his multiplicand untouched, but as he had invented neither point nor final cypher he could not use them, and presumably trusted to his memory to remind him that his lowest unit was now 60 times what it was before.

The multipliers comprised in these tables are forty-six in number; they begin with 2 to 6, 8, 9, 12, 18, and end with 3000, 160,000, 162,000 and 180,000. They are a curious series, the choice of which it is not easy to see. Hilprecht has attempted, not very convincingly, to derive them from the division by certain numbers of the number 3600×3600 or 12,960,000, which he believes to be Plato's geometric number.¹

Side by side with these multiplication tablets must be considered four tablets which, according to Hilprecht, give the division of this large number 12,960,000 by various numbers from $1\frac{1}{2}$ to 81. These tables are certainly divisions, but Thureau-Dangin has rightly pointed out² that the number divided need not be that supposed by Hilprecht, for owing to the absence of a point ("sexagesimal point" we should have to call it) in the Babylonian system we cannot determine what is the unit. Thus the number translated by Hilprecht as 1,080,000 is on the tablet actually represented by the figure 5; he takes this as 5×60 cubed, but it might equally well be $5 \times$ any other power of 60, such as 60^2 , 60, 1, $\frac{1}{60}$ or $\frac{1}{60^2}$. Indeed, here lies the very beauty of these tables. Just as our 12-times table will serve us to multiply 12, or 12,000 or .000012, so this table will divide for us not only 12,960,000, which is 60^4 , but also 60^2 , $\frac{1}{60^2}$, and so on. If this remarkable elasticity of unit was really intended by the Sumerians, they had certainly far surpassed the Egyptians in the matter of notation.

Tables of squares may be regarded as extracts from the multiplication tables. Nippur has furnished three running from 1 to 50, and an earlier discovered tablet runs from 1 to 60. Hilprecht's argument (*op. cit.*, 24, end of note 3 continued from p. 23) that squares of numbers from 31 to 60 were obtained by the formula $(a+b)^2 = a^2 + 2ab + b^2$, where a is 30, seems to me

¹ *Op. cit.*, 20 ff.

² *Revue d'Assyriologie*, XVIII, 124.

a simple *non sequitur*. No table of squares has yet been found in Egypt, though a technical term for square occurs in the Moscow papyrus in the problem of the truncated pyramid (see p. 20).

Side by side with the tables of squares we have from Babylonia several tables of square roots and one of cube roots. Egypt has yielded nothing of this kind, and though the conception of square root was well known there that of cube root has not yet been found.

5. WEIGHTS AND MEASURES.

With regard to Babylonian weights and measures we have a mass of evidence which can scarcely be glanced at here. Suffice it to say that Babylonia had a complete system of measurements for length, area, volume, capacity, liquid content and weight. In general these offer few points of comparison with the Egyptian, the characteristic Egyptian division by halves, quarters, etc. being conspicuously absent in Babylonia, where subdivisions based on the sexagesimal notation are more usual.

The Babylonian unit of length, the cubit, as determined by the measurements of the statue of Gudea, *patesi* of Lagash, and by the dimensions of the great tower at Babylon, was just under 496 mm. in length. This compares fairly closely with the Egyptian royal cubit of 523 mm., but no conclusion must be drawn from this, for the arm, or more exactly the forearm, is an obvious primitive unit of measurement for any people. In Sumerian the finger-breadth is one-thirtieth of the cubit; in Egyptian it is one-twenty-eighth. The Sumerian measure of the foot of 20 fingers, or $\frac{2}{3}$ of a cubit, does not occur in Egyptian.

Thureau-Dangin has sought to prove that a definite relation existed between the units of length, capacity and weight in Babylonia.¹ He finds that the *qa*,² the unit of capacity, is $\frac{1}{144}$ of the cubic cubit, and that the *mina*³ is the weight of a volume of water of $\frac{1}{240}$ of the cubic cubit. This will certainly need more convincing proof than has been given so far, and it would seem *a priori* a little unlikely that so artificial a system should have been adopted, though the modern decimal system affords an obvious parallel. However this may be, no evidence is as yet available for determining whether any relation of this kind exists between the various units in the Egyptian system.⁴

6. GEOMETRY.

Our knowledge of Babylonian geometry is slight. For the determination of areas there seems to be but a single document, a tablet of the second Dynasty of Ur, now in the Museum at Constantinople.⁵ On this are plans of fields accompanied by certain lengths and areas, from which it would appear that the following determinations of area were correctly known to the Babylonians: the area of the rectangle (and as a special case that of a square), the area of a right-angled triangle determined by half the product of the two sides enclosing the right-angle, and the area of a trapezoid, namely the product of the altitude and half the sum of the parallel sides.

Of these the first was known to the Egyptians, and indeed must be known to all peoples who have evolved the conception of square measure. Whether the second and third were correctly known in Egypt depends on the interpretation of the word *mryt* in Nos. 51 and 52, see pp. 91 ff.

¹ *Revue d'Assyriologie*, XVIII, 127-132; *Journal Asiatique*, 1909, 79 ff.

² Found from the silver vase of Entemena and other evidence to be about 404 millilitres.

³ The ancient *mina* was about 404 grammes, the later 505.

⁴ THUREAU-DANGIN, *op. cit.*, 107-8, comes to this conclusion.

⁵ THUREAU-DANGIN, *Revue d'Assyriologie*, IV, 16 ff.; OPPERT, *op. cit.*, IV, 28 ff. For later literature see HILPRECHT, *op. cit.*, 11, note 9. On this tablet an irregular area is divided for purposes of measurement into a series of right-angled triangles and approximate rectangles.

If Thureau-Dangin is correct with regard to the relation between the *qa*, the *mina*, and the cubic cubit, it follows that the Babylonians were capable of estimating the volume of a cube. We might indeed have inferred this from the existence of a fully developed system of measures of volume, which obviously presupposes the determination of the volume of a parallelepiped.¹

THE GREEKS ON EGYPTIAN MATHEMATICS.

The Greeks looked up to the Egyptians as the originators of their mathematics and more particularly of geometry. Herodotus² tells us a story on which perhaps all later references in Greek literature are based. Speaking of Sesostris, a king who would appear to be a combination of Ramesses II of the XIXth Dynasty with at least one of the great Senusrets of the XIIth, he says: "This king divided up the land among the Egyptians, giving an equal square plot to each man; from this he derived his revenue, imposing a rent to be paid each year. But if the river carried away a portion of any man's lot he would come to him and report what had happened. And the king would send men to examine and to measure by how much the land had been diminished, in order that he might pay only a proportionate amount of the rent. It appears to me that geometry was discovered in this way and that it afterwards came over into Greece."

The statement with regard to the equal division of land between the people has probably little historical value, but we need have no doubts as to the soundness of the derivation of Greek geometry from Egypt. Geometry is an intensely practical science and would naturally first appear in a country where land was of very great value, in other words, in a highly agricultural country. The two obvious places of origin are Mesopotamia and Egypt.

Herodotus' story with regard to the carrying away of land by the river is not altogether convincing, and it is probable that there is here a confusion with the fact that the Nile when it rises year by year to some extent obliterates the boundaries between field and field, a fact which in Strabo's Geography and in the Summary of Proclus is given as the origin of geometry in Egypt. As Strabo remarks,³ the land "had to be measured again and again."

Later Greek writers do not add much to Herodotus' story.⁴ Diodorus,⁵ however, tells us that the Egyptians themselves claimed astronomy and geometry as Egyptian discoveries, and Plato in the Phaedrus⁶ makes Socrates say that he has heard that the god Thoth was the inventor of arithmetic, calculation, geometry and astronomy. Aristotle,⁷ however, diverges from the common story in attributing the discovery of these sciences in Egypt not to the fact that there was need for land-measuring there, but to the fact that there was a leisured class of priests who had time to spare for such pursuits. In reference to this he it said that there is no particle of evidence that in early times Egyptian mathematics were in any sense in the hands of the priests, whatever may have been true of Aristotle's day.

The actual introduction into Greece of Egyptian geometry is attributed by the writer of the Summary of Proclus to Thales, "who first went to Egypt and from there introduced the study into Greece." This dependence upon Egypt is made very clear by Iamblichus' Life of Pythagoras, where Thales, after teaching his young pupil all he knew, advised him to go and study with the Egyptian priests, which indeed he did, spending no fewer than 22 years in the temples of Egypt learning astronomy and geometry.

In this connection an interesting problem is raised by Democritus' reference to the

¹ HILPRECHT, *op. cit.*, 36-38, draws attention to a possible case of the determination of the volume of a cylinder.

² II, 109.

³ XVII, 3.

⁴ The two letters of Rhabdas of Smyrna, 1341 A.D., are perhaps of greater interest than value in this connection.

⁵ I, 69, 81.

⁶ 274, C.

⁷ *Metaphysics*, A. 1, 981 b, 23.

harpedonaptai of Egypt.¹ The philosopher boasted that no one of his time had surpassed him in constructing figures from lines and in proving their properties, not even the so-called *harpedonaptai* of Egypt. Who were these *harpedonaptai*? More than one historian of mathematics has supposed that they were land-measurers. The literal meaning of the word is "rope-stretchers," and it is suggested² that they were acquainted with the fact that a triangle whose sides were 3, 4 and 5 contained a right-angle, and that they constructed right-angles accordingly, as did the Chinese and the Indians.

For this last statement I can find no foundation whatsoever; nothing in Egyptian mathematics suggests that the Egyptians were acquainted even with special cases of Pythagoras' theorem concerning the squares on the sides of a right-angled triangle. That the *harpedonaptai* were land-measurers on the other hand is most probable, indeed we can even see such persons at work in the pictures on the walls of Egyptian tombs. In the tomb of Khaemhet at Thebes we see a number of men equipped with ropes and writing material measuring a field, and there is a similar scene in the tomb of Menena.³ In either case the persons engaged in the work might most suitably be described as "rope-stretchers"; the very unit by which fields were measured was a "reel of rope" of 100 cubits in length (see p. 24).

This process of land-measuring with a rope, the Egyptian name for which is not known, has been confused by historians of mathematics with the ceremony called *pd šš* "the stretching of the cord." This was one of the initial ceremonies at the foundation of a temple.⁴ The king or whoever represented him took a sighting of the pole-star through a cleft stick, another person standing north of him with a plumb-bob attached to a wooden arm.⁵ Each then drove a stake into the ground in front of him, and a cord stretched between the two gave a true north and south line and enabled the four corners of the temple to be fixed. Here the stretching of the cord is used not necessarily in measurement, but in the fixing of the orientation.⁶ Possibly it was something of this kind which Democritus had in mind when he spoke of the skill of the *harpedonaptai*.

¹ Clemens Alexandrinus, *Stromata*, ed. Potter, I, 357.

² HEATH, I, 122.

³ WRESZINSKI, *Atlas zur altägyptischen Kulturgeschichte*, Taf. 191 and 232. For a statue of an "overseer of land" holding a measuring-rope see *Ä.Z.*, 42, 72.

⁴ I have to thank Dr. Blackman for putting at my disposal his admirable unpublished notes on this ceremony.

⁵ Examples of these two instruments, the diopter and the arm with a plumb-line, have been found; see Borchardt in *Ä.Z.*, 37, 10 ff. They are doubtless the instruments called *ὀρολόγιον* and *φοίνιξ ἀστρολογίας* respectively in Clemens Alexandrinus, *Stromata*, VI, 4, 35. It is significant that the priest who uses them is here called not *ἀρπεδονάπτης* but *ὀροσκόπος*.

⁶ The cord may have been used in the subsequent measurements.

TRANSLATION AND COMMENTARY

TITLE AND INTRODUCTION. Plate A.

"Rules for enquiring into nature, and for knowing all that exists, [every] mystery, every secret. Behold this roll was written in Year 33, month 4 of the inundation season, [under the majesty of the King of Upper] and Lower Egypt Aauserrē, endowed with life, in the likeness of a writing of antiquity made in the time of the King of Upper and Lower Egypt Nemarē. It was the scribe Ahmōse who wrote this copy."

Title and
Intro-
duction.

The small black cross under the word *h:t* indicates that the words *m ht* in the second column are to be inserted here. Eisenlohr through failing to observe this has missed the sense of the whole passage.

The papyrus begins with a formal title and indication of its contents which has unfortunately been damaged. This title is written in vertical lines and begins in red ink. It is followed by the dating and the name of the scribe who made the copy.

tp hsb n ht m iht. The concluding words *ht m iht* must mean "going into" or "probing" things. *iht* probably has the same general sense which it bears in the phrase *rh iht* "a wise man," one who knows facts or things. I can find no good parallel to the figurative use of *ht m*. In Nos. 35-38 *ht r* appears to be used in the literal sense of "go (down) into," but no attempt should be made to contrast a metaphorical use with *m* and a literal with *r*, for both prepositions are used with *ht* in its literal sense in the Old and Middle Kingdoms. It now remains for us to fix the meaning of *tp hsb*. The best known examples of the phrase occur in the Eloquent Peasant, B 1, ll. 98, 148, 161, 274, 311, 325, and B 2, l. 94. Vogelsang's notes to these passages should be consulted.¹ From a comparison of the various uses of the phrase in this text it is clear that it means in general "accuracy," not only in actual reckoning but also in conduct, that is to say moral as well as intellectual. It also occurs in a rather difficult sentence² in Prisse 5, 7, and again in 8, 5. The most frequent use of the phrase is in the title *ss ikr n tp hsb* (*Urk.*, IV, 122, 964), "Excellent scribe of correct reckoning," where of course it is used in the more concrete sense, meaning that the scribe is accurate in copying down and in the computation of accounts. There can be little doubt that this intellectual sense is to be given to *tp hsb* in the present passage and that it simply means "accuracy," or still more concretely, "accurate method of," i.e. "rules of." The whole phrase must therefore mean "Rules for" or "Correct method of enquiring into facts" or "into nature."

snkt. This reading is certain. The word is a slightly unusual one for "dark." See BRUGSCH, *Wörterbuch*, 379 and 1255. For the noun *snkt* "darkness" see LACAU, *Textes Religieux*, LXXXIX; we seem to have a masculine form in *Ä.Z.*, 1864, 2. *snkt* occurs in the metaphorical sense of obscurity of speech in *J.E.A.*, IV, 34, Pl. VIII, line 7. The following word was doubtless *nbt* "every."

DIVISION OF 2 BY ODD NUMBERS. Plates A—E.

Plates I to VI and part of VII in *B.M. Facs.* are occupied, after the title, by a problem which has attracted mathematicians of all ages, namely the expression of each of the various

Division
of 2.

¹ VOGELSANG, *Kommentar zu den Klagen des Bauern*, 94, 212, and 219.

² The passage 8, 5 would seem to mean "The judgment-hall works by rule (*tp hsb*), all its dealings are according to the measuring tape," i.e. strictly accurate.

³ *tp hsb*, if Griffith is right in restoring the words so, must have a similar sense in *K.P.*, Pl. VIII, 30. Cf. also Louvre stela C 14.

Division of 2. fractions whose numerator is 2 and whose denominator is one of the odd numbers from 3 to 101 as the sum of two or more aliquot parts, *i.e.* fractions whose numerator is unity.¹ Thus:—

$$\frac{2}{49} = \frac{1}{24} + \frac{1}{240} + \frac{1}{320}$$

The purpose of this process is not hard to find. The Egyptian, as we have seen above, had no notation for and disliked dealing with fractions other than aliquot parts, with the single exception of $\frac{2}{3}$. When brought face to face with such fractions he reduced them at once to a more workable form. Now any proper fraction can be reduced at once to the sum of a number of fractions whose numerators are either 1 or 2. Thus, to take a simple example, $\frac{7}{11} = \frac{1}{11} + 3(\frac{2}{11})$. From this it follows that if we can resolve into aliquot parts all the fractions whose numerators are 2 we can ultimately break up any proper fraction whatsoever into the sum of a series of aliquot parts. Thus:—

$$\frac{7}{11} = \frac{1}{11} + 3(\frac{2}{11}) = \frac{1}{11} + 3(\frac{1}{6} + \frac{1}{66}) = \frac{1}{11} + \frac{1}{2} + \frac{1}{22}$$

For this purpose the Egyptian reckoner was accustomed to keep by him a table of "the division of 2," or, in modern terms, of the resolution of fractions whose numerators are 2. Among the Kahun Papyri is one (Pl. VIII) which gives a series of such resolutions from $\frac{2}{3}$ to $\frac{2}{101}$. The Rhind Papyrus goes much farther and carries us up to $\frac{2}{101}$.

By what process did the Egyptian arrive at his results? This may best be understood by examining a typical specimen of a resolution, namely that of $\frac{2}{7}$, which may be paraphrased in modern terms as follows:—

Problem: Express $2 \div 7$ as the sum of a series of aliquot parts.

Answer: $1\frac{1}{2} + \frac{1}{4} = \frac{1}{4}$ th of 7; $\frac{1}{4} = \frac{1}{28}$ th of 7.

i.e. $\frac{2}{7} = \frac{1}{4} + \frac{1}{28}$

Proof:

	1	7	
	$\frac{1}{2}$	$3\frac{1}{2}$	
	$\frac{1}{4}$	$1\frac{1}{2} + \frac{1}{4}$	
— 4	$\frac{1}{28}$	$\frac{1}{4}$ (Proof of this step)	1 7
			2 14
			— 4 28

There is no doubt as to what takes place here. The 2 is broken up into two parts, namely $(1\frac{1}{2} + \frac{1}{4})$ and $\frac{1}{4}$. The first of these is then shown to be $\frac{1}{4}$ of 7 by the simple process of dividing 7 by 2 and then by 2 again, while the second is shown to be $\frac{1}{28}$ of 7 by multiplying 7 by 4 and obtaining 28. Thus $2 \div 7 = \frac{1}{4} + \frac{1}{28}$.

As a proof this is satisfactory, but it does not throw the slightest light on the one feature of interest in these problems, namely the manner in which the Egyptian obtained his answer, which, be it noted, is not worked out at all, but merely assumed and then proved. To arrive at the method by which the answer was obtained it is necessary to examine the whole series of resolutions from $\frac{2}{3}$ to $\frac{2}{101}$, and to try to discern in them any signs of the employment of a general formula.

The fraction $\frac{2}{3}$ is not resolved, since for the Egyptian reckoner it presented no difficulties (see above, p. 15). In the case of all other fractions whose denominator is a multiple of 3 a very obvious resolution presented itself, for the numerator 2 could be broken up into $1\frac{1}{2}$ and $\frac{1}{2}$, and since $1\frac{1}{2}$ divides exactly into 3 and all its multiples the problem was at once solved. Thus: $\frac{2}{9} = \frac{1\frac{1}{2} + \frac{1}{2}}{9} = \frac{1}{6} + \frac{1}{18}$.³

¹ Cantor treats this portion of the papyrus at great length, 24-33.

² Strictly speaking, we ought to write $2 \div 9$ and not $\frac{2}{9}$, since the Egyptian had no notation to correspond to the latter.

³ For the formula here used, *viz.* $\frac{1}{a} \times \frac{2}{3} = \frac{1}{2a} + \frac{1}{6a}$, compare No. 61B.

Similarly those fractions whose denominator was 5 or a multiple thereof could be dealt with by breaking up the numerator 2 into $1\frac{2}{3}$ and $\frac{1}{3}$. Thus $\frac{2}{25} = \frac{1\frac{2}{3} + \frac{1}{3}}{25} = \frac{1}{15} + \frac{1}{75}$. In this way the denominators 5, 25, 65 and 85 were dealt with, 15, 45 and 75 having been already treated as multiples of 3, 35 being treated irregularly, 55 as a multiple of 11, and 95 as a multiple of 19.

When the denominator was divisible by 7 the 2 was broken up into $1\frac{3}{4}$ (or $1 + \frac{1}{2} + \frac{1}{4}$ as the Egyptian called it) and $\frac{1}{4}$. Thus $\frac{2}{49} = \frac{(1 + \frac{1}{2} + \frac{1}{4}) + \frac{1}{4}}{49} = \frac{1}{28} + \frac{1}{196}$. In this way were resolved 7, 49 and 77; 21 and 63 were treated as multiples of 3, and 35 and 91 were dealt with irregularly.

In the case of 11 and its multiple 55 the 2 was resolved into $1\frac{2}{3} + \frac{1}{6}$ (i.e. $1\frac{5}{6}$) and $\frac{1}{6}$.

Up to this point it may be said that the method has been marked by considerable regularity. We are now left with the prime numbers between 13 and 97. A modern mathematician would probably treat these, as indeed all numbers, by some such formula as that suggested by Griffith,¹ namely:—

$$\frac{2}{n} = \frac{1}{a} + \frac{1}{na}, \text{ where } a = \frac{n+1}{2},$$

which has the advantage of resolving each 2-fraction into two aliquot parts only, but the disadvantage of giving a second fraction with a very high denominator. The Egyptian was bound by no fetters of this kind, for the simple reason that he reached his results not by formula but by trial. An inspection of them is sufficient to show this. Even in the treatment of multiples of the lower prime numbers we have already seen that there was some irregularity, and this is only emphasized when we come to the higher prime numbers.

Eisenlohr has attempted to embrace the Egyptian results under a series of rules which he enunciates as follows:—

1. Resolution into three fractions was preferred to resolution into four.
2. If a resolution existed (i.e. could be found) in which the denominator of the first root-fraction was the product of factors which when separately multiplied by the denominator of the original 2-fraction give the denominators of the remaining root-fractions, if, that is to say, $\frac{2}{n}$ could be broken up into $\frac{1}{ab} + \frac{1}{an} + \frac{1}{bn}$ or into $\frac{1}{abc} + \frac{1}{an} + \frac{1}{bn} + \frac{1}{cn}$, then this resolution was chosen. Otherwise that in which the denominator of the first root-fraction was not ab but $\frac{ab}{2}$, $\frac{ab}{3}$, $\frac{ab}{4}$ or $\frac{ab}{8}$ was adopted.
3. High factors of the original denominator were avoided.

It is true that as a matter of actual fact the resolutions given by the Egyptian do to some extent conform to these rules. Thus the resolutions of 17, 31, 37, 43, 47, 59, 67, 73 and 97 conform to the simple formula given above; in the case of 19, 41, 71, 79 and 83 the denominator of the first root-fraction is $\frac{ab}{2}$, in the case of 53 it is $\frac{ab}{3}$, in the case of 13, 29 and 89 it is $\frac{ab}{4}$, and in the case of 61 it is $\frac{ab}{8}$. But even here Eisenlohr's rules are by no means consistently carried out. Thus in dealing with 13 the denominator of the first root-fraction is $\frac{1}{8}$ where the rule would prescribe $\frac{1}{16}$; in 19 the simple formula is disregarded and one in which the first denominator is $\frac{1}{12}$ is used; 29, 71 and 89 are similar exceptions; in the case of 73 a resolution into three fractions is preferred to that into two, while 91, instead of obeying the formula, is resolved into only two fractions.

The fact is that Eisenlohr is here employing a method of analysis which ought not to be applied to Egyptian mathematics. Even could we show that all the results corresponded to a formula this would not prove that the Egyptian worked by this formula, and if Eisenlohr means to imply that he did he is undoubtedly wrong. The Egyptian, far from employing a formula, probably had no conception that the resolution could be accomplished by a single

¹ Badly misprinted in *P.S.B.A.*, XVI, 202 and 203.

Division of 2. method in all cases. He had indeed observed that where the denominator of the 2-fraction was a multiple of 3 the same resolution could be used as for 3 itself, and similarly for 5, 7, and even 11; but when he came to the higher prime numbers he had no formula to help him.

His method was undoubtedly that of trial. He had grasped the fact that the problem consisted in breaking up 2 into the sum of several quantities each of which would divide without remainder into the given denominator. The first of these quantities was always greater than unity, preferably greater than $1\frac{1}{2}$, and was such that when reduced to an improper fraction (to use a modern term) its numerator was precisely the given denominator. Thus to resolve 19 the 2 is broken up into quantities the first of which is $1 + \frac{1}{2} + \frac{1}{12}$ or $\frac{19}{12}$. If we can now find one or more aliquot parts which when added on will make this up to 2 the problem is solved, for $\frac{19}{12}$ must divide exactly into 19, and any aliquot part when divided by a whole number, namely the given denominator, remains an aliquot part. In the present case the obvious fractions to be added are $\frac{1}{4}$ and $\frac{1}{6}$.

There are two points here which call for explanation. If the Egyptian had no conception¹ of the quantity $1 + \frac{1}{2} + \frac{1}{12}$ under the form $\frac{19}{12}$, how did he obtain it, and how did he know it to be a twelfth of 19? Undoubtedly by trial divisions of the original denominator 19, making use of his favourite processes of successive division by 2 or of multiplication by $\frac{2}{3}$, followed by halvings of this, i.e. of division by 3. Thus the first step of the proof in the case of 19 gives us a correct idea of the method by which the result was originally arrived at:—

1	19
$\frac{2}{3}$	$12\frac{2}{3}$
$\frac{1}{3}$	$6\frac{1}{3}$
$\frac{1}{6}$	$3\frac{1}{6}$
$\frac{1}{12}$	$1\frac{1}{2} + \frac{1}{12}$

Here on arriving at $\frac{1}{12}$ of the original denominator he finds it to be $1\frac{1}{2} + \frac{1}{12}$, a very suitable number for the first of his parts of 2, and of course $\frac{1}{12}$ of the denominator 19. The other parts, which must be aliquot parts, are easily found, for the simple reason that the reckoner has a set of tables, constructed perhaps for this very purpose, namely the *skm*-tables, Nos. 21–23 below. He there finds that to complete 2 from $1\frac{1}{2} + \frac{1}{12}$ we must add $\frac{1}{4} + \frac{1}{6}$. Two short multiplications show that $\frac{1}{4}$ is $\frac{1}{76}$ th of 19 and $\frac{1}{6}$ is $\frac{1}{114}$ th of it.

In this case then the proof not only shows the correctness of the result but gives us a clue as to how the result was actually obtained. The method was to write down the denominator, to begin by halving or taking two-thirds of it, and then continuing to halve until a number greater than unity (often greater than $1\frac{1}{2}$) but less than 2 was arrived at. If this could by the use of *skm*-tables be made up to 2 by the addition of two or three aliquot parts, the problem was solved; if not, another trial must be made, starting this time by taking $\frac{2}{3}$ instead of $\frac{1}{2}$, or *vice versa*. Neither Cantor nor Eisenlohr in their elaborate analysis of this table has realized how much the Egyptians were here at the mercy of their inability to divide by numbers other than 2, 10 and 3, this last only through multiplication by $\frac{2}{3}$. If the denominators of the first aliquot parts into which the 2-fractions with prime denominators from 11 to 97 are resolved be examined it will be found that with two exceptions, 42 and 56, they contain only 2, 3 and 10 as factors. The two multiples of 7, namely 42 and 56, merely serve to emphasize the complete dependence of the table on trial, and its lack of regularity. Cantor is doubtless right in insisting that it was a gradual empirical accumulation.

The following table will enable both the results and the method employed in obtaining them to be seen at a glance. In the first column is the fraction to be resolved, in the second are the parts into which its numerator 2 is resolved, and in the third the solution.

¹ See, however, p. 16.

	RESOLUTION OF 2.	ANSWER.	Division of 2.
2 ÷ 5	$1\frac{2}{3} \cdot \frac{1}{3}$	$\frac{1}{3} + \frac{1}{15}$	
7	$1\frac{3}{4} \cdot \frac{1}{4}$	$\frac{1}{4} + \frac{1}{28}$	
9	$1\frac{1}{2} \cdot \frac{1}{2}$	$\frac{1}{6} + \frac{1}{18}$	
11	$1\frac{2}{3} + \frac{1}{6} \cdot \frac{1}{6}$	$\frac{1}{6} + \frac{1}{66}$	
13	$1\frac{1}{2} + \frac{1}{8} \cdot \frac{1}{4} \cdot \frac{1}{8}$	$\frac{1}{8} + \frac{1}{52} + \frac{1}{104}$	
15	$1\frac{1}{2} \cdot \frac{1}{2}$	$\frac{1}{10} + \frac{1}{30}$	
17	$1\frac{1}{3} + \frac{1}{12} \cdot \frac{1}{3} \cdot \frac{1}{4}$	$\frac{1}{12} + \frac{1}{51} + \frac{1}{68}$	
19	$1\frac{1}{2} + \frac{1}{12} \cdot \frac{1}{4} \cdot \frac{1}{6}$	$\frac{1}{12} + \frac{1}{76} + \frac{1}{114}$	
21	$1\frac{1}{2} + \frac{1}{2}$	$\frac{1}{14} + \frac{1}{42}$	
23	$1\frac{2}{3} + \frac{1}{4} \cdot \frac{1}{12}$	$\frac{1}{12} + \frac{1}{276}$	
25	$1\frac{2}{3} \cdot \frac{1}{3}$	$\frac{1}{15} + \frac{1}{75}$	
27	$1\frac{1}{2} \cdot \frac{1}{2}$	$\frac{1}{18} + \frac{1}{54}$	
29	$1\frac{1}{6} + \frac{1}{24} \cdot \frac{1}{2} \cdot \frac{1}{6} \cdot \frac{1}{8}$	$\frac{1}{24} + \frac{1}{58} + \frac{1}{174} + \frac{1}{232}$	
31	$1\frac{1}{2} + \frac{1}{20} \cdot \frac{1}{4} \cdot \frac{1}{5}$	$\frac{1}{20} + \frac{1}{124} + \frac{1}{155}$	
33	$1\frac{1}{2} \cdot \frac{1}{2}$	$\frac{1}{22} + \frac{1}{66}$	
35	$1\frac{1}{6} \cdot \frac{2}{3} + \frac{1}{6}$	$\frac{1}{30} + \frac{1}{42}$	
37	$1\frac{1}{2} + \frac{1}{24} \cdot \frac{1}{3} \cdot \frac{1}{8}$	$\frac{1}{24} + \frac{1}{111} + \frac{1}{296}$	
39	$1\frac{1}{2} \cdot \frac{1}{2}$	$\frac{1}{26} + \frac{1}{78}$	
41	$1\frac{2}{3} + \frac{1}{24} \cdot \frac{1}{6} \cdot \frac{1}{8}$	$\frac{1}{24} + \frac{1}{246} + \frac{1}{328}$	
43	$1\frac{1}{42} \cdot \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{7}$	$\frac{1}{42} + \frac{1}{86} + \frac{1}{129} + \frac{1}{301}$	
45	$1\frac{1}{2} \cdot \frac{1}{2}$	$\frac{1}{30} + \frac{1}{90}$	
47	$1\frac{1}{2} + \frac{1}{15} \cdot \frac{1}{3} \cdot \frac{1}{10}$	$\frac{1}{30} + \frac{1}{141} + \frac{1}{470}$	
49	$1\frac{1}{2} + \frac{1}{4} \cdot \frac{1}{4}$	$\frac{1}{28} + \frac{1}{196}$	
51	$1\frac{1}{2} \cdot \frac{1}{2}$	$\frac{1}{34} + \frac{1}{102}$	
53	$1\frac{2}{3} + \frac{1}{10} \cdot \frac{1}{6} \cdot \frac{1}{15}$	$\frac{1}{30} + \frac{1}{318} + \frac{1}{795}$	
55	$1\frac{2}{3} + \frac{1}{6} \cdot \frac{1}{6}$	$\frac{1}{30} + \frac{1}{330}$	
57	$1\frac{1}{2} \cdot \frac{1}{2}$	$\frac{1}{38} + \frac{1}{114}$	
59	$1\frac{1}{2} + \frac{1}{12} + \frac{1}{18} \cdot \frac{1}{4} \cdot \frac{1}{9}$	$\frac{1}{36} + \frac{1}{236} + \frac{1}{531}$	
61	$1\frac{1}{2} + \frac{1}{40} \cdot \frac{1}{4} \cdot \frac{1}{8} \cdot \frac{1}{10}$	$\frac{1}{40} + \frac{1}{244} + \frac{1}{488} + \frac{1}{610}$	
63	$1\frac{1}{2} \cdot \frac{1}{2}$	$\frac{1}{42} + \frac{1}{126}$	
65	$1\frac{2}{3} \cdot \frac{1}{3}$	$\frac{1}{39} + \frac{1}{195}$	
67	$1\frac{1}{2} + \frac{1}{8} + \frac{1}{20} \cdot \frac{1}{5} \cdot \frac{1}{8}$	$\frac{1}{40} + \frac{1}{335} + \frac{1}{736}$	
69	$1\frac{1}{2} \cdot \frac{1}{2}$	$\frac{1}{46} + \frac{1}{138}$	
71	$1\frac{1}{2} + \frac{1}{4} + \frac{1}{40} \cdot \frac{1}{8} \cdot \frac{1}{10}$	$\frac{1}{40} + \frac{1}{368} + \frac{1}{710}$	
73	$1\frac{1}{6} + \frac{1}{20} \cdot \frac{1}{3} \cdot \frac{1}{4} \cdot \frac{1}{5}$	$\frac{1}{60} + \frac{1}{219} + \frac{1}{292} + \frac{1}{365}$	
75	$1\frac{1}{2} \cdot \frac{1}{2}$	$\frac{1}{50} + \frac{1}{150}$	
77	$1\frac{1}{2} + \frac{1}{4} \cdot \frac{1}{4}$	$\frac{1}{44} + \frac{1}{308}$	
79	$1\frac{1}{4} + \frac{1}{15} \cdot \frac{1}{3} \cdot \frac{1}{4} \cdot \frac{1}{10}$	$\frac{1}{60} + \frac{1}{237} + \frac{1}{316} + \frac{1}{790}$	
81	$1\frac{1}{2} \cdot \frac{1}{2}$	$\frac{1}{54} + \frac{1}{162}$	
83	$1\frac{1}{3} + \frac{1}{20} \cdot \frac{1}{4} \cdot \frac{1}{5} \cdot \frac{1}{6}$	$\frac{1}{60} + \frac{1}{332} + \frac{1}{415} + \frac{1}{498}$	
85	$1\frac{2}{3} \cdot \frac{1}{3}$	$\frac{1}{51} + \frac{1}{255}$	
87	$1\frac{1}{2} \cdot \frac{1}{2}$	$\frac{1}{58} + \frac{1}{174}$	
89	$1\frac{1}{3} + \frac{1}{10} + \frac{1}{20} \cdot \frac{1}{4} \cdot \frac{1}{6} \cdot \frac{1}{10}$	$\frac{1}{60} + \frac{1}{356} + \frac{1}{354} + \frac{1}{890}$	
91	$1\frac{1}{3} + \frac{1}{10} \cdot \frac{2}{3} + \frac{1}{30}$	$\frac{1}{70} + \frac{1}{130}$	
93	$1\frac{1}{2} \cdot \frac{1}{2}$	$\frac{1}{62} + \frac{1}{186}$	
95	$1\frac{1}{2} + \frac{1}{12} \cdot \frac{1}{4} \cdot \frac{1}{6}$	$\frac{1}{60} + \frac{1}{380} + \frac{1}{570}$	
97	$1\frac{1}{2} + \frac{1}{8} + \frac{1}{14} + \frac{1}{28} \cdot \frac{1}{7} \cdot \frac{1}{8}$	$\frac{1}{56} + \frac{1}{679} + \frac{1}{776}$	
99	$1\frac{1}{2} \cdot \frac{1}{2}$	$\frac{1}{66} + \frac{1}{198}$	
101	$1 \cdot \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{6}$	$\frac{1}{101} + \frac{1}{202} + \frac{1}{303} + \frac{1}{606}$	

Division
of 2.

In the sums which follow there are endless scribe's errors, the most usual being the omission of the tick which denotes that a certain line in a multiplication is used in the addition by which the answer is obtained (see above, p. 13), and the incorrect omission or insertion of the fractional dot. This last is a common error throughout the papyrus, and in order to save space it has been corrected without further comment in these division sums.

It is further to be noted that in the earlier sums the number by which 2 is divided has the unity dot in front of it, e.g. 23, indicating that it is to be used again as the first line of the multiplication which follows, 1 23. After the sum $2 \div 29$ this dot is almost always omitted, but it is undoubtedly to be understood, and for the sake of clearness we have inserted it in rendering.

DIVIDE 2 by 3. (Pl. A.)

2 is $\frac{2}{3}$ rds.

No proof is necessary or indeed possible.

DIVIDE 2 by 5. (Pl. A.)

$1\frac{2}{3}$ is $\frac{1}{3}$ rd, $\frac{1}{3}$ is $\frac{1}{15}$ th.

Working out:

$$\begin{array}{r} 1 \qquad 5 \\ \frac{2}{3} \qquad 3\frac{1}{3} \\ \hline \frac{1}{3} \qquad 1\frac{2}{3}^1 \\ \hline \frac{1}{15} \qquad \frac{1}{3} \end{array}$$

As this sum lies in the top register of the papyrus the rubrics *nīs* ... *hnt* "Divide ... by" and *ššmt* "Working out" are inserted. Throughout the table they are omitted except in such sums as begin a new page. They must be regarded as repeated with every sum.

The answer to the sum is $\frac{1}{3} + \frac{1}{15}$, which is stated by implication in the second line. Note that $\frac{1}{3}$ of 5 is reached through $\frac{2}{3}$, as always. For the division see above, pp. 13 and 20.

DIVIDE 2 by 7. (Pl. A.)

$1\frac{3}{4}$ is $\frac{1}{4}$ th, $\frac{1}{4}$ is $\frac{1}{28}$ th.

Working out:

$$\begin{array}{r} 1 \qquad 7 \\ \frac{1}{2} \qquad 3\frac{1}{2} \\ \hline \frac{1}{4} \qquad 1\frac{1}{2} + \frac{1}{4} \\ \hline \frac{1}{28}^2 \qquad \frac{1}{4} \end{array} \qquad \begin{array}{r} 1 \qquad 7 \\ 2 \qquad 14 \\ \hline 4 \qquad 28 \end{array}$$

The figure 4 in front of the multiplier $\frac{1}{28}$ indicates that 4 is the number by which 7 must be multiplied to produce the required 28. This is proved in the short piece of working on the right.

DIVIDE 2 by 9. (Pl. A.)

$1\frac{1}{2}$ is $\frac{1}{6}$ th, $\frac{1}{2}$ is $\frac{1}{18}$ th.

Working out:

$$\begin{array}{r} 1 \qquad 9 \\ \frac{2}{3} \qquad 6 \\ \hline \frac{1}{3} \qquad 3 \\ \hline \frac{1}{6} \qquad 1\frac{1}{2} \\ \hline \frac{1}{18} \qquad \frac{1}{2} \end{array}$$

The papyrus is damaged, but the reading certain.

¹ Wrongly given as $1\frac{1}{3}$ in the plate.

² Actually 28. The Egyptian was very inconsistent with regard to the fractional dot in these cases: two different statements of the same fact were in his mind, viz. $4 \times 7 = 28$ and the resulting $\frac{1}{4}$ is $\frac{1}{28}$ th of 7.

DIVIDE 2 by 11. (Pl. A.)

 $1\frac{2}{3} + \frac{1}{6}$ is $\frac{1}{6}$ th, $\frac{1}{6}$ is $\frac{1}{66}$ th.Division
of 2.

Working out:

[1	11	[1	1]1	
$\frac{2}{3}$	$7\frac{1}{3}$	[2	2]2	
$\frac{1}{3}$	$3\frac{2}{3}$	[4	4]4	
— $\frac{1}{6}$	$1\frac{2}{3} + \frac{1}{6}$	— 6	[6]6	$\frac{1}{6}$

This restoration is practically certain and fits the traces on the torn edges of the papyrus perfectly. Oddly enough Griffith (*P.S.B.A.*, XVI, 207) restores only the right-hand portion of the above working, and notes "this leaves the first fraction $\frac{1}{6}$ $1\frac{2}{3} \frac{1}{6}$ stupidly unexplained." The very position of this right-hand portion (left in the papyrus) shows that it was preceded by something else, *viz.* the working out of $1\frac{2}{3} + \frac{1}{6}$ is $\frac{1}{6}$ th. (*Cf.* the division of 2 by 7, or indeed any which involve the use of fractions with high denominators.)

DIVIDE 2 by 13. (Pl. A.)

 $1\frac{1}{2} + \frac{1}{8}$ is $\frac{1}{8}$ th, $\frac{1}{4}$ is $\frac{1}{32}$ nd, $\frac{1}{8}$ is $\frac{1}{104}$ th.

Working out:

	1	13
	$\frac{1}{2}$	$6\frac{1}{2}$
	$\frac{1}{4}$	$3\frac{1}{4}$
	— $\frac{1}{8}$	$1\frac{1}{2} + \frac{1}{8}$
— 4	$\frac{1}{32}$	$\frac{1}{4}$
— 8	$\frac{1}{104}$	$\frac{1}{8}$

For the 4 and the 8 in the last two lines *cf.* the division of 2 by 7 and remarks there. Note the tacit assumption that $1\frac{1}{2} + \frac{1}{8}$ added to $\frac{1}{4}$ and $\frac{1}{8}$ gives 2. Here we see an example of the usefulness of the completion tables Nos. 21-23.

DIVIDE 2 by 15. (Pl. A.)

 $1\frac{1}{2}$ is $\frac{1}{10}$, $\frac{1}{2}$ is $\frac{1}{30}$ th.

Working out:

	1	15
	— $\frac{1}{10}$	$1\frac{1}{2}$
	— $\frac{1}{30}$	$\frac{1}{2}$

DIVIDE 2 by 17. (Pl. A.)

 $1\frac{1}{3} + \frac{1}{12}$ is $\frac{1}{12}$ th, $\frac{1}{3}$ is $\frac{1}{36}$ st, $\frac{1}{4}$ is $\frac{1}{68}$ th.

Working out:

	1	17	
	$\frac{2}{3}$	$11\frac{1}{3}$	
	$\frac{1}{3}$	$5\frac{2}{3}$	
	$\frac{1}{6}$	$2\frac{1}{2} + \frac{1}{3}$	
	— $\frac{1}{12}$	$1\frac{1}{4} + \frac{1}{6}$	
		— Remainder $\frac{1}{3} + \frac{1}{4}$	
	1	$1\frac{1}{7}$	
	2	$\frac{1}{34}$	
	— 3	$\frac{1}{51}$	$\frac{1}{3}$
	— 4	$\frac{1}{68}$	$\frac{1}{4}$

Here we have a slight change in method. After the first ticked line the worker sums up his position. Of the three terms $1\frac{1}{3} + \frac{1}{12}$, $\frac{1}{3}$ and $\frac{1}{4}$ of which the answer is composed he has accounted only for the first. The $\frac{1}{3}$ and $\frac{1}{4}$ he describes as "remainder" and proceeds to deal with them separately. In the last multiplication he has turned the multiplicands from whole

Division of 2. numbers into fractions by dotting them: strictly speaking this is not quite correct, for the digits on the left are multipliers and not divisors (see footnote to $2 \div 7$ and *cf.* next sum).

Note the tacit assumption that $1\frac{1}{2} + \frac{1}{6}$ is equivalent to $1\frac{1}{3} + \frac{1}{12}$.

DIVIDE 2 by 19. (Pl. A.)

$1\frac{1}{2} + \frac{1}{4}$ is $\frac{1}{12}$ th, $\frac{1}{4}$ is $\frac{1}{76}$ th, $\frac{1}{6}$ is $\frac{1}{114}$ th.

Working out:

1	19	
$\frac{2}{3}$	$12\frac{2}{3}$	
$\frac{1}{3}$	$6\frac{1}{3}$	
$\frac{1}{6}$	$3\frac{1}{6}$	
$\frac{1}{12}$	$1\frac{1}{2} + \frac{1}{12}$	
	Remainder $\frac{1}{4} + \frac{1}{6}$	
1	19	
2	38	
— 4	76	$\frac{1}{4}$
	Remainder $\frac{1}{6}$	
1	19	
2	38	
4	76	
— 6	114	$\frac{1}{6}$

For the method compare the preceding sum.

DIVIDE 2 by 21. (Pl. A.)

$1\frac{1}{2}$ is $\frac{1}{14}$ th, $\frac{1}{2}$ is $\frac{1}{42}$ nd.

Working out:

1	21	
— $\frac{2}{3}$	14	$1\frac{1}{2}$
— 2	42	$\frac{1}{2}$

Note that the Egyptian is well aware that the inverse of $\frac{2}{3}$ is $1\frac{1}{2}$.

DIVIDE 2 by 23. (Pl. A.)

$1\frac{2}{3} + \frac{1}{4}$ is $\frac{1}{12}$ th, $\frac{1}{12}$ is $\frac{1}{276}$ th.

Working out:

1	23	
$\frac{2}{3}$	$15\frac{1}{3}$	
$\frac{1}{3}$	$7\frac{2}{3}$	
$\frac{1}{6}$	$3\frac{1}{2} + \frac{1}{3}$	
— $\frac{1}{12}$	$1\frac{1}{2} + \frac{1}{4} + \frac{1}{6}$	
	Remainder $\frac{1}{12}$	
1	23	
— 10	230	
— 2	46	
	Total $\frac{1}{276}$	$\frac{1}{12}$

Note the assumption $1\frac{2}{3} + \frac{1}{4} + \frac{1}{6} = 1\frac{2}{3} + \frac{1}{4}$

DIVIDE 2 by 25. (Pl. B.)

$1\frac{2}{3}$ is $\frac{1}{15}$ th, $\frac{1}{3}$ is $\frac{1}{75}$ th.

Working out:

1	25	
— $\frac{1}{15}$	$1\frac{2}{3}$	
— 3	$\frac{1}{75}$	$\frac{1}{3}$

DIVIDE 2 by 27. (Pl. B.)

 $1\frac{1}{2}$ is $\frac{1}{18}$ th, $\frac{1}{2}$ is $\frac{1}{54}$ th.Division
of 2.

Working out:

1	27	
$\frac{2}{3}$	$\frac{1}{18}$	$1\frac{1}{2}$
$\frac{1}{2}$	$\frac{1}{54}$	$\frac{1}{2}$

DIVIDE 2 by 29. (Pl. B.)

 $1\frac{1}{6} + \frac{1}{24}$ is $\frac{1}{24}$ th, $\frac{1}{2}$ is $\frac{1}{58}$ th, $\frac{1}{6}$ is $\frac{1}{174}$ th, $\frac{1}{8}$ is $\frac{1}{232}$ nd.

Working out:

1	29	
$\frac{1}{24}$ (sic)	$\frac{1}{58}$	$1\frac{1}{6} + \frac{1}{24}$
$\frac{1}{2}$	$\frac{1}{174}$	$\frac{1}{2}$
$\frac{1}{6}$	$\frac{1}{232}$	$\frac{1}{6}$
$\frac{1}{8}$		$\frac{1}{8}$

The multiplier 1 (a dot) with a tick is obviously an error, for these digits on the left are those by which 29 must be multiplied to give the denominators of the fractions which respectively follow them. No working is given for the step against which it stands.

DIVIDE 2 by 31. (Pl. B.)

 $1\frac{1}{2} + \frac{1}{20}$ is $\frac{1}{20}$ th, $\frac{1}{4}$ is $\frac{1}{124}$ th, $\frac{1}{5}$ is $\frac{1}{155}$ th.

Working out:

1	31	
$\frac{1}{20}$ (sic)	$\frac{1}{124}$	$1\frac{1}{2} + \frac{1}{20}$
$\frac{1}{4}$	$\frac{1}{155}$	$\frac{1}{4}$
$\frac{1}{5}$		$\frac{1}{5}$

The dot 1 in front of $\frac{1}{20}$ is an error (*cf.* last sum). The working is highly condensed as in many of the sums from here onward.

DIVIDE 2 by 33. (Pl. B.)

 $1\frac{1}{2}$ is $\frac{1}{22}$ nd, $\frac{1}{2}$ is $\frac{1}{66}$ th.

Working out:

1	33	
$\frac{2}{3}$	$\frac{1}{22}$	$1\frac{1}{2}$
$\frac{1}{2}$	$\frac{1}{66}$	$\frac{1}{2}$

DIVIDE 2 by 35. (Pl. B.)

 $1\frac{1}{6}$ is $\frac{1}{30}$ th, $\frac{2}{3} + \frac{1}{6}$ is $\frac{1}{42}$ nd.

6 7 5

Working out:

1	35	
$\frac{1}{30}$	$\frac{1}{42}$	$1\frac{1}{6}$
$\frac{1}{42}$	$\frac{2}{3} + \frac{1}{6}$	

The digits 6 (in red), 7 and 5 are unexpected here. The 7 and the 5 clearly indicate the number of sixths in $1\frac{1}{6}$ and in $\frac{2}{3} + \frac{1}{6}$ respectively, and the 6, to be read not under the 35 but at the beginning of the line containing the 7 and 5, indicates that these are actually sixths.

Division of 2. DIVIDE 2 by 37. (Pl. B.)

$1\frac{1}{2} + \frac{1}{24}$ is $\frac{1}{24}$ th, $\frac{1}{3}$ is $\frac{1}{111}$ th, $\frac{1}{8}$ is $\frac{1}{296}$ th.

Working out:

1	37
$\frac{2}{3}$	$24\frac{2}{3}$
$\frac{1}{3}$	$12\frac{1}{3}$
$\frac{1}{6}$	$6\frac{1}{6}$
$\frac{1}{12}$	$3\frac{1}{2}$
$\frac{1}{24}$	$1\frac{1}{2} + \frac{1}{24}$
	Remainder $\frac{1}{3} + \frac{1}{8}$
1	37
2	74
3	111 $\frac{1}{3}$
	Remainder $\frac{1}{8}$
1	37
2	74
4	148
8	296 $\frac{1}{8}$

DIVIDE 2 by 39. (Pl. B.)

$1\frac{1}{2}$ is $\frac{1}{28}$ th, $\frac{1}{2}$ is $\frac{1}{78}$ th.

Working out:

1	39
$\frac{2}{3}$	$1\frac{1}{2}$
2	$\frac{1}{2}$

DIVIDE 2 by 41. (Pl. B.)

$1\frac{2}{3} + \frac{1}{24}$ is $\frac{1}{24}$ th, $\frac{1}{6}$ is $\frac{1}{246}$ th, $\frac{1}{8}$ is $\frac{1}{328}$ th.

Working out:

1	41
$\frac{2}{3}$	$27\frac{1}{3}$
$\frac{1}{3}$	$13\frac{2}{3}$
$\frac{1}{6}$	$6\frac{2}{3} + \frac{1}{6}$
$\frac{1}{12}$	$3\frac{1}{3} + \frac{1}{12}$
$\frac{1}{24}$	$1\frac{2}{3} + \frac{1}{24}$
	Remainder $\frac{1}{6} + \frac{1}{8}$
1	41
2	82
4	164
Total 6	246 $\frac{1}{6}$
8	328 $\frac{1}{8}$

DIVIDE 2 by 43. (Pl. B.)

$1\frac{1}{42}$ is $\frac{1}{42}$ nd, $\frac{1}{2}$ is $\frac{1}{86}$ th, $\frac{1}{3}$ is $\frac{1}{129}$ th, $\frac{1}{7}$ is $\frac{1}{301}$ st.

Working out:

1	43
Found $\frac{1}{42}$	$1\frac{1}{42}$
2	$\frac{1}{86}$
3	$\frac{1}{129}$
7	$\frac{1}{301}$

The sign here translated "found" is the *gmī*-bird, an abbreviation for some part of the

verb "to find." An examination of the twenty-two cases of its occurrence in this section of the papyrus shows that it is used to mark a step the rough working of which is suppressed, or the result of which has been derived from some outside source. Thus it is inserted here before the division of 43 by 42, and later before that of 53 by 30, the result of which is $1\frac{2}{3} + \frac{1}{10}$. Griffith notes that it is sometimes omitted when we most expect it, *e.g.* in $25 \div 15 = 1\frac{2}{3}$ (division of 2 by 25), in $29 \div 24 = 1\frac{1}{6} + \frac{1}{24}$ (division of 2 by 29). Conversely we are surprised to find it in $62 \div 93 = \frac{2}{3}$ and $66 \div 99 = \frac{2}{3}$, for to the Egyptian the taking of $\frac{2}{3}$ is as straightforward a process as taking an aliquot part. In these cases the symbol perhaps indicates that recourse has been had to tables.

DIVIDE 2 by 45. (Pl. B.)

$1\frac{1}{2}$ is $\frac{1}{30}$ th, $\frac{1}{2}$ is $\frac{1}{90}$ th.

Working out:

	1	45
— $\frac{2}{3}$	$\frac{1}{30}$	$1\frac{1}{2}$
— 2	$\frac{1}{90}$	$\frac{1}{2}$

DIVIDE 2 by 47. (Pl. B.)

$1\frac{1}{2} + \frac{1}{15}$ is $\frac{1}{30}$ th, $\frac{1}{3}$ is $\frac{1}{141}$ st, $\frac{1}{10}$ is $\frac{1}{470}$ th.

Working out:

	1	47
Found	$\frac{1}{30}$	$1\frac{1}{2} + \frac{1}{15}$
— 3	$\frac{1}{141}$	$\frac{1}{3}$
— 10	$\frac{1}{470}$	$\frac{1}{10}$

DIVIDE 2 by 49. (Pl. B.)

$1\frac{1}{2} + \frac{1}{4}$ is $\frac{1}{28}$ th, $\frac{1}{4}$ is $\frac{1}{196}$ th.

Working out:

	1	49
Found	$\frac{1}{28}$	$1\frac{1}{2} + \frac{1}{4}$
— 4	$\frac{1}{196}$	$\frac{1}{4}$

DIVIDE 2 by 51. (Pl. B.)

$1\frac{1}{2}$ is $\frac{1}{34}$ th, $\langle \frac{1}{2} \rangle$ is $\frac{1}{102}$ nd.

Working out:

	1	51
— $\frac{2}{3}$	34	$1\frac{1}{2}$
— 2	102	$\frac{1}{2}$

DIVIDE 2 by 53. (Pl. B.)

$1\frac{2}{3} + \frac{1}{10}$ is $\frac{1}{30}$ th, $\frac{1}{6}$ is $\frac{1}{318}$ th, $\frac{1}{15}$ is $\frac{1}{795}$ th.

Working out:

	1	53
Found	$\frac{1}{30}$	$1\frac{2}{3} + \frac{1}{10}$
— 6	$\frac{1}{318}$	$\frac{1}{6}$
		Remainder $\frac{1}{15}$
	1	53
— 10	530	
— 5	265	
Total	15	$\frac{1}{795}$ $\frac{1}{15}$

There is a dot (= multiplier 1) after "found" which should not be there. Cf. $2 \div 29$ and $2 \div 31$.

Division of 2. DIVIDE 2 by 55. (Pl. B.)

$1\frac{2}{3} + \frac{1}{6}$ is $\frac{1}{30}$ th, $\frac{1}{6}$ is $\frac{1}{330}$ th.

Working out:

	1	55
Found	$\frac{1}{30}$	$1\frac{2}{3} + \frac{1}{6}$
— 6	$\frac{1}{330}$	$\frac{1}{6}$

DIVIDE 2 by 57. (Pl. C.)

$1\frac{1}{2}$ is $\frac{1}{36}$ th, $\frac{1}{2}$ is $\frac{1}{114}$ th.

Working out:

	1	57	
	$\frac{2}{3}$	38	$1\frac{1}{2}$
— 2	114		$\frac{1}{2}$

DIVIDE 2 by 59. (Pl. C.)

$1\frac{1}{2} + \frac{1}{12} + \frac{1}{18}$ is $\frac{1}{36}$ th, $\frac{1}{4}$ is $\frac{1}{236}$ th, $\frac{1}{9}$ is $\frac{1}{531}$ st

Working out:

	1	59
Found	$\frac{1}{36}$	$1\frac{1}{2} + \frac{1}{12} + \frac{1}{18}$
— 4	$\frac{1}{236}$	$\frac{1}{4}$
— 9	$\frac{1}{531}$	$\frac{1}{9}$

DIVIDE 2 by 61. (Pl. C.)

$1\frac{1}{2} + \frac{1}{40}$ is $\frac{1}{40}$ th, $\frac{1}{4}$ is $\frac{1}{244}$ th, $\frac{1}{8}$ is $\frac{1}{488}$ th, $\frac{1}{10}$ is $\frac{1}{610}$ th.

Working out:

	1	61
Found	$\frac{1}{40}$	$1\frac{1}{2} + \frac{1}{40}$
— 4	$\frac{1}{244}$	$\frac{1}{4}$
— 8	$\frac{1}{488}$	$\frac{1}{8}$
— 10	$\frac{1}{610}$	$\frac{1}{10}$

DIVIDE 2 by 63. (Pl. C.)

$1\frac{1}{2}$ is $\frac{1}{42}$ nd, $\frac{1}{2}$ is $\frac{1}{126}$ th.

Working out:

	1	63
	$\frac{2}{3}$	$1\frac{1}{2}$
— 2	$\frac{1}{126}$	$\frac{1}{2}$

DIVIDE 2 by 65. (Pl. C.)

$1\frac{2}{3}$ is $\frac{1}{39}$ th, $\frac{1}{3}$ is $\frac{1}{195}$ th.

Working out:

	1	65
Found	$\frac{1}{39}$	$1\frac{2}{3}$
— 3	$\frac{1}{195}$	$\frac{1}{3}$

DIVIDE 2 by 67. (Pl. C.)

$1\frac{1}{2} + \frac{1}{8} + \frac{1}{20}$ is $\frac{1}{40}$ th, $\frac{1}{5}$ is $\frac{1}{335}$ th, $\frac{1}{8}$ is $\frac{1}{336}$ th.

Working out:

	1	67
Found	$\frac{1}{40}$	$1\frac{1}{2} + \frac{1}{8} + \frac{1}{20}$
— 5	$\frac{1}{335}$	$\frac{1}{5}$
— 8	$\frac{1}{536}$	$\frac{1}{8}$

DIVIDE 2 by 69. (Pl. C.)

 $1\frac{1}{2}$ is $\frac{1}{46}$ th, $\frac{1}{2}$ is $\frac{1}{138}$ th.

Working out:

	1	69
$\frac{2}{3}$	$\frac{1}{46}$	$1\frac{1}{2}$
2	$\frac{1}{138}$	$\frac{1}{2}$

DIVIDE 2 by 71. (Pl. C.)

 $1\frac{1}{2} + \frac{1}{4} + \frac{1}{40}$ is $\frac{1}{40}$ th, $\frac{1}{8}$ is $\frac{1}{568}$ th, $\frac{1}{10}$ is $\frac{1}{710}$ th.

Working out:

	1	71
Found	$\frac{1}{40}$	$1\frac{1}{2} + \frac{1}{4} + \frac{1}{40}$
8	$\frac{1}{568}$	$\frac{1}{8}$
10	$\frac{1}{710}$	$\frac{1}{10}$

DIVIDE 2 by 73. (Pl. C.)

 $1\frac{1}{6} + \frac{1}{20}$ is $\frac{1}{60}$ th, $\frac{1}{3}$ is $\frac{1}{219}$ th, $\frac{1}{4}$ is $\frac{1}{292}$ th, $\frac{1}{5}$ is $\frac{1}{365}$ th.

Working out:

	1	73
Found	$\frac{1}{60}$	$1\frac{1}{6} + \frac{1}{20}$
3	$\frac{1}{219}$	$\frac{1}{3}$
4	$\frac{1}{292}$	$\frac{1}{4}$
5	$\frac{1}{365}$	$\frac{1}{5}$

DIVIDE 2 by 75. (Pl. C.)

 $1\frac{1}{2}$ is $\frac{1}{50}$ th, $\frac{1}{2}$ is $\frac{1}{150}$ th.

Working out:

	1	75
$\frac{2}{3}$	$\frac{1}{50}$	$1\frac{1}{2}$
2	$\frac{1}{150}$	$\frac{1}{2}$

DIVIDE 2 by 77. (Pl. C.)

 $1\frac{1}{2} + \frac{1}{4}$ is $\frac{1}{44}$ th, $\frac{1}{4}$ is $\frac{1}{308}$ th.

Working out:

	1	77
Found	$\frac{1}{44}$	$1\frac{1}{2} + \frac{1}{4}$
4	$\frac{1}{308}$	$\frac{1}{4}$

DIVIDE 2 by 79. (Pl. C.)

 $1\frac{1}{4} + \frac{1}{15}$ is $\frac{1}{60}$ th, $\frac{1}{3}$ is $\frac{1}{237}$ th, $\frac{1}{4}$ is $\frac{1}{316}$ th, $\frac{1}{10}$ is $\frac{1}{790}$ th.

Working out:

	1	79
Found	$\frac{1}{60}$	$1\frac{1}{4} + \frac{1}{15}$
3	$\frac{1}{237}$	$\frac{1}{3}$
4	$\frac{1}{316}$	$\frac{1}{4}$
10	$\frac{1}{790}$	$\frac{1}{10}$

In the working read $\frac{1}{237}$ and $\frac{1}{316}$.

DIVIDE 2 by 81. (Pl. C.)

 $1\frac{1}{2}$ is $\frac{1}{54}$ th, $\frac{1}{2}$ is $\frac{1}{162}$ th.

Working out:

	1	81
$\frac{2}{3}$	$\frac{1}{54}$	$1\frac{1}{2}$
2	$\frac{1}{162}$	$\frac{1}{2}$

Division
of 2.

Division of 2. DIVIDE 2 by 83. (Pl. C.)

$1\frac{1}{3} + \frac{1}{20}$ is $[\frac{1}{60}]$ th, $\frac{1}{4}$ is $\frac{1}{332}$ nd, $\frac{1}{5}$ is $\frac{1}{415}$ th, $\frac{1}{6}$ is $\frac{1}{498}$ th.

Working out:

	1	83
Found	$\frac{1}{60}$	$1\frac{1}{3} + \frac{1}{20}$
$\diagdown 4$	$\frac{1}{332}$	$\frac{1}{4}$
$\diagdown 5$	$\frac{1}{415}$	$\frac{1}{5}$
$\diagdown 6$	$\frac{1}{498}$	$\frac{1}{6}$

DIVIDE 2 by 85. (Pl. C.)

$1\frac{2}{3}$ is $\frac{1}{51}$ st, $\frac{1}{3}$ is $\frac{1}{255}$ th.

Working out:

	1	85
Found	$\frac{1}{51}$	$1\frac{2}{3}$
$\diagdown 3$	$\frac{1}{255}$	$\frac{1}{3}$

DIVIDE 2 by 87. (Pl. D.)

$1\frac{1}{2}$ is $\frac{1}{58}$ th, $\frac{1}{2}$ is $\frac{1}{174}$ th.

Working out:

	1	87
$\diagdown \frac{2}{3}$	$\frac{1}{58}$	$[1]\frac{1}{2}$
$\diagdown 2$	$\frac{1}{174}$	$\frac{1}{2}$

DIVIDE 2 by 89. (Pl. D.)

$[1\frac{1}{3}] + \frac{1}{10} + \frac{1}{20}$ is $\frac{1}{60}$ th, $[\frac{1}{4}]$ is $\frac{1}{356}$ th, $\frac{1}{6}$ is $\frac{1}{5314}$ th, $[\frac{1}{10}]$ is $\frac{1}{890}$ th.

Working out:

	1	89
Found	$\frac{1}{60}$	$1\frac{1}{3} + \frac{1}{10} + \frac{1}{20}$
$\diagdown 4$	$\frac{1}{356}$	$\frac{1}{4}$
$\diagdown 6$	$\frac{1}{534}$	$\frac{1}{6}$
$\diagdown 10$	$\frac{1}{890}$	$\frac{1}{10}$

DIVIDE 2 by 91. (Pl. D.)

$1\frac{1}{3} + \frac{1}{10}$ is $\frac{1}{70}$ th, $\frac{2}{3} + \frac{1}{30}$ is $\frac{1}{30}$ th.

Working out:

	1	91
Found	$\frac{1}{70}$	$1\frac{1}{3} + \frac{1}{10}$
Found	$\frac{1}{30}$	$\frac{2}{3} + \frac{1}{30}$

The working is little more than a restatement of the answer.

DIVIDE 2 by 93. (Pl. D.)

$1\frac{1}{2}$ is $\frac{1}{62}$ nd, $\frac{1}{2}$ is $\frac{1}{186}$ th.

Working out:

	1	93
Found	$\frac{1}{62}$	$1\frac{1}{2}$
$\diagdown 2$	$\frac{1}{186}$	$\frac{1}{2}$

DIVIDE 2 by 95. (Pl. D.)

$1\frac{1}{2} + \frac{1}{12}$ is $\frac{1}{60}$ th, $[\frac{1}{4}]$ is $\frac{1}{380}$ th, $[\frac{1}{6}$ is $\frac{1}{570}$ th.]¹

Working out:

	1	95
Found	$\frac{1}{60}$	$1\frac{1}{2} + \frac{1}{12}$
$\diagdown 4$	$\frac{1}{380}$	$\frac{1}{4}$
$\diagdown 6$	$\frac{1}{570}$	$\frac{1}{6}$

¹ The 5 perhaps on a B.M. fragment. See p. 48.

DIVIDE 2 by 97. (Pl. D.)

$1\frac{1}{2} + \frac{1}{8} + \frac{1}{14} + \frac{1}{28}$ is $\frac{1}{36}$ th, $\frac{1}{7}$ is $\frac{1}{679}$ th, $[\frac{1}{8}]$ is $\frac{1}{7716}$ th.

Working out:

	1	97
Found	$\frac{1}{36}$	$1\frac{1}{2} + \frac{1}{8} + \frac{1}{14} + \frac{1}{28}$
— 7	$\frac{1}{679}$	$\frac{1}{7}$
— 8	$\frac{1}{7716}$	$\frac{1}{8}$

Division
of 2.

DIVIDE 2 by 99. (Pl. D.)

$1\frac{1}{2}$ is $\frac{1}{66}$ th, $\frac{1}{2}$ is $\frac{1}{198}$ th.

Working out:

	1	99
Found	$\frac{2}{3}$	$1\frac{1}{2}$
— 2	$\frac{1}{66}$	$\frac{1}{2}$
	$\frac{1}{198}$	

[DIVIDE 2 by 101.] (Pl. D.)

1 is $[\frac{1}{101}]$ st, $\frac{1}{2}$ is $\frac{1}{202}$ nd, $[\frac{1}{3}]$ is $\frac{1}{303}$ rd, $[\frac{1}{6}]$ is $\frac{1}{606}$ th.]

Working out:

	[1	101] ¹	
— 2	[202]		
— 3	303	$\frac{1}{3}$	
— 6	606	$\frac{1}{6}$	

It has always been supposed that the division of 2 in this papyrus ran from 3 to 99. The New York fragments make it clear that in the top register in the gap in the London papyrus (see below, p. 48) stood the division of 2 by 101. Mathematically the result is surprising and disappointing, for one of the fractions in the resolution is clearly $\frac{1}{101}$, i.e. half the fraction to be resolved; in other words a type of resolution is here adopted which has been purposely avoided throughout the long table. It may be surmised from this feeble ending that the mathematician was here at the extreme range of his ability, and that the resolution-tables of this period hardly went beyond this figure.

THE GAP BETWEEN THE TWO PAPYRI. Plate E.

On Plate VII of the *B.M. Facs.* we find a vertical gap down the centre of the page. On the right we have the left-hand edge of the recto of Papyrus 10058, and on the left the right-hand edge of the recto of Papyrus 10057. It has been generally assumed that these two papyri were originally one, and that they were cut apart in antiquity. This assumption was based mainly on the fact that both are clearly by the same hand, and that on either side of the gap the ruled horizontal registers correspond almost exactly in depth. The New York fragments, as has been pointed out in the Introduction, place this beyond doubt, for we now see that in the top register above the first of the division of bread sums stood the division of 2 by 101. As the last problem on the recto of 10058 was the division of 2 by 99, it would be idle to deny that the two papyri were originally one. Fortunately the New York fragments enable us almost completely to fill up the lacuna. Before placing them, however, we must rectify some errors on the edges of the papyrus in the British Museum facsimile.

The edge of 10057 has been carelessly patched, and three alterations are needed in the facsimile. On the edge of the second horizontal register is a misplaced fragment which

¹ This step was perhaps omitted.

Gap
between
the
papyri.

must be transferred to the edge of the sixth (bottom) register. Similarly the strip on the edge of the fourth register fits, when inverted, on to the edge of part of the fifth and sixth registers. Finally, the long strip which forms the main edge of registers 1-5 and protrudes at the top should be swung a little to the left round its lowest point, so as to take the place of the blank strip which follows it in the facsimile. See Plate E.

On the other side of the gap, *i.e.* on the edge of Papyrus 10058, there are two rectifications to be made. In the fourth register from the top there is a fragment with the fractional figures for 600 and 70 in red, which must be placed one register lower, in the division of 97, where it forms part of the number $\frac{1}{570}$. On the edge of the third register is a fragment with a sign in red for a fractional number of hundreds, though what number is not quite clear. Griffith read this as 100, and would place it on the edge of the sixth register as part of $\frac{1}{98}$. This is now impossible, for New York fragments 24 and 25 must be placed here. There are only two positions left for a fragment giving fractional hundreds, one is in the top register, $\frac{1}{34}$, and the other in the fourth, $\frac{1}{570}$. Our fragment is too deep for the top register, and should therefore come from the fourth.¹ Unfortunately it is now partly concealed beneath the frame and the mounting, and it is impossible to be certain as to its reading.

Having rectified the edges on each side of the gap, we may proceed to place the New York fragments, beginning with the left-hand side, *i.e.* Papyrus 10057. The large fragment 1 fits on exactly in registers 4, 5 and 6, and fragments 16 and 17 are obvious additions to it. Fragment 4 can be completed by means of fragments 8 and 10-14.² It is clear that it comes from the top and second registers, and if we look at line 1 of the second we see that between the verb *psš* of the fragment and the numeral 1 of 10057 we need only the word *t*; for loaves, which in the other sums takes up on the average 15 mm. Fragment 4 with its additions can thus be placed within a millimetre or two.

Fragment 3 with the sign $\langle$ occurring three times is needed nowhere but in register 2 and top line of 3; on to it fit fragments 2, 7, and 15. For fragment 9, which gives 2-times and 4-times $\frac{1}{3}$, room can be found only in registers 2 or 3. In register 2 it would be out of place, for the 8-times line, which is preserved, gives not $1\frac{1}{3} + \frac{1}{3} + \frac{1}{15}$ but $\frac{2}{3} + \frac{1}{10} + \frac{1}{30}$. It must therefore come from register 3, and it proves that the sum which stood there was the division not, as Eisenlohr supposed, of 3 loaves but of 2.

The top part of fragment 6 with its rubric *ššmt* can only come from the top register, and is thus part of the division of 2 by 101. Its position can be fixed by an examination of the bottom line of the register. Here we see on fragment 6 the multiplier 2 (two dots); on the right edge of fragment 4 is the sign for $\frac{1}{2}$, and between this and the 2 we have to place only the numeral 202, which would occupy anything from 12 to 20 mm., let us say 15 as an average. Thus we can place this large fragment within about 5 mm.

Turning to Pap. 10058 and the other side of the gap we can at once place in the two bottom registers fragments 20-25. Fragment 5 with its red $\frac{1}{890}$ can only come from the top register, and fragment 19 joins it. All we now have to do to estimate the whole width of the gap is to decide the spaces between this fragment 19 and the edge of 10058, and between fragments 5 and 6 on the left, for we have already bound up fragment 6 with 10057 with a possible error of only a few millimetres. Dealing first with 19 we notice that it bears in red the 4 of the fraction $\frac{1}{34}$: to the right of this must have come the fraction $\frac{1}{4}$ in black, corresponding to the $\frac{1}{34}$ in red on Pap. 10058. It is difficult to estimate the space taken up by the missing numbers, for the spacing in the top lines of these sums is very variable. It could hardly be less than 30 mm. and perhaps not more than 35. Turning to the left side we have to supply to the left of fragment 5 the rest of the sign for 90 and the black $\frac{1}{10}$ which must have followed. As the scribe was inclined to crowd here (witness the proximity

¹ Where it is placed, with a query, in Plate E.

² Fragment 13 might conceivably have come from one of two other places, top line of division of 2 by 101 ($\frac{1}{2}$ is $\frac{1}{505}$ th) or register 4 of 10058, $\frac{1}{2}$ is $\frac{1}{570}$ th (division of 2 by 95).

of the red 4 and the black $\frac{1}{6}$ on fragment 19) the space occupied was perhaps as little as 15 mm. These signs end the sum (division of 2 by 89), and as none of the five divisions under it is likely to have extended farther to the left, we may say that this point marks the farthest extension to the left of the page of division of 2 sums. The only question now remaining is how much space the scribe left before beginning the rubric of the division of 2 by 101 and the table of tenths below it. To this it may be replied that our scribe never wastes an inch, and always begins a fresh "page" at the point where the longest line of the preceding page ends. Indeed, he sometimes anticipates this, and lets a long line from the last page cut into a new one. Here, however, his longest line of the last page was the top line, and we shall therefore be fairly safe in assuming that the rubric of the division of 2 by 101 followed immediately on the $\frac{1}{10}$ which ends the division of 2 by 89. The result of this supposition is shown in the reconstruction of Plate E. This brings the nearest gummings on 10057 and 10058 to exactly 390 mm. apart, and as this is a fair size for a page on the recto of the papyrus (see above, p. 3) it corroborates the arrangement of the fragments here proposed.

The placing of some of the minute fragments dealt with above may seem somewhat arbitrary to the general reader. It must be remembered, however, that in a mathematical papyrus there is much less choice of position for a particular sign than in a literary document, and except in cases where doubt is expressed it may be regarded as morally certain that the fragments are correctly placed, with the obvious reservation that some of the more disconnected may be a few millimetres out of position, *e.g.* 20-25.

The fragments that remain are for the most part uninspiring (Pl. E, right). Some of them other scholars will doubtless place, if anyone cares to take the trouble, but a few will remain strays, unless the examination of the fibres in the originals gives some help. The chief interest lies in fragments 31-34 which clearly contain portions of something entirely different from the division of 2 and the bread sums. On two of these, 32 and 34, we see the 3rd Singular Masculine ending *f*, *i.e.* "he" or "it," and 31 is to be restored *snn pw n* "it is a copy of," which reminds us of the introduction to the whole papyrus. These fragments might possibly come from the verso. If they must be placed on the recto it may be that they are to be connected with the mysterious *m'itt* which occurs in a baffling position in front of the division of 8 loaves.

Gap
between
the
papyri

BOOK I. ARITHMETIC.

Nos. 1-6. DIVISION OF LOAVES. Plate F.

Nos. 1-6. LEAVING the long table of the division of 2 we pass on to a series of purely arithmetical problems, to each of which Eisenlohr has given a number which for reference purposes it is clearly advisable to preserve. The first six of these are concerned with the division of certain numbers of loaves of bread among 10 men, each man's share to be expressed in aliquot parts. Thus in the division of 7 loaves among 10 men each man receives $\frac{1}{2} + \frac{1}{5}$ of a loaf.

Expressed abstractly and in modern terms these examples are the resolution of the various proper fractions whose denominators are 10 into the sum of two or more aliquot parts. From New York fragment 6 (Pl. F, top left) it is clear that they were preceded by a table which ran as follows:—

$\frac{1}{10}$	$\frac{2}{3} + [\frac{1}{30}]$
$\frac{1}{5}$	$\frac{2}{3} + \frac{1}{10} + \frac{1}{30}$
$\frac{1}{5} + \frac{1}{10}$	$\frac{2}{3} + \frac{1}{5} + \frac{1}{30}$
$\frac{1}{3} + \frac{1}{15}$	
$\frac{1}{2}$	
$\frac{1}{2} + \frac{1}{10}$	

This is nothing more than the expression in aliquot parts of the tenths from $\frac{1}{10}$ to $\frac{9}{10}$, and the results, with the exception of $\frac{3}{10}$, $\frac{4}{10}$ and $\frac{5}{10}$, are actually proved in the problems which follow.

These comprise the division between 10 men of 1, 2, 6, 7, 8 and 9 loaves respectively. It is easy to understand the omission from the series of 4 loaves, for $\frac{4}{10}$ is equivalent to $\frac{2}{5}$, which by the preceding tables can be resolved at once into $\frac{1}{3} + \frac{1}{15}$. But why omit the division of 3 loaves and include that of 1, the answer to which, $\frac{1}{10}$, is already in the required form of an aliquot part? Here we are face to face with one of those anomalies which from time to time baffle all our attempts to see things with the mind of the Egyptian mathematician. Obviously 1 loaf is not treated merely for the sake of completeness, for 3 and 4 are omitted, and it is hard to conceive a mind which, while ready to accept as obvious the reduction of $\frac{4}{10}$ to $\frac{2}{5}$ and thence to $\frac{1}{3} + \frac{1}{15}$, felt a necessity to prove by four lines of arithmetic that if ten men divided a loaf each would receive one-tenth of it. Careless omissions by the scribe may be the simple explanation.

The examples are in the main a continued application of the resolutions of 2-fractions or division of 2 with which the papyrus has been hitherto occupied. This will be clearly seen if any one of the sums be written out in full. Thus in No. 3, the division of 6 loaves among 10 men, we are first given the answer $\frac{1}{2} + \frac{1}{10}$, and the sum is proved by multiplying this by 10 and showing that the result is 6, thus:—

$$\begin{aligned}
 & \frac{1}{2} + \frac{1}{10} \\
 \text{— Twice } & (\frac{1}{2} + \frac{1}{10}) = 1 + \frac{1}{5} \\
 \text{Four times } & (\frac{1}{2} + \frac{1}{10}) = 2 + \frac{2}{5} = 2 + (\frac{1}{3} + \frac{1}{15})^1 \\
 \text{— Eight times } & (\frac{1}{2} + \frac{1}{10}) = 4 + \frac{2}{3} + \frac{2}{15} = 4 + \frac{2}{3} + (\frac{1}{10} + \frac{1}{30})^2 \\
 \text{Total, ten times } & (\frac{1}{2} + \frac{1}{10}) = 6
 \end{aligned}$$

Here it will be seen that the multipliers are invariably 2, and that in order to multiply $\frac{1}{2} + \frac{1}{10}$ by 10 it is multiplied by 2, and by 2 again, and by 2 yet again, thus giving 4-times and 8-times. The 2-times and the 8-times are then added together, giving the required 10-times. In this process the only multiplier used is 2, and, as a consequence, in the working the only fractions excepting 1-fractions (aliquot parts) met with will be 2-fractions, which by the help of the preceding tables can be at once resolved into their 1-fractions.

¹ Using the table for $\frac{2}{3}$.

² Using the table for $\frac{2}{15}$.

It is thus easy to see why these bread calculations follow directly on the tables for the Nos. 1-6. resolution of 2-fractions. But this is not quite all. In the example given above the quantities $4\frac{2}{3} + \frac{1}{10} + \frac{1}{30}$ and $1 + \frac{1}{5}$ are added together and stated to give a total of 6. In other words, a process of addition of fractions is employed. And yet no working is given, the result being simply taken for granted. Presumably the same process was used as in Nos. 7-23. In No. 4, the division of 7 loaves, we are still further mystified, for there we find:—

$$\begin{array}{rcl} & \frac{2}{3} + \frac{1}{30} & \\ \text{— Twice} & ,, & \text{is } 1\frac{1}{3} + \frac{1}{15} \\ \text{4-times} & ,, & \text{is } 2\frac{2}{3} + \frac{1}{10} + \frac{1}{30} \text{ (by table)} \\ \text{— 8-times} & ,, & \text{is } 5\frac{1}{2} + \frac{1}{10} \end{array}$$

How is the 8-times obtained from the 4-times? To our minds the simplest process would be to double each term, and the answer would be $4 + 1\frac{1}{3} + \frac{1}{5} + \frac{1}{15}$. It is true that this is equivalent to $5 + \frac{1}{2} + \frac{1}{10}$, but the equivalence costs the modern mathematician a moment's thought, and was certainly not done out of hand by the Egyptian. Notice, too, that the form in which he gives the result, namely $5 + \frac{1}{2} + \frac{1}{10}$, is far less suitable than the obvious form $5\frac{1}{2} + \frac{1}{5} + \frac{1}{15}$ for the necessary addition to the 2-times, $1\frac{1}{3} + \frac{1}{15}$. How, moreover, does the reckoner add together $5\frac{1}{2} + \frac{1}{10}$ and $1\frac{1}{3} + \frac{1}{15}$ to form 7, a process for which most of us would have to employ a common denominator, even if only mental? Beyond all doubt he was working from tables of the addition of fractions, of which none have survived to us in the papyrus, but the existence of which might easily have been inferred from Nos. 21-23 below. The substitution of $5\frac{1}{2} + \frac{1}{10}$ for $5\frac{1}{3} + \frac{1}{5} + \frac{1}{15}$ is a more difficult point. Here again we surmise that tables of equivalents were used, and in the choice of the less suitable equivalent for addition purposes we may perhaps trace what one constantly suspects in the Egyptian, an implicit belief in results obtained by trial, and a certain intellectual dishonesty in constructing a proof of them.

No. 1. (Pl. F.)

No. 1.

“Example of dividing 1 [loaf]¹ among 10 men.

[You] are to mult[iply] $\frac{1}{10}$ by 10.

The doing [as it occurs:]

$$\begin{array}{rcl} \text{— } 2 & \frac{1}{5} & \\ 4 & \frac{1}{3} + \frac{1}{15} & \\ \text{— } 8 & \frac{2}{3} + \frac{1}{10} + \frac{1}{30} & \end{array}$$

Total 1. This is the number in question.”

The method of all these sums is the same. In the second line the answer is given, and the working consists of multiplying this by 10 and showing that it gives the number of loaves to be divided.

For the phrase *irt mī hpr*, see above, pp. 23-24.

No. 2. (Pl. F.)

No. 2.

“To divide [2] loaves [among 10 men].

You are to multiply $\frac{1}{5}$ by 10].

The doing as it occurs:

$$\begin{array}{rcl} \text{— } 2 & \frac{1}{3} + \frac{1}{15} & \\ [4] & \frac{2}{3} + \frac{1}{10} + \frac{1}{30} & \\ \text{— } 8 & [1\frac{1}{3} + \frac{1}{5}] + \frac{1}{15} & \end{array}$$

[Total 2.] This is [the number in question].

¹ Square brackets [] indicate lacunae in the text. They are never used in this volume as mathematical symbols.

No. 2. Eisenlohr has restored this sum as the division of 3 loaves, the answer to which would be, according to the table of fractions which precedes these sums, $\frac{1}{5} + \frac{1}{10}$. His proof is as follows:—

$$\begin{array}{rcl} 2 \times (\frac{1}{5} + \frac{1}{10}) & = & \frac{1}{2} + \frac{1}{10} \\ 4 & , & = 1\frac{1}{5} \\ 8 & , & = 2\frac{1}{3} + \frac{1}{15} \\ \text{Total } 10 & , & = 3 \end{array}$$

Now when the Egyptian multiplies $\frac{1}{5} + \frac{1}{10}$ by 2 he should get not $\frac{1}{2} + \frac{1}{10}$ but $\frac{1}{3} + \frac{1}{5} + \frac{1}{15}$. Yet it is clear from No. 35 that the equivalence of $2(\frac{1}{5} + \frac{1}{10})$ and $\frac{1}{2} + \frac{1}{10}$ was known to him. The sum therefore might quite conceivably deal with 3 loaves. It certainly does not deal with 4 loaves, i.e. $\frac{1}{3} + \frac{1}{15}$ to each man, for we should expect $\frac{1}{30}$ as last term of the 8-times line, whereas the text preserves $\frac{1}{15}$.¹ The division of 5 loaves would not account for this $\frac{1}{15}$, which must be our guide, and we are therefore forced back on to the belief that the number divided was 3 or 2, either of which, as seen from the reconstructions, would account for the $\frac{1}{15}$. The latter is shown to be correct by the New York fragment 9, which reads:—

$$\begin{array}{l} \frac{1}{3} + \frac{1}{15} \\ \frac{2}{3} + \frac{1}{10} + \frac{1}{30} \end{array}$$

with a horizontal line marking the division of registers immediately below. The fragment would thus be in the right position for the 2-times and 4-times lines, and there can be little doubt that this is its place.² The figures show that the division is that of 2 loaves, not 3.

The divisions of 3, 4 and 5 loaves were thus not given, despite the occurrence of their answers in the table of fractions preceding the problems. We might have expected the omission of 5 and 2, which give the answers $\frac{1}{2}$ and $\frac{1}{5}$ respectively, though indeed neither is quite so obvious as the division of 1 loaf, which is given. The omission of 3 loaves is hard to explain, and may be an error on the part of the scribe.

No. 3. No. 3. (Pl. F.)

“To divide 6 loaves among [10] men.

You are to multiply $[\frac{1}{2}] + \frac{1}{10}$ by 10.

The doing as it [occurs.]

$$\begin{array}{rcl} [1 & \frac{1}{2}] + \frac{1}{10} & \\ [2 & 1] \frac{1}{5} & \\ [4 & 2] \frac{1}{3} + \frac{1}{15} & \\ 8 & 4 [\frac{2}{3} + \frac{1}{10}] + \frac{1}{30} & \\ \text{Total } 6. & \text{This is [the number in question].} & \end{array}$$

Observe that in the multiplication by four $\frac{2}{3}$ is broken up into $\frac{1}{3} + \frac{1}{15}$ by table, and in the next line $\frac{2}{15}$ is resolved into $\frac{1}{10} + \frac{1}{30}$.

No. 4. No. 4. (Pl. F.)

“To divide 7 loaves among 10 men.

You are to multiply $\frac{2}{3} + \frac{1}{30}$ by 10: result [7].

$$\begin{array}{rcl} [1 & \frac{2}{3} + \frac{1}{30}] & \\ [2 & 1\frac{1}{3}] + \frac{1}{15} & \\ 4 & 2\frac{2}{3} + \frac{1}{10} + \frac{1}{30} & \\ 8 & 5\frac{1}{2} + \frac{1}{10} & \\ \text{Total } 7 \text{ loaves.} & \text{This is it.} & \end{array}$$

¹ There is, however, some irregular multiplication in Nos. 4-6; see above.

² There is indeed no other place for it: it is, however, a trifle lower (judging by the register line) than we should have expected from the multiplier 2 on fragment 2.

Note here in the multiplication by 8 that twice $\frac{2}{3} + \frac{1}{10} + \frac{1}{30}$ is taken to be $1\frac{1}{2} + \frac{1}{10}$, a No. 4. result used also in Nos. 5 and 6, but never proved. It was probably one of those pieces of information regarding the addition of fractions which the scribe knew by heart or by table.

nt pw. In this sum and in No. 6 the phrase *m'itt pw* "That is the same" with which the proof concludes, *i.e.* that is the number of loaves which was to be divided, is replaced by *nt pw*. The meaning must be the same, and *nt* must be a word meaning "it" or "that" or "the like" or something of that kind. The phrase occurs again in *Sinuhe*, B 115 and possibly B 126,¹ and also in Ebers, 99, 5. It is difficult to catch the exact sense in the *Sinuhe* passages, but it is certain that their solution must be approached from the equivalence of *nt* with *m'itt* afforded by Rhind. The Ebers passage translates quite straightforwardly: "For its (*i.e.* the heart's) ducts (lead) to each of his limbs; this it is (*i.e.* it is the heart) that speaks through the ducts of every limb."

No. 5. (Pl. F.)

No. 5.

"To divide 8 loaves among 10 men.

You are to multiply $\frac{2}{3} + \frac{1}{10} + \frac{1}{30}$ by 10: result 8.

$$\begin{array}{rcl} 1 & \frac{2}{3} + \frac{1}{10} + \frac{1}{30} & \\ -2 & 1[\frac{1}{2} + \frac{1}{10}] & \\ [4 & 3\frac{1}{5} & \\ -8 & 6\frac{1}{5} + \frac{1}{15} & \\ \text{Total} & 8. & \text{This is the number in question.} \end{array}$$

See note on No. 4 for line 2 of the multiplication.

The word *m'itt* standing in front of line 3 (see Plate E) is a complete puzzle. It certainly has nothing to do with this sum. See notes on the unplaced New York fragments, above, p. 49.

No. 6. (Pl. F.)

No. 6.

"To divide 9 loaves among 10 men.

You are to multiply $\frac{2}{3} + \frac{1}{5} + \frac{1}{30}$ by 10.

The doing as it occurs:

$$\begin{array}{rcl} 1 & \frac{2}{3} + \frac{1}{5} + \frac{1}{30} & \\ -2 & 1\frac{2}{3} + \frac{1}{10} + \frac{1}{30} & \\ 4 & 3\frac{1}{2} + \frac{1}{10} & \\ -8 & 7\frac{1}{5} & \\ \text{Total} & 8 \text{ loaves.} & \text{This is it.} \end{array}$$

Note that the words "the doing as it occurs" have slipped out of position, standing in the text before "you are to multiply." This is doubtless due to the scribe's efforts to compress into three lines (the top register being very shallow) a sum which in his original had an entirely different arrangement.

The second line of multiplication is got from the first by the resolution of $\frac{2}{3}$ into $\frac{1}{3} + \frac{1}{15}$ and of $\frac{1}{5}$ into $\frac{1}{10} + \frac{1}{30}$.

Nos. 7-20. FIRST GROUP OF COMPLETIONS (*skm*). Plate G.

The completion-calculations fall into two distinct groups of type so different that it is surprising to find the same verb *skm* used for both. In the first group, Nos. 7-20, it is important to notice that no problem is set. The reckoner begins with some fractional quantity and adds to it two of its fractional parts, either its half and its quarter, or its third and

¹ See GARDINER, *Notes on the Story of Sinuhe*, 46 and 158.

Nos. 7-20. its two-thirds, and gives the result. Thus in No. 13 we find a calculation which in modern times we should set down as follows:—

$$\begin{aligned} a &= \frac{1}{16} + \frac{1}{112} \\ \frac{1}{2}a &= \frac{1}{32} + \frac{1}{224} \\ \frac{1}{4}a &= \frac{1}{64} + \frac{1}{448} \\ \text{Total } a(1 + \frac{1}{2} + \frac{1}{4}) &= \frac{1}{8} \end{aligned}$$

This is in effect equivalent to multiplying $\frac{1}{16} + \frac{1}{112}$ by $1\frac{1}{2} + \frac{1}{4}$. What the Egyptian actually did was to experiment with various quantities, adding either their half and their quarter, or their third and their two-thirds, and recording the result as valuable when it happened to be an aliquot part. In fact, we are here face to face with some of those actual experimental methods on which so much of Egyptian mathematics is based. We are not solving set problems but discovering by trial useful facts for future use.

The process employed in the additions is virtually that of a common denominator, and has been fully discussed in the Introduction, pp. 17-19.

The best translation for *skm* would appear to be the perfectly literal "completion." Both the simple verb *km* and its causative *skm* appear in the papyrus. The former, although it is in Egyptian used both transitively¹ and intransitively, here occurs only in the latter sense "to be complete": thus in No. 22 we find *hr km* $\frac{2}{3} \frac{1}{5} \frac{1}{15} \frac{1}{15} r 1$, "Therefore $\frac{2}{3} + \frac{1}{5} + \frac{1}{15} + \frac{1}{15}$ is complete up to unity," i.e. adds up to unity.

No. 7. No. 7. (Pl. G.)

"Example of completion.

$$\begin{array}{r} 1 \quad \frac{1}{4} + \frac{1}{28} \\ \quad \text{~} \quad \text{~} \quad 1 \\ \frac{1}{2} \quad \frac{1}{8} + \frac{1}{56} \\ \quad \text{~} \quad \text{~} \quad \frac{1}{2} \\ \frac{1}{4} \quad \frac{1}{16} + \frac{1}{112} \\ \quad \text{~} \quad \text{~} \quad \frac{1}{2} + \frac{1}{4} \quad \frac{1}{4} \\ \text{Total } \frac{1}{2} \end{array}$$

Here $\frac{1}{4} + \frac{1}{28}$ is experimented with by the addition of its half and its quarter. The Egyptian sets down the given quantity, $\frac{1}{4} + \frac{1}{28}$, preceding it by a 1 (a mere dot) as is usual in the case of a quantity which is to be operated on by multiplication or division. He then takes $\frac{1}{2}$ of this quantity, which is $\frac{1}{8} + \frac{1}{56}$, and then $\frac{1}{4}$ of it, which is $\frac{1}{16} + \frac{1}{112}$. He then adds together the original quantity or unit, its half and its quarter, and finds the result to be $\frac{1}{2}$. The addition is done by means of reduction of all the separate 1-fractions to the common denominator 28. Under each fraction is placed in red (here shown by italics)² its value in terms of this common denominator, and in view of what has been said above concerning this process it will surprise no one to find such fractional values as $\frac{1}{2}$ and $1\frac{1}{2} + \frac{1}{4}$. Finally the values, written in red, are added up and found to amount to 14. The sum of the fractions is thus $\frac{14}{28}$ or $\frac{1}{2}$.

No. 7b. No. 7b. (Pl. G.)

$$\begin{array}{r} 1 \quad \frac{1}{4} + \frac{1}{28} \\ \frac{1}{2} \quad \frac{1}{8} + \frac{1}{56} \\ \frac{1}{4} \quad \frac{1}{16} + \frac{1}{112} \\ \quad \text{~} \quad \text{~} \quad 1\frac{1}{2} + \frac{1}{4} \quad \frac{1}{4} \\ \text{Total } \frac{1}{2} \end{array}$$

This is identical with No. 7: some of the red values are omitted.

¹ E.g. Shipwrecked Sailor, 127, and Pap. Petrograd 1116 A, recto, 101.

² Only in the case of these additions by common denominator has the attempt been made to indicate red ink in the translations.

No. 8. (Pl. G.)

No. 8.

$$\begin{array}{rcl}
 1 & & \frac{1}{4} \\
 & & 1\frac{1}{2} \\
 \frac{2}{3} & & \frac{1}{6} \\
 & & 3 \\
 \frac{1}{3} & & 1\frac{1}{2} \\
 & & 1\frac{1}{2} \\
 \text{Total} & & \frac{1}{2} \\
 & & 9
 \end{array}$$

Here the mathematician experiments with the fraction $\frac{1}{4}$, which he first sets down, preceded by the figure 1, to show that it is the unit which he is about to multiply or divide. He then takes two-thirds of it, which his tables tell him to be $\frac{1}{6}$, and then one-third, reached as always by halving two-thirds. He now adds the original fraction $\frac{1}{4}$ and the two obtained from it, using not the expected common denominator 12, but 18. Under each fraction is written in red its value in terms of the common denominator 18, which in two cases is not a whole number. The three values add up to .9, and since nine-eighteenths is $\frac{1}{2}$, the sum of the three fractions is $\frac{1}{2}$.¹

No. 9. (Pl. G.)

No. 9.

$$\begin{array}{rcl}
 1 & & \frac{1}{2} + \frac{1}{10} \\
 \frac{1}{2} & & \frac{1}{4} + \frac{1}{20} \\
 \frac{1}{4} & & \frac{1}{8} + \frac{1}{30} \\
 \text{Total} & & 1
 \end{array}$$

The quantity here experimented on is $\frac{1}{2} + \frac{1}{10}$. Its half and its quarter are added to it, and the result is stated to be 1. This is clearly incorrect. If the original fraction had been $\frac{1}{2} + \frac{1}{14}$ the answer would, however, have been right: the second fraction in the second line would have been $\frac{1}{28}$ and that in the third line $\frac{1}{36}$. Perhaps some inkling of the truth was in the writer's mind when he wrote $\frac{1}{30}$ instead of $\frac{1}{40}$. That he was not happy about the calculation is indicated by his failure to prove his result by the common denominator method, the red numerators under each fraction never having been inserted.

No. 10. (Pl. G.)

No. 10.

$$\begin{array}{rcl}
 1 & & \frac{1}{4} + \frac{1}{28} \\
 \frac{1}{2} & & \frac{1}{7} \\
 \frac{1}{4} & & 9 \\
 \text{Total} & & \frac{1}{2}
 \end{array}$$

In halving $\frac{1}{4} + \frac{1}{28}$ use has been made of the fact, known from the table of resolution of 2-fractions, that this quantity is equivalent to $\frac{2}{7}$. In the third line there is a gross error in the halving of $\frac{1}{7}$, the 2 and the 7 having been added instead of multiplied, giving 9 instead of $\frac{1}{14}$. The total is nevertheless correct, and the error is clearly due to a stupid copyist. Cf. No. 11.

No. 11. (Pl. G.)

No. 11.

$$\begin{array}{rcl}
 1 & & \frac{1}{7} \\
 \frac{1}{2} & & \frac{1}{9} \quad \frac{1}{14} \\
 \frac{1}{4} & & \frac{1}{18} \\
 \text{Total} & & \frac{1}{4}
 \end{array}$$

Here we have once more the error $7 \times 2 = 9$, but it has been noticed and the correct $\frac{1}{14}$ is placed after it in much lighter ink. The error, however, persists into the next line, where we find $\frac{1}{18}$ for $\frac{1}{28}$. Despite faulty working the total is given correctly. Cf. No. 10.

¹ The ignoring of the fact that to add to a number its third and its two-thirds is equivalent to doubling it is a further testimony to the experimental nature of these calculations.

No. 12. No. 12. (Pl. G.)

$$\begin{array}{r}
 1 \\
 \frac{1}{2} \\
 \frac{1}{4} \\
 \text{Total}
 \end{array}
 \begin{array}{r}
 \frac{1}{9} \\
 \frac{1}{28} \\
 \frac{1}{36} \\
 \frac{1}{8}
 \end{array}
 \begin{array}{r}
 \frac{1}{14}
 \end{array}$$

Here the greatest confusion seems to reign between multiples of 7 and those of 9. In order to give the correct sum, $\frac{1}{8}$, the series should be $\frac{1}{14}$, $\frac{1}{28}$, $\frac{1}{36}$. The correction $\frac{1}{14}$ is in lighter ink: in the second line the original reading was $\frac{1}{18}$, but this was altered to $\frac{1}{28}$ in the same light ink.

No. 13. No. 13. (Pl. G.)

$$\begin{array}{r}
 1 \\
 \frac{1}{2} \\
 \frac{1}{4} \\
 \text{Total}
 \end{array}
 \begin{array}{r}
 \frac{1}{16} + \frac{1}{112} \\
 \frac{1}{2} + \frac{1}{4} \\
 \frac{1}{32} + \frac{1}{224} \\
 \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \\
 \frac{1}{64} + \frac{1}{448} \\
 \frac{1}{4} + \frac{1}{8} + \frac{1}{16}
 \end{array}
 \begin{array}{r}
 \frac{1}{4} \\
 \frac{1}{8} \\
 \frac{1}{8} \\
 \frac{1}{8} \\
 \frac{1}{16} \\
 \frac{1}{16}
 \end{array}$$

The quantity $\frac{1}{16} + \frac{1}{112}$ is operated on, and it is found that when its half and its quarter are added to it it becomes $\frac{1}{8}$. The result is proved by reducing all the fractions to the common denominator 28, instead of to the L.C.M. 448. The values of the fractions so reduced are inserted in red, and their sum is seen on addition to be $3\frac{1}{2}$. Although these values are all fractional they involve no fractions save $\frac{1}{2}$ and its powers.

No. 14. No. 14. (Pl. G.)

$$\begin{array}{r}
 1 \\
 \frac{1}{2} \\
 \frac{1}{4} \\
 \text{Total}
 \end{array}
 \begin{array}{r}
 \frac{1}{18} \\
 1 \\
 \frac{1}{36} \\
 \frac{1}{2} \\
 \frac{1}{72} \\
 \frac{1}{4} \\
 \frac{1}{6}
 \end{array}$$

The fraction $\frac{1}{18}$ is here operated on, and it is stated that when its half and its quarter are added to it the result is $\frac{1}{6}$. This is manifestly false. The common denominator used in the addition is 18.

No. 15. No. 15. (Pl. G.)

$$\begin{array}{r}
 1 \\
 \frac{1}{2} \\
 \frac{1}{4} \\
 \text{Total}
 \end{array}
 \begin{array}{r}
 \frac{1}{32} + \frac{1}{228} \\
 \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \\
 \frac{1}{64} + \frac{1}{436} \\
 \frac{1}{4} + \frac{1}{8} + \frac{1}{16} \\
 \frac{1}{128} + \frac{1}{912} \\
 \frac{1}{8} + \frac{1}{16} + \frac{1}{32}
 \end{array}
 \begin{array}{r}
 \frac{1}{8} \\
 \frac{1}{16} \\
 \frac{1}{16} \\
 \frac{1}{16} \\
 \frac{1}{32} \\
 \frac{1}{32}
 \end{array}$$

Total $\frac{1}{6}$ WRONG.

The sum states that if the quantity $\frac{1}{32} + \frac{1}{228}$ be taken and its half and its quarter added to it the result is $\frac{1}{6}$. This is untrue, but the answer would have been correct had the original quantity read $\frac{1}{32} + \frac{1}{224}$. Either the scribe or a reviser of his work was aware of the error, for against the total is placed an abbreviated form of the verb *thi* "to be wrong," "to err."

No. 16. (Pl. G.)

No. 16.

1	$\frac{1}{2}$
$\frac{2}{3}$	$\frac{1}{3}$
$\frac{1}{3}$	$\frac{1}{6}$
Total	1

The quantity here operated on is $\frac{1}{2}$, and it is shown that if two-thirds of it and one-third of it be added to it the result is unity. The example is so simple that the addition of the three fractions is achieved without the use of a common denominator. The Egyptian has again, as in No. 8, ignored the fact that to add to a quantity two-thirds of it and one-third of it is simply to add three-thirds of it, or, in other words, to double it. Note that, as always, one-third of the quantity is reached not by direct division by 3, but by halving two-thirds.

No. 17. (Pl. G.)

No. 17.

1	$\frac{1}{3}$
$\frac{2}{3}$	$\frac{1}{6} + \frac{1}{18}$
$\frac{1}{3}$	$\frac{1}{9}$
Total	$\frac{2}{3}$

Here it is shown that if the quantity $\frac{1}{3}$ be taken and two-thirds of it and then one-third of it added to it the result is $\frac{2}{3}$. Again, the Egyptian has failed to see that to add to any quantity its third and its two-thirds is equivalent to doubling it. (Cf. No. 16.)

The working shows points of interest. Thus the second line is arrived at by means of a table of multiplication of fractions, and indeed two-thirds of $\frac{1}{3}$ is actually given in the table in No. 61 as $\frac{1}{6} + \frac{1}{18}$. The third line is obtained as usual by halving the second, but in doing this use has been made of the fact that $\frac{1}{6} + \frac{1}{18}$ is $\frac{2}{9}$ (see the table of the division of 2). The fractions are added without the use of an expressed common denominator. Probably the scribe used the fact that $\frac{1}{6} + \frac{1}{18}$ is $\frac{2}{9}$, then added the $\frac{1}{9}$ and saw that $\frac{3}{9}$ was $\frac{1}{3}$.

No. 18. (Pl. G.)

No. 18.

1	$\frac{1}{6}$
$\frac{2}{3}$	$\frac{1}{9}$
$\frac{1}{3}$	$\frac{1}{18}$
Total	$\frac{1}{3}$

To $\frac{1}{6}$ are added its two-thirds and its one-third and the result is found to be $\frac{1}{3}$. The second line is clearly taken from a table of multiplication of fractions, though this particular case does not occur in the short table in No. 61. In adding use was probably made of the fact that $\frac{1}{6} + \frac{1}{18} = \frac{2}{9}$.

No. 19. (Pl. G.)

No. 19.

1	$\frac{1}{12}$
	$1\frac{1}{2}$
$\frac{2}{3}$	$\frac{1}{18}$
	1
$\frac{1}{3}$	$\frac{1}{36}$
	$\frac{1}{2}$
Total	$\frac{1}{6}$

It is here found that if $\frac{1}{12}$ be taken and its two-thirds and its one-third added to it it is "completed" to $\frac{1}{6}$. Two-thirds is reached by the use of a multiplication of fractions table, one-third by halving two-thirds. In adding the three fractions the common denominator 18 is used instead of the L.C.M. 36. The values of the fractions so reduced are inserted below them in red and add up to 3, giving three-eighths or one-sixth.

No. 20. No. 20. (Pl. G.)

$$\begin{array}{rcl}
 1 & & \frac{1}{24} \\
 & & \frac{1}{2} + \frac{1}{4} \\
 \frac{2}{3} & & \frac{1}{36} \\
 & & \frac{1}{2} \\
 \frac{1}{3} & & \frac{1}{72} \\
 & & \frac{1}{4} \\
 \text{Total} & & \frac{1}{12}
 \end{array}$$

Similar to No. 19. In adding the fractions the common denominator used is again 18.

Nos. 21-23. SECOND GROUP OF COMPLETIONS (*skm*). Plate H.

Nos. 21-23. Although these three sums come under the same heading of *skm* or "completion" as the preceding fifteen, they are in reality entirely different. Thus in No. 21 we are asked, "What completes $\frac{2}{3} + \frac{1}{15}$ to 1?" and the answer is $\frac{1}{5} + \frac{1}{15}$, or as we should say $\frac{4}{15}$. The problem then is simply "Subtract $\frac{2}{3} + \frac{1}{15}$ from 1, expressing the answer in aliquot parts." In other words, the problem dealt with is subtraction of fractions.

These examples differ in one other important respect from the preceding group. In Nos. 21-23 we are set a definite problem to solve, while in Nos. 7-20 we started out with no problem, but merely operated on certain fractional quantities and recorded the results.

No. 21. No. 21. (Pl. H.)

"It is said to you 'What completes $\frac{2}{3} + \frac{1}{15}$ into 1?'"

10 1 Total 11: remainder 4.

Reckon with 15 to find 4.

$$\begin{array}{rcl}
 1 & 15 & \\
 \frac{1}{15} & 1\frac{1}{2} & \\
 \hline \frac{1}{3} & 3 & \\
 \hline \frac{1}{15} & 1 &
 \end{array}$$

Total 4 : then $\frac{1}{5} + \frac{1}{15}$ is what must be added to it.

Therefore $\frac{2}{3} + \frac{1}{3} + \frac{1}{15} + \frac{1}{15}$ is complete up to 1."

10 3 1 1

The problem is to subtract $\frac{2}{3} + \frac{1}{15}$ from 1, expressing the result in aliquot parts. First $\frac{2}{3} + \frac{1}{15}$ are reduced to the common denominator 15, their values 10 and 1 being written below them. The result is eleven(-fifteenths) and the remainder to make up unity is four(-fifteenths). All that is now needed is to express this in aliquot parts. This is done by trial by writing down various fractional parts of 15. It is observed that $\frac{1}{5}$ of 15 is 3, and that $\frac{1}{15}$ is 1. Since 3 and 1 = 4, therefore 4 must be $\frac{1}{5} + \frac{1}{15}$ of 15, or, in other words, $\frac{4}{15} = \frac{1}{5} + \frac{1}{15}$, which is the answer required.

The proof consists of adding the four fractions by reducing them to the common denominator 15. The values are filled in beneath their respective fractions, but the writer does not deign to state in writing the fact that they add up to 15.

The words in the top left-hand corner (in *B.M.Facs.*) of the section, *viz.* :

tp n slty ky $\frac{1}{5} \frac{1}{15}$ *m wsh* "Proof. Another. $\frac{1}{5} + \frac{1}{15}$ is the amount to be added,"

cannot be fitted into this problem. It is just possible to take in *tp n slty*, as Griffith suggests, before *hr km* etc., and the position of the words favours this. But they are unnecessary, and, what is more, if as is probable (see p. 22) they mean "proof," they are unsuitable, for the following words are a statement of result rather than of proof, the proof lying in the small

No. 23. red,¹ each beneath the fraction to which it belongs. The sum of the fractions is $\frac{23\frac{1}{2}}{45}$, and the amount needed to make this to up $\frac{2}{3}$ (i.e. $\frac{30}{45}$) is $\frac{6\frac{1}{2}}{45}$, i.e. $\frac{5+1\frac{1}{2}}{45}$, i.e. $\frac{1}{9} + \frac{1}{45}$. The whole of this step is, however, omitted, and the answer is simply stated.

The proof is unusual. In order to show that the sum of the fractions is $\frac{2}{3}$, $\frac{1}{3}$ is added to them and the result shown to be unity. This addition is done in the usual way by reduction to a common denominator, viz. 45, but again the sum of the numerators, which is of course 45, is never given.

Nos. 24-38. SOLUTION BY TRIAL OF EQUATIONS OF THE FIRST DEGREE. Plates H-M.

Nos. 24-38. These problems have acquired some notoriety in mathematical circles,² for they formed at one time the centre of a controversy which to us appears so stupid and futile that we shall do little more than indicate its nature. After the appearance of Eisenlohr's commentary in 1877, Rodet wrote in the *Journal Asiatique*, 1881, 184-232 and 390-459, an article, *Les prétendus problèmes d'algèbre du manuel du calculateur égyptien*, in which he accused Eisenlohr of having wrongly attributed a knowledge of algebra to the Egyptians. Cantor³ at once replied and was followed by Eisenlohr himself⁴ and by Revillout.⁵

The fact is that Eisenlohr and Cantor had lent themselves to Rodet's attack by a most unguarded use of algebraical symbols in their treatment of these problems. The Egyptians used no such symbols, they solved the problem by the trial-method of a "false supposition" followed by a proportion, not, however, definitely formulated by them as such (see below), and it is as foolish to ask whether this is algebra as it is to ask the same question with regard to many of our processes of solving modern arithmetical problems. The matter is not one of essence but of form, a fact well known to every schoolboy when he is warned to keep clear of *writing x* in his arithmetic examination.

The problems are divisible into three groups. The first group includes Nos. 24-29, where the solution is obtained by taking a trial number, treating it in the manner prescribed in the problem and finding the correct number by proportion. Thus in No. 24 we are asked to find a quantity which when its seventh part is added to it becomes 19. We take a trial number 7, which for obvious reasons is convenient. We add its seventh part and the result is 8. We have now to work out the proportion

$$8 : 7 :: 19 : x$$

or, as the Egyptian would say, we have to divide 19 by 8 and multiply the result by 7, for he never formulates a proportion as such. No. 28 differs from the rest in the form of its working out, but it is clear that this is due to an error or an omission of some kind.

The second group includes Nos. 30 to 34, and differs from the first not in the nature of its problems, though these are a little more complicated, but in the method by which they are solved. Here instead of the trial method a direct method by division is adopted. Thus in No. 31 we are required to find a quantity such that when $\frac{2}{3}$ and $\frac{1}{2}$ and $\frac{1}{7}$ of it are added to it it becomes 33. The solution is reached by directly multiplying $1\frac{2}{3} + \frac{1}{2} + \frac{1}{7}$ to find 33. In essence this method is much the same as the trial method, the trial figure here being unity, but the arrangement of the whole gives it a very different aspect.

The third group, Nos. 35-38, is of a similar nature except that the statement of the problem is clothed in concrete language. Instead of dealing with quantity in the abstract we deal with actual amounts of corn. For the method of proof in these cases the reader is referred to the discussions of the separate problems.

¹ $11\frac{1}{2}$ erroneously in black.

² They are generally referred to in mathematical treatises as the *hau-calculations* (*h'w*).

³ *Zeitschrift für Math. und Physik*, 27, 117.

⁴ *Journal Asiatique*, 1882, 515-8.

⁵ *Revue Égyptologique*, II, 287-303.

No. 24. (Pl. H.)

No. 24.

"A quantity whose seventh part is added to it¹ becomes 19.

— 1	7
— $\frac{1}{7}$	1
1	8
— 2	16
$\frac{1}{2}$	4
— $\frac{1}{4}$	2
— $\frac{1}{8}$	1
— 1	$2 + \frac{1}{4} + \frac{1}{8}$
— 2	$4\frac{1}{2} + \frac{1}{4}$
— 4	$9\frac{1}{2}$

The doing as it occurs:—The quantity is $16\frac{1}{2} + \frac{1}{8}$
 one seventh is $2\frac{1}{4} + \frac{1}{8}$
 Total 19 "

Abstractly expressed in modern terms the problem is: "If $x + \frac{1}{7}x = 19$, find x ." A trial number is first selected, and it is precisely that which we should choose ourselves, namely 7, for the simple reason that its seventh part is an integer and thus easily obtained. Now 7 plus one-seventh of 7 amounts to 8, and all we have now to do is, as we should say, to solve the proportion

$$8 : 7 :: 19 : x.$$

or, as the Egyptian says, to divide 19 by 8 and multiply by 7.

The Egyptian working is arranged as follows:—

Step 1: The trial number 7 is set down and one-seventh of it is added to it, giving 8.

Step 2: This 8 is operated on in the usual fashion to produce 19, or, as we put it, 19 is divided by 8. The result as shown by the ticks is $2 + \frac{1}{4} + \frac{1}{8}$.

Step 3: This last quantity is multiplied by 7, giving $16\frac{1}{2} + \frac{1}{8}$.

Proof: One seventh of this quantity is taken and added to it, the result being the required 19.

The working of this proof is omitted.

The term $h^c w$, literally, as the word-sign shows, "a heap," seems to be used here as a mathematical technical term equivalent to our "quantity." It is a good example of the concrete nature of Egyptian mathematics.

No. 25. (Pl. H.)

No. 25.

"A quantity whose half is added to it becomes 16.

1	2
$\frac{1}{2}$	1
— 1	3
2	6
— 4	12
$\frac{2}{3}$	2
— $\frac{1}{3}$	1
1	$5\frac{1}{3}$
— 2	$10\frac{2}{3}$

¹ The antecedent "a quantity" being undefined, the following clause, whatever its syntactical form, may qualify it relatively. In this case the f of $hpr.f$ picks up the pre-placed subject $h^c w$.

No. 25.

The doing as it occurs:—The quantity is $10\frac{2}{3}$
 a half is $5\frac{1}{3}$
 Total 16 "

The equation here solved is $x + \frac{1}{2}x = 16$. The method is similar to that of No. 24. The trial number taken is 2. This, when its half is added to it, becomes 3. This is the first step. In the second step 16 is divided by this 3, giving $5\frac{1}{3}$, and in the third step this last is multiplied by 2. Expressed as a proportion the sum is:—

$$3 : 2 :: 16 : x.$$

No. 26. No. 26. (Pl. H.)

"A quantity whose fourth part is added to it becomes 15.

Reckon with 4: you are to make their quarter, namely 1.

Total 5

Reckon with 5 to find 15.

1	5
2	10
The result is 3.	

Multiply 3 by 4.

1	3
2	6
4	12
The result is 12.	

1	12
$\frac{1}{4}$	3
Total	15

The quantity is 12

its quarter is 3

Total 15 "

The equation here solved is $x + \frac{1}{4}x = 15$. The method is that of proportion used in the two preceding examples with two small differences in the statement. In the first place, the first step is described in full instead of being merely stated in figures; and in the second place, the proof is stated twice over, first symbolically in figures and then in words: in neither case is it called *irt mi hpr* (cf. pp. 23-4 above).

The trial number is 4, which when increased by its quarter gives 5. The proportion to be solved is thus

$$5 : 4 :: 15 : x$$

and x is found by dividing 15 by 5 and multiplying the result by 4.

Note the treatment of the 4 as a plural ("You are to make *their* quarter"), and compare SETHE, *V.Z.Z.*, 44-51.

No. 27. No. 27. (Pl. J.)

"A quantity whose fifth part is added to it becomes 21.

1	5
$\frac{1}{5}$	1
Total	6

No. 27.

— 1	6
— 2	12
— $\frac{1}{2}$	3
Total	21

— 1	$3\frac{1}{2}$
2	7
— 4	15 (<i>sic</i>)

The quantity is $17\frac{1}{2}$
 its fifth is $3\frac{1}{2}$
 Total 21 "

The equation solved is $x + \frac{1}{5}x = 21$. The trial number chosen is 5, which, when its fifth is added, becomes 6. The proportion to be worked out is:—

$$6 : 5 :: 21 : x$$

In the second step 21 is divided by 6, and in the third the resulting $3\frac{1}{2}$ is multiplied by 5. In the last line of this step 15 has been erroneously written for 14.

No. 28. (Pl. J.)

No. 28.

"Two-thirds added and one-third taken away: 10 remains.

Make one-tenth of this ten¹: the result is 1: remainder 9.

Two thirds of it, namely 6, are added to it; total 15. A third of it is 5.

It was 5 that was taken away: remainder 10.

The doing as it occurs: "

This problem is incomplete and elliptically worded. Written in full it would run as follows:—

"A number: two-thirds of it is added to it and one-third of the total is subtracted. Result 10. Find the number. Answer 9."

In algebraical terms the equation to be solved is:—

$$x + \frac{2}{3}x - \frac{1}{3}(x + \frac{2}{3}x) = 10.$$

The working given is most singular. Instead of taking a trial number, working it in the manner indicated, and finishing with a proportion as in the other examples, one-tenth of the remainder 10 is taken from it and the resulting 9 is taken as the answer. Either the scribe knew the correct answer and botched up a "working" to obtain it, or he was aware that the result of the processes indicated in the setting out was equivalent to adding on one-ninth of itself to the original number.

There has been a considerable error in copying at this point of the text. After the words *irt mi hpr* we expect the proof of the example No. 28, instead of which we find what is clearly part of the working out of an entirely different problem, No. 29. The copyist has evidently missed out both the end of 28 and the beginning of 29. It is natural to suggest that his eye wandered from the *irt mi hpr* of the first sum to that of the second, and so missed out all that lay between, but the problem is not so straightforward as this (see notes on No. 29). Whether one or more whole problems are omitted as well as the portions of 28 and 29 we have no means of ascertaining.

The two signs $\wedge$ and Δ^2 signify, as the sense shows, addition and subtraction respectively. But of what are they abbreviations? Δ doubtless stands for the verb *pri*, which is used later in the sum for subtraction. If the sign for addition represents a verb of

¹ The *B.M.Facs.* has, erroneously, 20.

² Here made to face as in the hieratic.

- No. 28. motion at all and is not a mere mathematical symbol it probably stands for *h.i*, "to go down," which is frequently used in Egyptian in contrast to *pri* in its original meaning of "to go up."¹ If, however, *pri* gets its technical sense of "to be subtracted" from the later meaning "to go out," the addition sign may be an abbreviation of *k*, "to go in." Compare the technical use of the two verbs for "income" and "outgoing" in account papyri such as Bulaq 18.

The legs-sign in the sense of "to subtract" has been discussed by Spiegelberg in his *Rechnungen aus der Zeit Setis I*, Text, 40, but though he alludes to the Rhind passage he does not note that the sign there used faces the opposite way to those quoted by him from the account papyri, or, in other words, that the sign used in the latter for "subtract" actually stands in Rhind for "add." The same sign stands in the Moscow Papyrus for "to square."

- No. 29. No. 29. (Pl. J.)

1	10
$\frac{1}{4}$	$2\frac{1}{2}$
$\frac{1}{10}$	1
Total	$13\frac{1}{2}$
$\frac{2}{3}$	9
Total	$22\frac{1}{2}$
$\frac{1}{3}$	$7\frac{1}{2}$
Total	30
$\frac{2}{3}$	20
$\frac{1}{3}$	10

It has already been noted in dealing with No. 28 that here under No. 29 we have nothing more than the latter part of a problem. The original problem must have read as follows:—

"A number: two-thirds of it are added to it, and one-third of the sum is added. A third of the total is found to be 10. Find the number." The answer is $13\frac{1}{2}$.

In algebraical terms the equation to be solved was:—

$$\frac{1}{3}\{x + \frac{2}{3}x + \frac{1}{3}(x + \frac{2}{3}x)\} = 10.$$

Of the portion preserved the first four lines down to and including the total $13\frac{1}{2}$ in red ink are part of the working out. The rest constitute the *irt m.lpr* or proof.

These first four lines contain the multiplication of 10 by $1\frac{1}{4} + \frac{1}{10}$. What is this quantity $1\frac{1}{4} + \frac{1}{10}$, and how was it obtained? Had the scribe conscientiously worked out No. 28 we should have been able to answer this question exactly; as it is, our answer must be approximate. Presumably he began with a trial figure of 1. To this he added its two-thirds, obtaining $1\frac{2}{3}$. He then took one-third of this and added it on, obtaining $2\frac{2}{3}$. Dividing this by 3 he got $\frac{20}{27}$. All that remained was to solve the proportion

$$20 : 10 :: 27 : x$$

or, in other words, to divide 27 by 20 and multiply by 10, just as in the previous examples. The division was doubtless set out as follows:—

— 1	20	
$\frac{1}{2}$	10	
— $\frac{1}{4}$	5	
— $\frac{1}{10}$	2	Result $1\frac{1}{4} + \frac{1}{10}$

The multiplication of $1\frac{1}{4} + \frac{1}{10}$ by 10 has survived, and the result is $13\frac{1}{2}$.

¹ GARDINER, *Admonitions of an Egyptian Sage*, p. 51, and the examples there quoted, where there is no implication of movement up and down, but rather in and out or to and fro.

The proof, which has survived entire, consists of three steps, firstly the addition to $13\frac{1}{2}$ No. 29. of two-thirds of it, i.e. 9, giving $22\frac{1}{2}$, secondly the addition to this last of its third part $7\frac{1}{2}$, total 30, and finally the division of 30 by three, giving the correct 10.

It would have been interesting to see how the Egyptian managed the more than usually complicated multiplications and additions of fractions in the early steps of the working out. That he did not shirk the use of a common denominator is clear both from the preceding examples and from the form of his proportion, which merely conceals the use of the improper fraction $\frac{27}{10}$.

It is curious that the scribe should have preserved a portion of the working out as well as the *irt mi hpr*, for the simplest explanation of his error in copying is to suppose that his eye wandered from *irt mi hpr* in No. 28 to the same words in No. 29 and omitted all between. Yet here we have a small fragment (four lines) of what must have lain between. A possible solution of this difficulty may be found in the fact that in the remnant left the words *irt mi hpr*, which ought to follow the fourth line, do not occur. In other words, the blunder is actually due to haplography of *irt mi hpr*, but the copyist despite this included the last four lines of the working out of No. 29 because in his prototype they were so placed that he read them as following the words *irt mi hpr* of that sum instead of preceding them. Similar instances of ambiguous position, often due to compression into horizontal registers, are not difficult to find in our own copy.

It might possibly be suggested that the whole of what is left is the working out, and that the problem read: "A number: its $\frac{1}{4}$ and its $\frac{1}{10}$ are added to it, two-thirds of this sum are added, and the total, when divided by 3, is 10. Find the number." This is practically untenable, firstly because it would give a problem of too complicated a nature, and secondly because it would not account for the use of red ink in "Total $13\frac{1}{2}$." It must also be more than a coincidence that the first four lines are exactly what we need in the last step of the working out.

No. 30. (Pl. J.)

No. 30.

"If a scribe says to thee

'10 has become $\frac{2}{3} + \frac{1}{10}$ of what?'

Let him hear:—

You are to multiply $\frac{2}{3} + \frac{1}{10}$ to find 10:

$$\begin{array}{rcl} & 1 & \frac{2}{3} + \frac{1}{10} \\ & 2 & 1\frac{1}{3} + \frac{1}{5} \\ & 4 & 3\frac{1}{3} \\ & 8 & 6\frac{1}{3} + \frac{1}{5} \\ \text{Total } 13 & & \frac{1}{30} \end{array}$$

$\frac{1}{30}$ is multiplied 23 times to find $\frac{2}{3} + \frac{1}{10}$.

Total, this quantity that says it, $13\frac{1}{2}$.

$$\begin{array}{rcl} & 1 & 13\frac{1}{2} \\ & 2 & 8\frac{2}{3} + \frac{1}{6} + \frac{1}{15} \\ & 10 & 1\frac{1}{5} + \frac{1}{10} + \frac{1}{30} \\ \text{Total } 10 & & \end{array}$$

The problem here solved is $(\frac{2}{3} + \frac{1}{10})x = 10$, and the answer is $13\frac{1}{2}$. The method of solution is to make trial multiplications of $\frac{2}{3} + \frac{1}{10}$ in the usual way. The multipliers 1, 4 and 8, totalling 13, bring us to $9\frac{29}{30}$ which is within $\frac{1}{30}$ of our goal. The working out of the addition of the fractions is not shown, and the scribe has further omitted to mark the $\frac{1}{30}$ as *d:t* or "remainder" as he should have done. All that now remains is to find what fraction of $\frac{2}{3} + \frac{1}{10}$ amounts to $\frac{1}{30}$, and since $\frac{2}{3} + \frac{1}{10}$ is $\frac{23}{30}$, a step which is completely omitted, the reply must be $\frac{1}{23}$. Or, in other words, " $\frac{1}{30}$ must be multiplied 23 times to find $\frac{2}{3} + \frac{1}{10}$."

- No. 30. The final answer is thus $13\frac{1}{3}$. This is proved by actually taking $\frac{2}{3}$ and $\frac{1}{10}$ of this, adding them, and finding that they amount to 10. It is easy to see how these two fractional parts are obtained, but their addition is omitted.

The translation of this problem offers some difficulties. It is inconsequently stated, for, though it begins "If a scribe say to thee," yet later we read "this quantity that says it is," as if the opening statement had been "If a quantity says to thee $\frac{2}{3} + \frac{1}{10}$ of me is 10; what am I?" The *f* of *sdm.f* ought to refer to the scribe, but in a somewhat similar case in No. 37 it can only refer to the quantity which speaks, and in view of the mixed nature of the statement here it may originally have done so. Grammatically we could also translate "10 has become $\frac{2}{3} + \frac{1}{10}$: what obeys (this condition)?" In other words, what quantity obeys the condition that $\frac{2}{3} + \frac{1}{10}$ of it is 10? This may indeed be the correct translation.

For the *sdm.n.f* form of *hpr.n* cf. perhaps *h.n* in Nos. 45 and 46.

- No. 31. No. 31. (Pl. J.)

"A quantity to which its two-thirds, its half and its seventh are added becomes 33.

1	$1\frac{2}{3} + \frac{1}{2} + \frac{1}{7}$		
2	$4\frac{1}{3} + \frac{1}{4}$	$+ \frac{1}{28}$	
4	$9\frac{1}{6}$	$+ \frac{1}{18}$ (read $\frac{1}{14}$)	
8	$18\frac{1}{3}$	$+ \frac{1}{7}$	
$\frac{1}{2}$	$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{14}$		
$\frac{1}{4}$	$\frac{1}{4} + \frac{1}{6}$	$+ \frac{1}{8} + \frac{1}{28}$	
Total $32\frac{1}{2}$			
Remainder $\frac{1}{2}$			

$$\begin{array}{ccccccc} \frac{1}{7} & + & \frac{1}{8} & + & \frac{1}{14} & + & \frac{1}{28} & + & \frac{1}{28} \\ 6 & 5\frac{1}{4} & 3 & 1\frac{1}{2} & 1\frac{1}{2} & & & & \\ 17\frac{1}{4} & & & & & & & & \\ \text{〈Remainder〉 } 3\frac{1}{2} & + & \frac{1}{4} & & & & & & \\ \text{(since) } \frac{1}{2} & \text{is } 21. & & & & & & & \end{array}$$

1	42
$\frac{2}{3}$	28
$\frac{1}{2}$	21
$\frac{1}{7}$	6
Total 99 (read 97).	

$\frac{1}{97}$	$\frac{1}{42}$	1
$\frac{1}{56} + \frac{1}{679} + \frac{1}{776}$	$\frac{1}{21}$	2
$\frac{1}{194}$	$\frac{1}{84}$	$\frac{1}{2}$
$\frac{1}{388}$	$\frac{1}{168}$	$\frac{1}{4}$
Total 33."		

This problem is difficult to follow, partly because the arrangement of the working is extraordinary, and still more because a portion of it has been misplaced by the scribe in No. 38. The problem is $x + \frac{2}{3}x + \frac{1}{2}x + \frac{1}{7}x = 33$. We choose for our trial value 1, and set out to divide 33 direct by $1 + \frac{2}{3} + \frac{1}{2} + \frac{1}{7}$, or rather to multiply this quantity to find 33. When the multipliers 2, 4, 8 and $\frac{1}{4}$, total $14\frac{1}{4}$, have been applied the integers and the simpler fractions in the products are added up and found to give $32\frac{1}{2}$, which is $\frac{1}{2}$ short of the required 33. This $32\frac{1}{2}$, however, does not include the smaller fractions $\frac{1}{7} + \frac{1}{8} + \frac{1}{14} + \frac{1}{28} + \frac{1}{28}$ (which we have placed to the right of a vertical line for the sake of clearness), though the scribe when he writes "Total $32\frac{1}{2}$ " gives no hint of this. In Step 2, which in the papyrus is misplaced after Step 4, these smaller fractions are added, using 42 as common denominator, and come to $\frac{17\frac{1}{4}}{42}$. In other words, $1 + \frac{2}{3} + \frac{1}{2} + \frac{1}{7}$ when multiplied by $14\frac{1}{4}$ comes to $32\frac{1}{2} + \frac{17\frac{1}{4}}{42}$.

The amount still needed to make up the $\frac{1}{2}$ by which $32\frac{1}{2}$ falls short of 33 is clearly $\frac{1}{2} - \frac{17\frac{1}{2}}{42}$ No. 31. or $\frac{3\frac{1}{2} + \frac{1}{2}}{42}$. Thus to get the *exact* quotient of our original division we must still divide $\frac{3\frac{1}{2} + \frac{1}{2}}{42}$ by $1 + \frac{2}{3} + \frac{1}{2} + \frac{1}{4}$. This can only be done by expressing both divisor and dividend in terms of some common denominator, and as the dividend is already in forty-seconds the obvious thing is to express the divisor in terms of the same, which is done in Step 3, which by an error of the copyist had strayed into No. 38. The result is $\frac{97}{42}$ and we have now only to divide $3\frac{1}{2} + \frac{1}{4}$ by 97, or in Egyptian terms, to multiply $1 + 2 + \frac{1}{2} + \frac{1}{4}$ to find 97. Thus in Step 4, where the division is actually performed, we have in the right-hand column these four numbers 1, 2, $\frac{1}{2}$ and $\frac{1}{4}$; in the left-hand column are these same numbers each divided by 97, giving $\frac{1}{97}$, $\frac{2}{97}$ (or as the Egyptian's tables told him $\frac{1}{56} + \frac{1}{679} + \frac{1}{776}$), $\frac{1}{194}$ and $\frac{1}{388}$, while in the centre column are the same numbers 1, 2, $\frac{1}{2}$ and $\frac{1}{4}$ divided by 42, which of course are the actual products of the multiplicand $1\frac{2}{3} + \frac{1}{2} + \frac{1}{4}$ with the four fractions of the first column respectively.

We now add the ticked quantities in Steps 1 and 4. On the left we get $14\frac{1}{2} + \frac{1}{97} + \frac{1}{56} + \frac{1}{679} + \frac{1}{776} + \frac{1}{194} + \frac{1}{388}$, which will be our answer; and on the right $32\frac{1}{2} + (\frac{1}{7} + \frac{1}{8} + \frac{1}{14} + \frac{1}{28} + \frac{1}{28}) + \frac{1}{42} + \frac{1}{21} + \frac{1}{84} + \frac{1}{168}$, which, as the words "Total 33," misplaced in the papyrus, show, add up to the correct 33.

We can now explain the inconsequent arrangement of the sum in the papyrus. It consists of a division which, owing to its complexity, is done in two parts, Steps 1 and 4. In order to render the final addition easy the scribe of the original XIIth Dynasty document placed Step 4 under Step 1, and probably moved Steps 2 and 3 to the left. The copyist was puzzled by this arrangement, omitted or transferred to No. 38 the necessary Step 3, and squeezed in Step 2 after Step 4, adding to it the "Total 33," which should end the sum.

No. 32. (Pl. K.)

No. 32.

"A quantity whose third and whose quarter are added to it becomes 2.

1	$1\frac{1}{3} + \frac{1}{4}$	228
$\frac{2}{3}$	$1\frac{1}{18}$	152
$\frac{1}{3}$	$\frac{1}{2} + \frac{1}{36}$	76
$\frac{1}{6}$	$\frac{1}{4} + \frac{1}{72}$	38
$\frac{1}{12}$	$\frac{1}{8} + \frac{1}{144}$	19
(sic) $\frac{1}{228}$	$\frac{1}{144}$	1
(sic) $\frac{1}{114}$	$\frac{1}{72}$	2
Total $1\frac{1}{6} + \frac{1}{12} + \frac{1}{144} + \frac{1}{228}$ is this quantity that says it.		

$\frac{2}{3}$	$\frac{2}{3} + \frac{1}{9} + \frac{1}{18} + \frac{1}{72} + \frac{1}{36}$
$\frac{1}{3}$	$\frac{1}{3} + \frac{1}{18} + \frac{1}{36} + \frac{1}{72} + \frac{1}{18}$
$\frac{1}{2}$	$[\frac{1}{2}] + \frac{1}{12} + \frac{1}{24} + \frac{1}{24} + \frac{1}{48}$
$\frac{1}{4}$	$[\frac{1}{4}] + \frac{1}{24} + \frac{1}{48} + \frac{1}{48} + \frac{1}{96}$

Insert (?)

$\frac{1}{3}$ (sic)	144	1	12
$\frac{1}{3}$	48	2	24
$\frac{1}{4}$	36	4	48
Total	228	8	96
		Total	144

Proof:

1	$1\frac{1}{6}$	$+ \frac{1}{12} + \frac{1}{144} + \frac{1}{228}$
$\frac{1}{3}$	$\frac{1}{3}$	$+ \frac{1}{18} + \frac{1}{36} + \frac{1}{72} + \frac{1}{18}$
$\frac{1}{4}$	$\frac{1}{4}$	$+ \frac{1}{24} + \frac{1}{48} + \frac{1}{48} + \frac{1}{96}$
Total	$1\frac{1}{2} + \frac{1}{4}$	
Remainder	$\frac{1}{4}$	

No. 32.

$$\frac{1}{12} + \frac{1}{14} + \frac{1}{28} + \frac{1}{8} + \frac{1}{36} + \frac{1}{54} + \frac{1}{63} + \frac{1}{24} + \frac{1}{48} + \frac{1}{56} + \frac{1}{12}$$

76 8 4 50 $\frac{2}{3}$ 25 $\frac{1}{3}$ 2 $\frac{2}{3}$ 1 $\frac{1}{3}$ 38 19 2 1

Total 228,
i.e. 1a quarter.

912

 $\frac{1}{2}$

456

 $\frac{1}{4}$

228 "

The problem is $x(1 + \frac{1}{3} + \frac{1}{4}) = 2$, and the answer $1\frac{1}{6} + \frac{1}{2} + \frac{1}{14} + \frac{1}{28}$. The method of this sum is similar to that of the last and begins with a direct multiplication (Step 1) of $1 + \frac{1}{3} + \frac{1}{4}$ to find 2. There is, however, one slight difference in that here each product as it is obtained is expressed in terms of a common denominator 144 and the result placed in a third column in the multiplication. When the multipliers $\frac{2}{3}$, $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{2}$ have been tried it is noticed that the products corresponding to them in the third column add up to 285, or just 3 short of 288, which in terms of 144ths would be the required 2. It therefore remains to divide this remaining $\frac{3}{144}$ by $1\frac{1}{3} + \frac{1}{4}$, and since this last, in terms of 144ths, is 228 (obtained in the first line) this is equivalent to dividing 3 (i.e. $2 + 1$) by 228, and the quotient is obviously $\frac{1}{144} + \frac{1}{288}$. Thus the answer to the problem is $1\frac{1}{6} + \frac{1}{2} + \frac{1}{144} + \frac{1}{288}$.

The proof (Step 2) now begins, with no heading. It consists of adding to the number just found its third and its quarter and showing the result to be 2. We get the $\frac{1}{3}$ as always through the $\frac{2}{3}$, and the $\frac{1}{4}$ through the $\frac{1}{2}$: but instead of now proceeding to add up the whole, the third and the quarter the reckoner interrupts the proof for the insertion of Step 3, two pieces of rough working used in Step 1, *viz.* the reduction of $1\frac{1}{3} + \frac{1}{4}$ to $\frac{228}{144}$ (for $\frac{1}{3}$ in the first line read 1) and the multiplication of 12 by 12, as is perfectly clear from the ticking of the multipliers 4 and 8. This last piece of work is presumably the source of the common denominator used in Step 1, but it is not easy to see why that number should be obtained as the product of 12 and 12: we have indeed no evidence of the lines on which the Egyptians chose their common denominators. The sign 𓂏 placed in front of this third step probably indicates a 'stop' for the insertion of pieces of work omitted in Step 1 (see notes on No. 70).

Step 4 is headed *tp n sity*, probably "proof." In this step the quantity found, namely $1\frac{1}{6} + \frac{1}{2} + \frac{1}{144} + \frac{1}{288}$, is added to its third and its fourth parts, which were found in Step 2, and the result is shown to be the correct 2. The method is peculiar. First the simpler quantities $1\frac{1}{6}$, $\frac{1}{3}$ and $\frac{1}{4}$ are added, total $1\frac{1}{2} + \frac{1}{4}$. It is now only necessary to show that the more complicated fractions, placed by us to the right of a vertical line, add up to the remaining $\frac{1}{4}$. This is done by reducing them all to the common denominator 912. Under each fraction is placed as usual its numerator with respect to this denominator (shown here in italic figures). The total of these numerators is 228, which a simple sum shows to be $\frac{1}{4}$ of 912.

No. 33. No. 33. (Pl. K.)

"A quantity whose two-thirds, half and seventh are added to it becomes 37.

1	$1\frac{2}{3} + \frac{1}{2} + \frac{1}{7}$
2	$4\frac{1}{3} + \frac{1}{4} + \frac{1}{28}$
4	$9\frac{1}{6} + \frac{1}{14}$
8	$18\frac{1}{3} + \frac{1}{7}$
— 16	$36\frac{2}{3} + \frac{1}{4} + \frac{1}{28}$
	28 10 $\frac{1}{2}$ 1 $\frac{1}{2}$
1	42
<i>sic</i> $\frac{2}{3}$	28
$\frac{1}{2}$	21
— $\frac{1}{4}$	10 $\frac{1}{2}$
— $\frac{1}{28}$	1 $\frac{1}{2}$
Total	40; remainder 2

$$\begin{array}{rcl}
 \langle 1 & 42 \\
 \frac{2}{3} & 28 \\
 \frac{1}{2} & 21 \\
 \frac{1}{7} & 6 \\
 \text{Total} & 99 \text{ (sic: read 97)} \rangle^1
 \end{array}$$

$$\begin{array}{rcl}
 \frac{1}{56} + \frac{1}{679} + \frac{1}{776} & \frac{1}{42} & 1 \\
 & \frac{1}{21} & 2 \\
 \text{Total} & 37
 \end{array}$$

Proof:

$$\begin{array}{rcll}
 1 & 16 & + & \frac{1}{56} + \frac{1}{679} + \frac{1}{776} \\
 & & & 97 \quad 8 \quad 7 \\
 \frac{2}{3} & 10\frac{2}{3} & + & \frac{1}{84} + \frac{1}{1358} + \frac{1}{4074} + \frac{1}{1184} \text{ (read } 1164) \\
 & & & 64\frac{2}{3} \quad 4 \quad 1\frac{1}{2} \quad 4\frac{2}{3} \\
 \frac{1}{2} & 8 & + & \frac{1}{112} + \frac{1}{1358} + \frac{1}{1552} \\
 & & & 48\frac{1}{2} \quad 4 \quad 3\frac{1}{2} \\
 \frac{1}{7} & 2\frac{1}{4} + \frac{1}{28} & + & \frac{1}{392} + \frac{1}{4753} + \frac{1}{5432} \\
 & & & 13\frac{1}{2} + \frac{1}{4} + \frac{1}{14} + \frac{1}{28} \quad 1\frac{1}{7} \quad 1 \\
 \text{(Total)} & 36 + \frac{2}{3} + \frac{1}{4} + \frac{1}{28} & ; \text{ remainder } \frac{1}{28} + \frac{1}{84} \\
 & 3621\frac{1}{3} \quad 1358 \quad 194 & & 194 \quad 64\frac{2}{3}
 \end{array}$$

$$\begin{array}{rcl}
 1 & 5432 \\
 \frac{2}{3} & 3621\frac{1}{3} \\
 \frac{1}{2} & 2716 \\
 \frac{1}{4} & 1358 \\
 \frac{1}{28} & 194 \\
 \text{Total} & 5173\frac{1}{3} ; \\
 & \text{remainder } 258\frac{2}{3}''
 \end{array}$$

The problem is $x(1 + \frac{2}{3} + \frac{1}{2} + \frac{1}{7}) = 37$, and the answer is $16 + \frac{1}{56} + \frac{1}{679} + \frac{1}{776}$. The method is as follows. In Step 1 the quantity $1 + \frac{2}{3} + \frac{1}{2} + \frac{1}{7}$ is multiplied in order to make 37. The multiplier 16 almost gives this result, producing as it does $36 + \frac{2}{3} + \frac{1}{4} + \frac{1}{28}$. In Step 2 these fractions $\frac{2}{3}$, $\frac{1}{4}$, and $\frac{1}{28}$ are added in terms of the common denominator 42 and found to give $4\frac{0}{42}$, which falls short of 1 by $\frac{2}{42}$. We have now only to multiply $1 + \frac{2}{3} + \frac{1}{2} + \frac{1}{7}$ to find this $\frac{2}{42}$. In Step 3, which must be restored to its place here from No. 38, whither it has been wrongly transferred by the scribe,² the multiplicand $1 + \frac{2}{3} + \frac{1}{2} + \frac{1}{7}$ is reduced to $\frac{97}{42}$, and to get $\frac{2}{42}$ it is clear (Step 4) that the multiplier must be $\frac{97}{97}$, or $\frac{1}{56} + \frac{1}{679} + \frac{1}{776}$, a value found from the tables earlier in the papyrus. The answer is therefore $16 + \frac{1}{56} + \frac{1}{679} + \frac{1}{776}$.

In the proof this quantity is taken as multiplicand and multiplied successively by 1, $\frac{2}{3}$, $\frac{1}{2}$ and $\frac{1}{7}$. The more complicated fractions in the product (here placed to the right of a vertical line) are reduced to common denominator 5432 and their respective values in terms of that denominator are written in in red (here in italics) under them. The whole numbers and larger fractions ($36 + \frac{2}{3} + \frac{1}{4} + \frac{1}{28}$) (to left of vertical line) are first set down and are stated to fall short of the required 37 by $\frac{1}{28} + \frac{1}{84}$ (no proof of this is given and the fact must have been drawn from tables or worked out elsewhere). We have now only to show that the fractions on the right of the vertical line add up to $\frac{1}{28} + \frac{1}{84}$. These last, when reduced to the common denominator 5432, give 194 and $64\frac{2}{3}$, the sum of which is $258\frac{2}{3}$, which will be found identical with the sum of the italicized numerators (in terms of the same denominator 5432) on the right of the vertical line. Thus the fractions on both sides of the line add up to $\frac{5432}{5432}$ or 1, and the total product is 37, as it should be.

¹ Supplied from No. 38.

² Perhaps this step may have been omitted here even in the original MS. owing to its having already occurred in No. 31, *q.v.*

- No. 33. A further check is obtained by reducing the fractions on the left of the vertical line, viz. $\frac{2}{3}$, $\frac{1}{4}$ and $\frac{1}{28}$, to the same common denominator 5432, and adding their numerators (Step 6). These come to 5173 $\frac{1}{2}$ which, when subtracted from 5432, again gives 258 $\frac{2}{3}$. In this step the multiples $\frac{2}{3}$, $\frac{1}{4}$, and $\frac{1}{28}$ ought to be marked with a tick.

No. 34. No. 34. (Pl. L.)

"A quantity whose half and whose quarter are added to it becomes 10.

$$\begin{array}{rcl}
 \text{—} 1 & 1\frac{1}{2} + \frac{1}{4} & \\
 2 & 3\frac{1}{2} & \\
 \text{—} 4 & 7 & \\
 \text{—} \frac{1}{7} & \frac{1}{4} & \\
 \frac{1}{4} + \frac{1}{28} & \frac{1}{2} & \\
 \text{—} \frac{1}{2} + \frac{1}{14} & 1 & \\
 \text{Total this quantity} & 5\frac{1}{2} + \frac{1}{7} + \frac{1}{14} &
 \end{array}$$

Proof:

$$\begin{array}{rcl}
 \text{—} 1 & 5\frac{1}{2} & + \frac{1}{7} + \frac{1}{14} \\
 \text{—} \frac{1}{2} & 2\frac{1}{2} + \frac{1}{4} & + \frac{1}{14} + \frac{1}{28} \\
 \text{—} \frac{1}{4} & 1\frac{1}{4} + \frac{1}{8} & + \frac{1}{28} + \frac{1}{56} \\
 \text{Total} & 9\frac{1}{2} + \frac{1}{8} & \\
 \text{Remainder} & \frac{1}{4} + \frac{1}{8} & \\
 \frac{1}{4} & \text{is} & 14 \\
 \frac{1}{8} & & 7 \\
 \text{Total} & 21 &
 \end{array}$$

$$\begin{array}{cccccc}
 \frac{1}{7} + \frac{1}{14} + \frac{1}{14} + \frac{1}{28} + \frac{1}{28} + \frac{1}{56} & & & & & \\
 8 & 4 & 4 & 2 & 2 & 1
 \end{array}$$

The problem is $x(1 + \frac{1}{2} + \frac{1}{4}) = 10$, and the answer is $5\frac{1}{2} + \frac{1}{7} + \frac{1}{14}$. The method is as in the two preceding examples. The quantity $1\frac{1}{2} + \frac{1}{4}$ is multiplied by trial to find 10. The exact result is achieved by the use of the multipliers 1, 4, $\frac{1}{7}$ and $\frac{1}{2} + \frac{1}{14}$. Note that the multiplier $\frac{1}{4} + \frac{1}{28}$ is simply the Egyptian way of writing $\frac{3}{7}$ (the table earlier in the papyrus being here used) and is obtained direct from $\frac{1}{7}$ by doubling.

In the proof the answer $5\frac{1}{2} + \frac{1}{7} + \frac{1}{14}$ is simply halved and quartered, the results are added to it and the whole is shown to amount to 10. For convenience, however, the whole numbers and simpler fractions, here placed on the left of a vertical line, are added first and seen to amount to $9\frac{1}{2} + \frac{1}{8}$. The quantity still needed to make up 10 is thus $\frac{1}{4} + \frac{1}{8}$, and it only remains to show that the fractions to the right of the line are equivalent to this $\frac{1}{4} + \frac{1}{8}$. This is done by reducing all the fractions to the common denominator 56. In terms of this denominator $\frac{1}{4}$ is 14 and $\frac{1}{8}$ is 7, total 21. In the last line the fractions $\frac{1}{7} + \frac{1}{14} + \frac{1}{14} + \frac{1}{28} + \frac{1}{28} + \frac{1}{56}$ are set down, and under each is placed in red (here in italics) its numerator in terms of the common denominator 56. These clearly add up to 21, and the proof is complete.

No. 35. No. 35. (Pl. L.)

"I go three times into the *hekat*-measure; my third part is then added to me and I return fully satisfied.

What is it that says this?

The doing as it occurs:

$$\begin{array}{rcl}
 \text{—} 1 & 1 & \\
 \text{—} 2 & 2 & \\
 \text{—} \frac{1}{3} & \frac{1}{3} & \\
 \text{Total} & 3\frac{1}{3} &
 \end{array}$$

You are to divide 1 by $3\frac{1}{3}$:

	1	$3\frac{1}{3}$
<i>sic</i>	$\frac{1}{10}$	$\frac{1}{3}$
<i>sic</i>	$\frac{1}{5}$	$\frac{2}{3}$
	Total	1

Proof:

	1	$\frac{1}{3} + \frac{1}{10}$
	2	$\frac{1}{2} + \frac{1}{10}$
<i>sic</i>	$\frac{1}{3}$	$\frac{1}{10}$
	Total	1

Proof:

1	320
$\frac{1}{10}$	32
$\frac{1}{5}$	64
Total	96

amounting in corn to

1	96	1	$(\frac{1}{4} + \frac{1}{32} + \frac{1}{64})$ hekat and 1 ro
2	192	2	$(\frac{1}{2} + \frac{1}{16} + \frac{1}{32})$ hekat and 2 ro
$\frac{1}{3}$	32	$\frac{1}{3}$	$(\frac{1}{16} + \frac{1}{32})$ hekat and 2 ro
Total	320	Total	1 hekat

The problem is on the same lines as Nos. 31-34, except that the amount to be made up is a concrete unit, the *hekat* of capacity. The equation to be solved is $x(3 + \frac{1}{3}) = 1$ hekat, and the answer in its final form is $(\frac{1}{4} + \frac{1}{32} + \frac{1}{64})$ hekat plus 1 ro.

In the first step, curious as it may seem to us, unity is solemnly multiplied by three and its third part is added to it, the result being $3\frac{1}{3}$. Next 1 is divided by $3\frac{1}{3}$, or, in other words, $3\frac{1}{3}$ is multiplied by various trial fractions to find 1. The fractions $\frac{1}{10}$ and $\frac{1}{5}$ (the latter from the former by doubling) are seen to give the required result, and the answer is therefore $\frac{1}{3} + \frac{1}{10}$ of a hekat.

This answer is now proved in terms of three separate units:—

- (1) In terms of pure fractions of the hekat ($\frac{1}{5}$, $\frac{1}{10}$, etc.).
- (2) In terms of ro, the ro being $\frac{1}{320}$ of a hekat (see p. 25).
- (3) In terms of the $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, etc. of a hekat, expressed in the customary Horus-eye notation (see page 25).

It is very much as if we were to solve a problem in Avoirdupois Weight, and then to test our result firstly in pure fractions of a ton, secondly in ounces, and thirdly in hundredweights, quarters, pounds, and ounces.

The first proof, marked *tp n sity*, needs little comment: it consists in multiplying $\frac{1}{3} + \frac{1}{10}$ by $3\frac{1}{3}$ and showing it to give 1. In the multiplication by 2 we expect $\frac{2}{3}$ to be resolved by the table into $\frac{1}{3} + \frac{1}{15}$, yet this is not the case, some more direct table being used which gave the result of multiplying $\frac{1}{3} + \frac{1}{10}$ by 2 in the useful form $\frac{1}{2} + \frac{1}{10}$. A tick is omitted before $\frac{1}{3}$.

The second and third proofs are arranged in parallel lines and the heading *tp n sity* doubtless refers to both. First of all the hekat is written down as 320 ro and $\frac{1}{10} + \frac{1}{5}$ of it shown to be 96 ro. This number when multiplied by $3\frac{1}{3}$ gives 320 ro, or 1 hekat. The parallel column is headed "amounting in corn to," doubtless because it was in this notation that grain was actually measured in Egypt and not in various fractions of a hekat or in ro. The amounts given in this column are certainly obtained indirectly from the numbers of ro in the parallel column, by means of the table for converting the $\frac{1}{2}$, $\frac{1}{4}$, etc. of the hekat into ro

- No. 35. and *vice versa*. Note that the first line of the column gives for the first time the answer to the problem in the form in which an Egyptian would need it for practical purposes. It must not be forgotten that to him the $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, and $\frac{1}{64}$ of a *hekat* were specific measures, each with its own name and notation, just as much as our quarter, pound, ounce, etc. See above, p. 25.

Though the sense of the problem is of the clearest its translation has been much discussed. Everything turns on the precise meaning of *hw·i mh·kw·i*. If we translate these words "I am filled" or "I am full" we make nonsense of the problem, for we then represent the unknown quantity as filling itself $3\frac{1}{3}$ times and then saying "I am filled." Griffith¹ suggested avoiding this by taking *mh·kw·i* as example of the rare survival of the old active-transitive use of the pseudo-participle, "If I go down 3 times into the *hekat* measure I fill it." But such a use, besides being a little improbable in a Middle Kingdom text (though the Rhind does contain archaisms), would require the addition of the Old Independent Pronoun *š* or *š·i* for "it."

Schack-Schackenburg² seeks to avoid the difficulty by reading *hw·kw·i* in a metaphorical sense and translating "Ich bin dreimal genommen um die Masseinheit zu erreichen, ein Drittel von mir zu mir hinzu, dann bin ich zur Einheit complettiert." He supports this metaphorical translation of *hw·kw·i* by the statement that "to go down into the *hekat* measure" in the literal sense would require the preposition *m*, *r* not being used for "into" after *hw·i* in its literal concrete meaning, at least in the Pyramid Texts. (This is untrue: see below.)

As a matter of fact both Griffith and Schack-Schackenburg are attempting to force on the text a degree of logic which it does not possess. The scribe who stated the problem in its present form seems neither to have had before him a perfectly clear concrete picture of a measure being dipped into another so many times, nor on the other hand to be speaking entirely in the abstract terms of mathematics. He may have aimed at the latter, but an Egyptian rarely succeeded in completely dissociating himself from the concrete.

Thus we need not fear to translate *hw·kw·i r h·k·t*, "I go into³ the *hekat*-measure." This rendering is perfectly literal, but at the same time preserves just the ambiguity between abstract and concrete which exists in the Egyptian as it stands. Coming now to *hw·i mh·kw·i*, it is clear that *hw·i* is not a mere auxiliary, but the counterpart to *hw·kw·i* and means "I return," which again favours a literal sense for *hw·kw·i*. The pseudo-participle *mh·kw·i* involves that use of *mh* in the sense of "to pay in full" or "to satisfy" (in the matter of payment) which has been illustrated by Gardiner,⁴ and the words *hw·i mh·kw·i* are normal Egyptian for "I return fully satisfied," *scilicet*, with my full *hekat*. For the rare geminated form *hw·kw·i* see SETHE, *Verbum*, II, § 116.

No. 36. (Pl. L.)

"I go 3 times, and my third and my fifth are then added to me. I return fully satisfied. What is the quantity that says this?"

1	1
1	1
1	1
$\frac{1}{3}$	$\frac{1}{3}$
$\frac{1}{5}$	$\frac{1}{5}$

¹ P.S.B.A., XVI, 234.

² A.Z., 41, 79-80.

³ Examples of *hw·i* in the literal sense followed by *r* are not uncommon in the Middle Egyptian, e.g. Pap. Westcar, 3, 2; Eloquent Peasant, R. 7; Cairo stela 20,007; Shipwrecked Sailor, 25.

⁴ A.Z., 43, 34.

1	106
$\frac{1}{2}$	53
$\frac{1}{4}$	26 $\frac{1}{2}$
$\frac{1}{106}$	1
$\frac{1}{53}$	2
$\frac{1}{212}$	$\frac{1}{2}$
Total	1

1	$\frac{1}{4} + \frac{1}{53} + \frac{1}{106} + \frac{1}{212}$
2	$\frac{1}{2} + \frac{1}{30} + \frac{1}{318} + \frac{1}{795} + \frac{1}{53} + \frac{1}{106}$
$\frac{1}{3}$	$\frac{1}{12} + \frac{1}{159} + \frac{1}{318} + \frac{1}{636}$
$\frac{1}{5}$	$\frac{1}{20} + \frac{1}{265} + \frac{1}{530} + \frac{1}{1060}$

$\frac{1}{53} + \frac{1}{106} + \frac{1}{212}$	
20 10 5	35

$\frac{1}{30} + \frac{1}{318} + \frac{1}{795} + \frac{1}{53} + \frac{1}{106}$	
35 $\frac{1}{3}$ 3 $\frac{1}{3}$ 1 $\frac{1}{3}$ 20 10	70

$\frac{1}{12} + \frac{1}{159} + \frac{1}{318} + \frac{1}{636}$	
88 $\frac{1}{3}$ 6 $\frac{2}{3}$ 3 $\frac{1}{3}$ 1 $\frac{2}{3}$	100

$\frac{1}{20} + \frac{1}{265} + \frac{1}{530} + \frac{1}{1060}$	
53 4 2 1	80 (read 60)

265 $\frac{1}{4}$

$\frac{1}{2}$	530
$\frac{1}{4}$	265
$\frac{1}{4}$	265
Total	1060 "

This problem is exactly similar to the last. The equation to be solved is $x(3 + \frac{1}{3} + \frac{1}{5}) = 1$ hekat, and the answer is $(\frac{1}{4} + \frac{1}{53} + \frac{1}{106} + \frac{1}{212})$ hekat.

We begin with the (to us unnecessary) step of multiplying 1 by $3\frac{1}{3} + \frac{1}{5}$. The product is then added by means of the common denominator 30 (we expect 15) and found to come to $\frac{106}{30}$, but the working of this is suppressed. We have now to divide $\frac{106}{30}$ into 1, or, in Egyptian fashion, to operate on 106 to find 30. The multipliers $\frac{1}{4}$, $\frac{1}{106}$, $\frac{1}{53}$ and $\frac{1}{212}$ give the required product, but the scribe, instead of giving the total of the products as 30, gives it as 1. This must not be regarded as a mere error; although working in terms of the denominator 30 the reckoner has not lost sight of the logical goal of his step, namely the multiplication of $\frac{106}{30}$ to find 1, and the unexpected substitution throws an interesting light on the psychology of the Egyptian treatment of fractions. The answer is thus $\frac{1}{4} + \frac{1}{53} + \frac{1}{106} + \frac{1}{212}$ hekat.

The proof has no heading. The answer is multiplied in the usual way by 1, by 2, by $\frac{1}{3}$ and by $\frac{1}{5}$ (total $3\frac{1}{3} + \frac{1}{5}$) and the products are added in terms of the common denominator 1060. But in setting out the step the scribe forgot to leave spaces for the entry in red of the numerators (in terms of this denominator) under each fraction. He therefore sets out the products afresh with more space, omitting, however, the two simplest fractions $\frac{1}{4}$ and $\frac{1}{2}$. He now enters his numerators in red and adds them for each line separately. The four totals add up to 265, which, as the first two lines of the short calculation below demonstrate, is $\frac{1}{4}$ of 1060. The total of the fractions is thus $\frac{1}{4}$, which, with the neglected $\frac{1}{2}$ and $\frac{1}{4}$, adds up to 1. In order to apply an additional check the reckoner now uses the first two lines of the final step above referred to to reduce $\frac{1}{2}$ and $\frac{1}{4}$ to the common denominator 1060. To these he adds another $\frac{1}{4}$, represented by 265, and finds that the three numerators amount to 1060, thus doubly proving the result.

No. 36. In the main multiplication of fractions, which constitutes the first part of the proof, note in the second line the resolution by table of $\frac{2}{3}$ into $\frac{1}{30} + \frac{1}{315} + \frac{1}{795}$. In the third and fourth lines we need not assume that $\frac{1}{3}$ and $\frac{1}{5}$ are obtained otherwise than in the orthodox manner through $\frac{2}{3}$ and $\frac{1}{10}$ respectively.

No reduction to *ro* and to the Horus-eye parts of the *hekat* is given. Perhaps the reckoner's heart failed him before the complicated nature of the reduction in this case: perhaps the scribe is responsible for the omission.

No. 37. No. 37. (Pl. M.)

"I go 3 times into the *hekat*; a third of me is added to me, a third of a third of me is added to me, and a ninth of me is added to me. I return fully satisfied. What is it that says this?

Let it¹ hear:

	1	1
	2	2
	$\frac{1}{3}$	$\frac{1}{3}$
$\frac{1}{3}$ of its	$\frac{1}{3}$	$\frac{1}{9}$
its	$\frac{1}{9}$	$\frac{1}{9}$
Total	$3\frac{1}{2} + \frac{1}{18}$	

Divide 1 by $3\frac{1}{2} + \frac{1}{18}$:

	1	$3\frac{1}{2} + \frac{1}{18}$
	$\frac{1}{2}$	$1\frac{1}{2} + \frac{1}{4} + \frac{1}{36}$
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{72}$
$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{144}$
$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{288}$
$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \frac{1}{576}$
Total	1	

Addition ² :	$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \frac{1}{128}$	Total	$\frac{1}{8}$
	8 36 18 9 1		72

Proof:

	1	$\frac{1}{4} + \frac{1}{32}$
	2	$\frac{1}{2} + \frac{1}{16}$
$\frac{1}{3}$ of it	$\frac{1}{3}$	$\frac{1}{12} + \frac{1}{96}$
$\frac{1}{3}$ of its $\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{36} + \frac{1}{288}$
$\frac{1}{9}$ of it	$\frac{1}{9}$	$\frac{1}{36} + \frac{1}{288}$
Total	1	

Addition ² :	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{32}$	$\frac{1}{16}$	$\frac{1}{12}$	$\frac{1}{96}$	$\frac{1}{36}$	$\frac{1}{288}$	$\frac{1}{36}$	$\frac{1}{288}$	Total	$\frac{1}{4}$
	9	18	24	3	8	1	8	1			72	

Total	320
$\frac{1}{2}$	160
$\frac{1}{4}$	80
$\frac{1}{8}$	40
$\frac{1}{16}$	20
$\frac{1}{32}$	10
Total	90

¹ Cf. No. 30 and notes thereto.

² Literally "completion."

Proof:

— 1	90
— 2	180
sic $\frac{1}{3}$	30
sic $\frac{1}{3}$ of $\frac{1}{3}$	10
— $\frac{1}{9}$ of it	10
Total	320

amounting in corn to

No. 37.

— 1	$\frac{1}{4} + \frac{1}{32}$ hekat
— 2	$\frac{1}{2} + \frac{1}{16}$ hekat
— $\frac{1}{3}$	$\frac{1}{16} + \frac{1}{32}$ hekat
— $\frac{1}{3}$ of $\frac{1}{3}$	$\frac{1}{32}$ hekat
— $\frac{1}{9}$ of it	$\frac{1}{32}$ hekat
Total	$\frac{1}{2} + \frac{1}{8} + \frac{1}{4} + \frac{1}{8}$ hekat"

The problem is of the same type as the two preceding. The equation for solution is $x\{3 + \frac{1}{3} + (\frac{1}{3} \times \frac{1}{3}) + \frac{1}{9}\} = 1$ hekat, and the answer is $(\frac{1}{4} + \frac{1}{32})$ hekat.

The method is clear. First it is shown that the various operations to be performed on the unknown measure amount to multiplying it $3\frac{1}{2} + \frac{1}{18}$ times. To obtain the measure we must therefore divide 1 hekat by $3\frac{1}{2} + \frac{1}{18}$. This is done by the usual trials, and the multipliers $\frac{1}{4}$ and $\frac{1}{32}$ are ticked off as giving the correct amount. The addition of the two products corresponding to these multipliers is called *km*. The translation "addition" for this is perhaps a little bold, for what is actually done is to add only the last five fractions, omitting the first three, which are $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{8}$. The common denominator 576 is used and the numerators of the five fractions when reduced to this denominator are placed beneath them and found to add up to 72, which is $\frac{1}{8}$ of 576. This, together with the $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{8}$ neglected completes 1. Thus what is actually done is to show that the five fractions "complete" $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{8}$ to make unity, and in view of this *km* ought strictly speaking to be translated "completion" rather than "addition" (see Nos. 21-23 and pp. 12-13).

Thus $\frac{1}{4} + \frac{1}{32}$ of a hekat is the required amount. First comes a proof in ordinary fractions, involving a "completion" similar to that above. The answer is now reduced to *ro*, giving 90, and a proof in parallel columns follows, on the left in *ro* and on the right in terms of the $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, etc. of the hekat in their Horus-eye notation.

No. 38. (Pl. M.)

No. 38.

"I go three times into the hekat; a seventh of me is added to me and I return fully satisfied.

— 1	1
— 2	2
— $\frac{1}{7}$	$\frac{1}{7}$
Total	$3\frac{1}{7}$

Divide 1 by $3\frac{1}{7}$:

1	$3\frac{1}{7}$
$\frac{1}{22}$	$\frac{1}{7}$, since $\frac{1}{7}$ is multiplied 22 (times) to make $3\frac{1}{7}$
$\frac{1}{11}$	$\frac{1}{4} + \frac{1}{28}$
$\frac{1}{6} + \frac{1}{66}$	$\frac{1}{2} + \frac{1}{14}$
Total	1

Proof:

1	$\frac{1}{6} + \frac{1}{11} + \frac{1}{22} + \frac{1}{66}$
2	$\frac{1}{2} + \frac{1}{11} + \frac{1}{33} + \frac{1}{66}$
$\frac{1}{7}$	$\frac{1}{22}$, since $\frac{1}{22}$ is multiplied 7 times to find the top group of fractions.
Total	1

No. 38.

$$\begin{array}{r}
 1 \quad 320 \\
 \frac{2}{3} \quad 213\frac{1}{3} \\
 \frac{1}{3} \quad 106\frac{2}{3} \\
 \frac{1}{6} \quad 53\frac{1}{3} \\
 \frac{1}{11} \quad 29\frac{1}{11} \\
 \frac{1}{22} \quad 14\frac{1}{2} + \frac{1}{22} \\
 \frac{1}{66} \quad 4\frac{2}{3} + \frac{1}{6} + \frac{1}{66} \\
 \text{Total} \quad 101\frac{2}{3} + \frac{1}{11} + \frac{1}{22} + \frac{1}{66}
 \end{array}$$

Proof:

$$\begin{array}{r}
 \frac{1}{3} \quad 101\frac{2}{3} + \frac{1}{11} + \frac{1}{22} + \frac{1}{66} \\
 \frac{2}{3} \quad 203\frac{1}{3} + \frac{1}{11} + \frac{1}{33} + \frac{1}{66} \\
 \frac{1}{7} \quad 14\frac{1}{2} + \frac{1}{22} \\
 \text{Total} \quad 1
 \end{array}$$

$$\begin{array}{r}
 \text{amounting in corn to} \\
 1 \quad (\frac{1}{4} + \frac{1}{16}) \text{ hekat} + (1\frac{2}{3} + \frac{1}{11} + \frac{1}{22} + \frac{1}{66}) \text{ ro} \\
 2 \quad (\frac{1}{2} + \frac{1}{8}) \text{ hekat} + (3\frac{1}{2} + \frac{1}{11} + \frac{1}{33} + \frac{1}{66}) \text{ ro} \\
 \frac{1}{7} \quad \frac{1}{32} \text{ hekat} + (4\frac{1}{2} + \frac{1}{22}) \text{ ro} \\
 \text{Total} \quad 319\frac{2}{3} + \frac{1}{11} + \frac{1}{11} + \frac{1}{22} + \frac{1}{22} + \frac{1}{33} + \frac{1}{66} + \frac{1}{66} \\
 \quad \quad \quad 6 \quad 6 \quad 3 \quad 3 \quad 2 \quad 1 \quad 1 \\
 \text{Total} \quad 22 \\
 \quad \quad \quad \frac{1}{3}''
 \end{array}$$

The problem is $x(3\frac{1}{7}) = 1$ hekat, and the answer is $(\frac{1}{4} + \frac{1}{16})$ hekat plus $(1\frac{2}{3} + \frac{1}{11} + \frac{1}{22} + \frac{1}{66})$ ro.

The method is that of the preceding examples. First the operations described in the problem are shown to amount to multiplying the unknown quantity by $3\frac{1}{7}$. Next 1 is divided by $3\frac{1}{7}$, giving $\frac{1}{6} + \frac{1}{11} + \frac{1}{22} + \frac{1}{66}$ (note the resolution in the fourth multiplier of $\frac{1}{11}$ into $\frac{1}{6} + \frac{1}{66}$). The answer is first proved in ordinary fractions by multiplying $\frac{1}{6} + \frac{1}{11} + \frac{1}{22} + \frac{1}{66}$ by $3\frac{1}{7}$ and showing that the result is 1 (hekat). The answer is then reduced to ro and the usual double proof follows, firstly in ro and their fractions, and secondly in terms of the Horus-eye parts of the hekat. As there are already fractional parts of the ro in the first of these proofs they naturally persist into the second. In the final addition of fractions in the latter proof the common denominator 66 is used, the fraction $\frac{2}{3}$ neglected, and the sum of the rest shown to be $\frac{22}{66}$ or $\frac{1}{3}$, thus making up the 320 ro or 1 hekat.

Two points in this sum need special notice. These are the phrases *irt pw* $\frac{1}{7}$ *(spw)* 22 r gmt 3, and *irt pw* $\frac{1}{22}$ spw 7 r gmt t: i:t hrt. The first of these is placed in black ink after the step $\frac{1}{22}$ $\frac{1}{7}$, and is clearly an explanation of how that result is obtained. In these multiplications and divisions we are accustomed to meet only simple multipliers like 2, $\frac{2}{3}$ and $\frac{1}{2}$. Here, however, we are suddenly confronted by $\frac{1}{22}$ as a multiplier, and therefore some word of explanation is felt to be necessary.¹ But though the meaning is fairly obvious the syntax is difficult. Since the second word is *pw* the first should normally be a noun-equivalent and it can therefore only be an Infinitive or the Neuter Perfect Participle. The first is the more likely, and the translation in this case would be "It is the multiplying" $\frac{1}{7}$ 22 times ("times" can be supplied from the parallel phrase) to find $3\frac{1}{7}$." This gives good sense grammatically, though it is not quite the turn of phrase which we expect here, something like "since $\frac{1}{7}$ must be multiplied 22 times to find $3\frac{1}{7}$ " being what seems to be needed. Nevertheless it is the only rendering, for it is impossible to get a grammatical translation on the supposition that *irt* is a neuter participle either active or passive.³

The parallel phrase undoubtedly refers to the step $\frac{1}{22}$ in the multiplication of $\frac{1}{6} + \frac{1}{11} + \frac{1}{22} + \frac{1}{66}$ by $3\frac{1}{7}$, though owing to the exigencies of arrangement in narrow horizontal registers it has been separated from it by the words "Total 1." It is clear from the context

¹ It is seldom given, however.

² *irt* followed by *spw* is used frequently in the papyrus for "multiply."

³ If passive, we should expect *iryt* not *irt*.

that the translation must be "It is the multiplying of $\frac{1}{22}$ 7 times to find this above *i:t*," No. 38. where *i:t* must be a word standing for the quantity $\frac{1}{6} + \frac{1}{11} + \frac{1}{22} + \frac{1}{66}$, and meaning "a group" or "sum of fractions," or even more generally "a quantity." To combine *t* and *i:t* into one word and to identify the result, as Eisenlohr does, with the *i:t* of No. 61 is impossible.¹

Nos. 39 & 40. DIVISION OF LOAVES IN UNEQUAL PROPORTIONS.

No. 39. (Pl. M.)

No. 39.

"Method of finding the difference of share. A hundred loaves to 10 men, fifty to 6 and fifty to 4. What is the difference of share?"

1	4	1	6
— 10	40	2	12
sic 2	8	4	24
— $\frac{1}{2}$	2	— 8	48
		— $\frac{1}{3}$	2

$12\frac{1}{2}$	$8\frac{1}{3}$
$12\frac{1}{2}$	$8\frac{1}{3}$
$12\frac{1}{2}$	$8\frac{1}{3}$
$12\frac{1}{2}$	$8\frac{1}{3}$
	$8\frac{1}{3}$
	$8\frac{1}{3}$

Difference of share $4\frac{1}{6}$ "

This example brings us to a fresh type of problem, the division of loaves between men in unequal proportions. We are now introduced to a new technical term, *tnnw*, whose meaning must be guessed from the context. Here 100 loaves are divided into two fifties, one of which is further divided among 4 men at the rate of $12\frac{1}{2}$ each, and the other among 6 men at the rate of $8\frac{1}{3}$ each. The *tnnw* is stated to be $4\frac{1}{6}$, and it can therefore hardly be other than the difference between the share which any one of the 4 men receives and that which any one of the 6 men receives. It seems curious that a special technical term should exist in mathematics for this quantity (see, however, below, on No. 40).

The working explains itself. First 4 is multiplied to find 50, giving the larger share as $12\frac{1}{2}$. Then 6 is multiplied to find 50, and the smaller share is seen to be $8\frac{1}{3}$. The four larger shares and the six smaller shares are then set out in full, and the *tnnw* stated to be $4\frac{1}{6}$. No working of this step is shown, but the subtraction of $\frac{1}{3}$ from $\frac{1}{2}$ was one with which the mathematician was probably perfectly well acquainted.

The word *tnnw* is unknown outside this papyrus. A verb *tnn*, determined in the same way, occurs in Ebers Medical Pap., 101, 12-13, and in Pap. Harris Mag., 8, 6, but I am unable to seize its meaning in either passage.² The ox persists even in the plant-name *tnn*, Hearst Medical Pap., 8, 4, and BRUGSCH, *Wörterbuch, Suppl.*, 1315. Doubtless the same root is involved in the town-name *Mtnn* of *Medum*, Pl. XIX (determined by a lassoed³ ox), and the noun *mtwn* of Pap. Millingen, 1, 10.

¹ It is just conceivable that *irt pw* is a passive form of the *sdm.f pw* of the Ebers Papyrus (SETHE, *Verbum*, II, § 142), though there are no passive examples known, and we should in any case expect the geminated form (*op. cit.*, II, § 332). It occurs again in No. 62.

² For further examples see *A.Z.*, 57, 38.

³ See, however, *A.Z.*, 43, 74-6.

No. 40. No. 40. (Pl. M.)

"A hundred loaves to 5 men, one-seventh of the three first men to the two last.¹

What is the difference of share?

The doing as it occurs supposing the difference of share to be $5\frac{1}{2}$:

— 1	23
— 1	$17\frac{1}{2}$
— 1	12
— 1	$6\frac{1}{2}$
— 1	1
Total	60

— 1	60
— $\frac{2}{3}$	40
(Total	100)

You are to count with $1\frac{2}{3}$

23	times, it becomes	$38\frac{1}{3}$
$17\frac{1}{2}$	" "	$29\frac{1}{6}$
12	" "	20
$6\frac{1}{2}$	" "	$10\frac{2}{3} + \frac{1}{6}$
1	" ²	$1\frac{2}{3}$
Total		100 "

The problem consists in dividing 100 loaves among 5 men in such a way that the shares are in arithmetical progression and that the sum of the two smallest shares is one-seventh the sum of the three greatest. The first of these two essential conditions is not mentioned in the problem as set by the Egyptian, but it is possible that it was implied by the mention of *townw*. In dealing with No. 39 we found it difficult to believe that a special technical term should have been invented for the "difference of share" in the very simple sense in which it occurs in that problem, and it is possible that the real technical meaning of *townw* is that in which it is used here, namely the "common difference" in an arithmetical series.

The method is as follows. A hypothetical series in arithmetical progression, namely 1, $6\frac{1}{2}$, 12, $17\frac{1}{2}$, 23, is taken, the common difference or *townw* of which is $5\frac{1}{2}$, while its sum is seen to be 60. This series has the further property that the sum of its two lowest terms is one-seventh the sum of its three highest. Thus the trial numbers chosen are not really arbitrary,³ but are chosen because they were already known to be suitable for the purpose. The fact probably is that it had been noticed that in this set of numbers in arithmetical progression with a common difference of $5\frac{1}{2}$ the sum of the last two was just one-seventh of that of the three first, while the total of all five was 60. This suggested the setting of problems involving a division on the lines demanded in our example, but in which the total of loaves is not 60 but some other number, thus involving a small sum in proportion in addition to the knowledge of the 60-series. In other words, the sum was, like those in many modern examination papers, set from the answer. Otherwise it is impossible that the Egyptian could have obtained the 60-series and its *townw* of $5\frac{1}{2}$, which involve the preparation and solution of two simultaneous equations, which we know to have been beyond his reach.

To return to the working. Having copied down his 60-series the scribe notes that the total he needs is not 60 but 100, and as 100 is $60 + (60 \times \frac{2}{3})$ he must multiply all the shares

¹ Or, "one seventh of the three superiors to the two inferiors."

² The papyrus here has "Total 60," erroneously repeated from above.

³ Obviously when *two* trial numbers are chosen (in this case the first or last term of the series and the common difference) they cannot be arbitrary, for they bear a numerical relation the one to the other.

in the 60-series by $1\frac{2}{3}$. This gives him the required series whose total is 100, and whose *tnnw*, No. 40. which, oddly enough, he never works out, is $9\frac{1}{6}$.

It is worth while to notice the knowledge of proportion displayed in this example. The Egyptian had realized that if the total of a number of proportionate shares was increased that of each separate share would be increased in the same proportion. This is indeed the method by which we now solve problems in arithmetic where there is only one unknown and where we are anxious to avoid the open use of an algebraic symbol. He does not reveal to us whether he had also realized the fact that the *tnnw* would also be increased in the same proportion and could therefore be obtained direct from $5\frac{1}{2}$ by multiplying by $1\frac{2}{3}$. From the fact of his having given us the actual shares and not the *tnnw* it is not fair to argue that he did not, for despite the fact that the problem asks for the *tnnw* the important point in practice—and these problems are almost always practical—is to find the actual share of each person.

BOOK II. MENSURATION.

PART I. VOLUMES AND CUBIC CONTENT. Nos. 41-47. (Plates M—O.)

Nos. 41-47. WITH these problems begins the second great section or book of our papyrus, that which treats of mensuration. Problems 41-47 deal with the volume, or more strictly, as might be expected from the concrete Egyptian mind, the content in corn of certain containers or spaces of varying shape. The word used for container is š^{c} , which, as Griffith notes, might mean not a bin or granary but rather a three dimensional space or figure in the mathematical sense. The word is unknown elsewhere.¹ The solids represented by it in these problems seem to be regular figures varying in shape at the base. In Nos. 41-43 the š^{c} is circular at the base, i.e. it is a cylinder: in No. 44 it is ifd , square, and as the height is equal to the sides of the base the whole is a cube. In No. 46 the š^{c} has no epithet, and is seen to be either square or rectangular in base (in No. 45 it is square), while in No. 47 it is distinguished from the noun dbn , which can be nothing but a round container (cylinder), and it must therefore stand for a figure on a rectangular or square base. Thus the word when used alone would seem to carry the idea of rectangularity, though when qualified by the adjective dbn (41-43) it indicates a cylinder.

No. 41. No. 41. (Pl. M.)

"Example of working out a circular container of diameter 9 and height 10.

You are to subtract a ninth of 9, namely 1; remainder 8.

Multiply 8 eight times, result 64.

You are to multiply 64 ten times; it becomes 640.

Its half is now added to it; it becomes 960.

<This is>² its content in *khar*. You are to take a twentieth of 960, namely 48. This is the amount which will go into it in quadruple-*hekat*, namely 48 hundreds of quadruple-*hekat* of corn.

Form of its working:

1	8
2	16
4	32
— 8	64
1	64
— 10	640
— $\frac{1}{2}$	320
Total	960
$\frac{1}{10}$	96
$\frac{1}{20}$	48 "

In this problem the base of the š^{c} is circular and 9 cubits in diameter, the figure being in fact a cylinder. The diameter is decreased by its ninth part and the result squared, which gives roughly the area of the base in square cubits.³ This is next multiplied by the height

¹ An obscure noun $\text{š}^{\text{c}}t$ with the house-determinative occurs LACAU, *Textes Religieux*, 86, 85-88. Cf.  in a list of buildings in the Golenischeff Glossary, 5, 15 (Gardiner).

² *pw* omitted by the scribe.

³ See No. 48. That the dimensions are in cubits is clear from No. 43.

of 10 cubits, and the resulting 640 stated to be the volume of the whole in cubic cubits. To No. 41. turn this into a measure of capacity we add its half, and the result is the content in *khar*, from which we may see at once that $1\frac{1}{2}$ *khar* is the capacity of a cubic cubit. We now divide the number of *khar* by 20, and the resulting 48 is the amount of corn which will go into the space, reckoned in hundreds of quadruple-*hekat*.

For the *khar* and its history see Introduction, p. 26.

The expression here arrived at for the volume of a cylinder is a very reasonable approximation. The correct value is given by

$$V = \pi r^2 h$$

where r is the radius and h the height. The Egyptian uses

$$V = \left(\frac{16}{9}r\right)^2 h$$

giving as the value of π $3\frac{1}{8}\frac{1}{4}$, which is not very far from the correct value 3.14159265..... (about $3\frac{1}{7}$).

The uses of the noun *rht* in this papyrus bear out the belief that its literal meaning is simply "number" (*nombre*, not *numéro*) and not "list" or "specification," as it is so often translated, perhaps in consequence of its obvious derivation from *rh*, "to know." It is true that the word often serves to head a list, but it will be found that, in the Middle Kingdom at least, the items in these lists are accompanied by numbers or quantities. See, for example, GRIFFITH, *K.P.*, VIII, 44, *Ä.Z.*, 57, 58 (from Pap. Bulaq 18), *Urk.*, IV, 664 and 893, and *Ä.Z.*, 37, 92. In the Rhind Papyrus it occurs eleven times. In Nos. 63, 74 and 82B it can mean nothing but "number" or "amount," while in 86 this meaning suits quite as well as "list." Elsewhere we have a slightly extended use of the meaning "number," for *rht* is used in 46 for "dimensions," in 44 for the content in *khar* of a certain space, in 50, 52 and 53 for the "area" of fields, and in 69 for the "flour-content" of a loaf, an idea expressed in No. 70 and elsewhere by *hrt*, "the share" of a bushel of flour to be found in each of a batch of loaves fixed at so many to the bushel. In No. 62 the phrase *rht.f pw*, whether we take it to refer simply to the words "Total 84" or to the separate values in *shaty* of the three metals, can hardly mean anything but "amount" or "value."

No. 42. (Pl. N.)

No. 42.

"A circular container of 10 by 10.¹

You are to subtract a ninth of 10, namely $1\frac{1}{9}$; remainder $8\frac{2}{3} + \frac{1}{6} + \frac{1}{18}$.

You are to multiply $8\frac{2}{3} + \frac{1}{6} + \frac{1}{18}$ by $8\frac{2}{3} + \frac{1}{6} + \frac{1}{18}$; result $79\frac{1}{108} + \frac{1}{324}$.

You are to multiply $79\frac{1}{108} + \frac{1}{324}$ by 10; it becomes $790\frac{1}{18} + \frac{1}{27} + \frac{1}{54}$.

Its half is added to it; it becomes 1185.

Multiply 1185 by $\frac{1}{20}$, giving $59\frac{1}{4}$. This is the amount that will go into it in quadruple-*hekat*, namely $59\frac{1}{4}$ hundreds of quadruple-*hekat* of corn.

Form of its working:

1	$8\frac{2}{3} + \frac{1}{6} + \frac{1}{18}$
2	$17\frac{2}{3} + \frac{1}{9}$
4	$35\frac{1}{2} + \frac{1}{18}$
— 8	$71\frac{1}{9}$
— $\frac{2}{3}$	$5\frac{2}{3} + \frac{1}{6} + \frac{1}{18} + \frac{1}{27}$
— $\frac{1}{3}$	$2\frac{2}{3} + \frac{1}{6} + \frac{1}{12} + \frac{1}{36} + \frac{1}{54}$
— $\frac{1}{6}$	$1\frac{1}{3} + \frac{1}{12} + \frac{1}{24} + \frac{1}{72} + \frac{1}{108}$
— $\frac{1}{18}$	$\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{108} + \frac{1}{324}$
Total	$79\frac{1}{108} + \frac{1}{324}$

¹ i.e. of diameter 10 and height 10.

No. 42.

$$\begin{array}{rcl}
 1 & 79\frac{1}{108} + \frac{1}{324} & \\
 10 & 790\frac{1}{18} + \frac{1}{27} + \frac{1}{54} & \\
 \frac{1}{2} & 395\frac{1}{36} + \frac{1}{54} + \frac{1}{108} & \\
 \text{Total} & 1185 & \\
 \\
 \frac{1}{10} & 118\frac{1}{2} & \\
 \frac{1}{20} & 59\frac{1}{4} & "
 \end{array}$$

This example is precisely similar to the last except that the numbers involved are more complicated. There is a slight error in the working, $79\frac{1}{108} + \frac{1}{324}$ (which is in effect $79\frac{1}{81}$) when multiplied by 10 yielding not $790\frac{1}{18} + \frac{1}{27} + \frac{1}{54}$ (which is $790\frac{1}{9}$) but $790\frac{1}{9}$, a difference of $\frac{1}{81}$.

No. 43. No. 43. (Pl. N.)

"A circular container of 9 cubits in its height and 6 in its breadth. What is the amount that will go into it in corn?"

The doing as it occurs:

You are to take 1 from 9; remainder 8.

Operate on 8; you are to add its third part to it; it becomes $10\frac{2}{3}$.

Multiply $10\frac{2}{3}$ by $10\frac{2}{3}$; it becomes $113\frac{2}{3} + \frac{1}{9}$.

Multiply $113\frac{2}{3} + \frac{1}{9}$ by 4, this being two-thirds of the 6 cubits which are the breadth.

It becomes $455\frac{1}{9}$. This is its content in *khar*.

You are to find one-twentieth of its content in *khar*; it becomes $22\frac{1}{2} + \frac{1}{4} + \frac{1}{45}$ (*sic*).

This is the amount that will go into it in¹ quadruple-hekat, viz.:

corn, hundreds of quadruple-hekat ($22\frac{1}{2} + \frac{1}{4}$)

+ quadruple-hekat ($\frac{1}{2} + \frac{1}{32} + \frac{1}{64}$)

+ quadruple-ro ($2\frac{1}{2} + \frac{1}{4} + \frac{1}{36}$)

Form of working:

$$\begin{array}{rcl}
 \begin{array}{rcl}
 \frac{1}{3} & 8 & \\
 \frac{2}{3} & 5\frac{1}{3} & \\
 \frac{1}{3} & 2\frac{2}{3} & \\
 \text{Total} & 10\frac{2}{3} &
 \end{array}
 &
 &
 \begin{array}{rcl}
 1 & 10\frac{2}{3} & \\
 10 & 106\frac{2}{3} & \\
 \frac{2}{3} & 7\frac{1}{9} & \\
 \text{Total} & 113\frac{2}{3} + \frac{1}{9} &
 \end{array}
 \\
 \begin{array}{rcl}
 1 & 113\frac{2}{3} + \frac{1}{9} & \\
 2 & 227\frac{1}{2} + \frac{1}{18} & \\
 4 & 455\frac{1}{9} &
 \end{array}
 &
 &
 \begin{array}{rcl}
 1 & 455\frac{1}{9} & \\
 \frac{1}{10} & 45\frac{1}{2} + \frac{1}{90} & \\
 \frac{1}{20} & 22\frac{1}{2} + \frac{1}{4} + \frac{1}{45} \text{ (read } \frac{1}{80} \text{)} & "
 \end{array}
 \end{array}$$

This is one of the most difficult problems in the papyrus. It professes to give a method of finding the content of a regular figure in *khar* without first working out the volume in cubic cubits and multiplying it by $1\frac{1}{2}$ as in Nos. 41 and 42. The container is again a round one (*šš' dbn*), of which only two dimensions are given, a height of 9 cubits and a breadth of 6 cubits. Now the only suitable regular solid figure which can be determined by two dimensions alone is the cylinder, and the *šš'* of this problem is therefore again a cylinder, just as in the two previous examples. The "breadth" is the diameter of its base, to which, it will be remembered, no name was actually given in the other two problems. It will be seen later that the statement of the problem is incorrect, the 9 cubits being really the diameter and the 6 cubits the height of the cylinder.

The difficulty of the question lies in the fact that, though the container is precisely similar to that in Nos. 41 and 42, the result obtained for its content in *khar* is different from that which would be reached by the method of the other two examples. Clearly, in the attempt to find the result directly in *khar* without first finding the volume in cubic cubits

¹ The words *m šš' hkt* which follow are clearly a scribe's error and to be omitted.

and adding its half to it, some error has been introduced, and we have to ask where No. 43. it lies.

Eisenlohr treats the question at great length, but his reasoning is vitiated by his having translated *khar* incorrectly throughout and therefore having failed to grasp the real meaning of the problem. There is no need to try to solve the question by referring as he does to granaries of various irregular forms. The solution of the difficulty is due to the ingenuity of Schack-Schackenburg. To follow his reasoning we must go back to a somewhat difficult problem in the Kahun Papyri, Pl. VIII. It is set out as follows:—

	12				
8	$\bigcirc$				
	1365 $\frac{1}{3}$				
$\frac{2}{3}$	8	$\frac{1}{3}$	16	$\frac{1}{3}$	256
$\frac{1}{3}$	4	$\frac{1}{3}$	160	$\frac{2}{3}$	512
Total	16	$\frac{1}{3}$	80	$\frac{4}{3}$	1024
		Total	256	$\frac{1}{3}$	85 $\frac{1}{3}$
				Total	1365 $\frac{1}{3}$

There is no doubt as to what is being done here. First $1\frac{1}{3}$ times 12 is taken and found to be 16. The first line of the step, namely 1 12, is omitted because the 12 over the figure of the circle, used to indicate its diameter, also does duty for this first line. Next 16 is squared, giving 256, and finally 256 is multiplied by $5\frac{1}{3}$ which is $\frac{2}{3}$ of 8, the other given dimension of the circular figure.

Griffith (*K.P.*, Text, 16) surmised that the problem was to find the content of a cylindrical granary of height 12 cubits and diameter 8, but was unable to reconcile the working with this hypothesis.

Borchardt, writing in *Ä.Z.*, 35, 150-52, attempted to explain it as the finding of the cubic content of a hemisphere 8 cubits in diameter. In order to do this it was necessary to suppose that the 12 over the figure of the circle belonged to the working and not to the figure at all. This hypothesis certainly does violence to the position of the 12, and in *Ä.Z.*, 37, 78-9, Schack-Schackenburg proposed an alternative reading which avoids this difficulty and is certainly the correct one. According to him the problem is to determine the volume of a cylinder of diameter 12 and height 8 cubits. The diameter 12 is first multiplied by $1\frac{1}{3}$, producing 16. This is squared, giving 256, and this number is then multiplied by $5\frac{1}{3}$ which is two-thirds of the height. The result is $1365\frac{1}{3}$, and this result is in *khar*, though the papyrus, which is very terse in expression, does not make this point clear. It is in fact exactly the number of *khar* which would be given by the method of Nos. 41 and 42 in the Rhind, as will be clear from the following working:—

$$\begin{aligned}
 12 - \frac{1}{3} \cdot 12 &= 10\frac{2}{3} \\
 10\frac{2}{3} \times 10\frac{2}{3} &= 113\frac{7}{9} \\
 113\frac{7}{9} \times 8 &= 910\frac{2}{3} \\
 910\frac{2}{3} \times 1\frac{1}{2} &= 1365\frac{1}{3}
 \end{aligned}$$

In other words, the method of the Kahun Papyrus is a simplified method of finding the content of a cylinder direct in *khar* without working out its volume in cubic cubits and multiplying by $1\frac{1}{2}$ to turn it into *khar*.

Now this is precisely what No. 43 in the Rhind seems to be aiming at, yet it fails to get the right answer. This failure is due simply to the fact that the scribe, attempting to use the short method, mixed it up in his mind with the longer and more logical method, and put into the new method one of the steps of the old which is not needed in the new.

- No. 43. This is the first step, where from nine he takes away one, which is a ninth of it. From this error it results that his result is only $(\frac{8}{9})^2$ of what it should be, namely $455\frac{1}{9}$ instead of 576 *khar*.

There still remains a difficulty about Schack-Schackenburg's solution. In the statement of the problem the height is distinctly said to be 9 cubits and the diameter 6. But the cylinder actually worked out is one which has these two figures interchanged, 6 being the height and 9 the diameter. How are we to account for this? The most probable explanation seems to be that in the original papyrus the statement of the problem was correct, but the scribe made a mistake in the first line of working, as we have seen. A later scribe, seeing in the first line of the working the subtraction of a ninth of 9 from 9, just as in Nos. 41 and 42, very naturally concluded that this 9 must be the diameter and not the height, and so he transposed the two dimensions in the statement of the problem, his mathematical knowledge not taking him far enough to test the result with the statement in its new form.

There are several points to notice in the method. In the first place, after the statement of the problem comes the phrase *irt mī hpr*. This here refers to the full detailed working out, while the *ššmt* is kept as a heading for the rough working. See Introduction, p. 23. In the second place, there is an unfortunate mistake in both the *ššmt* and the *irt mī hpr*, $\frac{1}{45}$ being written instead of $\frac{1}{80}$ in the division of $455\frac{1}{9}$ by 20. Curiously enough this mistake disappears when the fraction is reduced to the Horus-eye notation of the quadruple-*hekat*, the amount taken being clearly $\frac{1}{80}$ and not $\frac{1}{45}$ of a hundred quadruple-*hekat*. The actual mistake is made, as will be observed, in the *ššmt*, where the calculator divides 90 by 2 instead of multiplying it. The fact that the answer is nevertheless correctly given would suggest that the answer was copied from the prototype, while the working was actually filled in by the copyist. Possibly the full *ššmt* (rough working in this case) was not in the prototype in all cases, though it is curious that here the same error appears in the *irt mī hpr*.

The statement of the answer is interesting. It is $22\frac{1}{2} + \frac{1}{4} + \frac{1}{80}$ hundreds of quadruple-*hekat*. Of this the $22\frac{1}{2} + \frac{1}{4}$ are left in units and ordinary fractions, as is correct in dealing with the hundred-of-*hekat* (whether simple, double or quadruple), the 22 preceding the $\frac{1}{2}$ and the $\frac{1}{4}$ following it. The other fraction $\frac{1}{80}$ is then reduced to the Horus-eye divisions (half, quarter, etc.) of one quadruple-*hekat*.

No. 44. No. 44. (Pl. N.)

"Example of reckoning out a square container of 10 in its length, 10 in its breadth and 10 in its height. What is the amount that will go into it in corn?"

Multiply 10 by 10, it becomes 100.

Multiply 100 by 10, it becomes 1000.

Take a half of 1000, that is 500, it becomes 1500; this is its content in *khar*.

You are to take a twentieth of 1500, it becomes 75. This is the amount that will go into it in quadruple-*hekat*, viz. 75 hundreds of quadruple-*hekat* of corn.

Working out:

					10
					10
1	10		1	100	
10	100		10	1000	
		1		1000	
		$\frac{1}{2}$		500	
		1		1500	
		$\frac{1}{20}$		75	
		$\frac{1}{20}$		75	

1	75	
10	750	
20	1500	
$\frac{1}{10}$ th		150
$\frac{1}{10}$ of $\frac{1}{10}$ th		15
$\frac{2}{3}$ of $\frac{1}{10}$ of $\frac{1}{10}$ th of it		10 "

We have now come to the determination of the volume of a rectangular parallelopiped, which in this particular case happens to be a cube. It is described as a $\dot{s}:\dot{s}$ *ifd*. The word *ifd* is generally translated "square" or "rectangular," but it is quite possible that we ought to extend its use to three dimensions to cover parallelopipedal. It is clearly a derivative of *fdw*, the numeral 4, and its original meaning must therefore have been four-sided, perhaps with the tacit addition of rectangular. In the present instance the meaning parallelopipedal¹ is conveyed by implication if not directly, for a $\dot{s}:\dot{s}$ is a figure in three dimensions, and the adjective accompanying it need only describe its base. Thus a $\dot{s}:\dot{s}$ *dbn* is a cylinder, and a $\dot{s}:\dot{s}$ *ifd* is a parallelopiped, or in special cases a cube.

This cube has a side of 10 cubits, and its volume is correctly calculated as 1000. This is then multiplied by $1\frac{1}{2}$ to obtain the number of *khar* contained, and the division by 20 reduces this last to 75 hundreds of quadruple-*hekat*. The three last lines constitute a proof by continued division, if indeed they have not merely strayed hither from No. 45.

There is nothing to note in the text except a slight confusion in the first line. The word 10 occurs four times, whereas we only need it three times. This is due to the fact that there is a confusion between two methods of statement.

$\dot{s}:\dot{s}$ *n mh* 10 *m* *swf mh* 10 *m* *shw.f mh* 10 *m* *k:i.f*
 $\dot{s}:\dot{s}$ *swf* 10 *shw.f* 10 *k:i.f* 10

We must delete the words *n* 10 *m* which follow *ifd*, or else insert an *m* between the second 10 and *shw.f* and another between the third 10 and *k:i.f*, finally deleting the fourth 10.

No. 45. (Pl. N.)

"A container into which corn has gone to the amount of 75 <hundreds of> quadruple-*hekat*. How much is it by how much?

Multiply 75 twenty times; it becomes 1500.

Operate on 1500: you are to take one-tenth of it, namely 150;

$\frac{1}{10}$ of $\frac{1}{10}$ of it, namely 15;

$\frac{2}{3}$ of $\frac{1}{10}$ of $\frac{1}{10}$ of it, namely 10.

Therefore it is 10 by 10 by 10.

1	75
10	750
20	1500: behold this is its content.

1	1500
$\frac{1}{10}$ th	150
$\frac{1}{10}$ of $\frac{1}{10}$ th of it	15
$\frac{2}{3}$ of $\frac{1}{10}$ of $\frac{1}{10}$ th of it	10 "

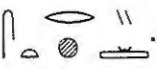
This problem is the reverse of the preceding. Here we are given the cubic content of a container in hundreds of quadruple-*hekat*, and we are asked to find its dimensions. It is assumed that the container is parallelopipedal, and even, as the answer shows, that it is a cube.

¹ In L. D., II, 134A *ifd* appears to be used of a parallelopipedal block of stone.

No. 45. The translation of this problem has given considerable difficulty on account of the incorrect reading of the words here rendered "How much is it by how much?" i.e. "What are its dimensions." Griffith (*K.P.*, Text, 57) read , and took the words to be a further description of the container "having side equal to side." He gave instances of a word *šdvo* meaning a "side" or "fence," and of the use of the preposition *r* in the sense in which he here required it. The difficulty is that the hieratic signs for  and  both occur elsewhere in the papyrus and are not made as they are here. Nor is Griffith's earlier reading  (*P.S.B.A.*, XVI, p. 235) any more satisfactory palaeographically. There is, however, a reading which seems to me free from every objection, namely . The group does not occur elsewhere in the papyrus, so that no internal comparison is possible, but forms not unlike this occur in Ebers and are quoted by Möller (*Hieratische Paläographie*, I, p. 19, note 1). If this reading be correct, what we have here is simply a late Middle Kingdom use of the Late Egyptian *wr* in the sense of "how much" or "how many" (Coptic *ⲱⲣ*), and the correct translation will be "How much is it by how much?" For the use of *ns* in giving the dimensions of an object see below in this same problem and also Shipwrecked Sailor, l. 62, and for the preposition *r* in the sense of "by" in measurements see No. 46 and also below in the present problem. This translation has the further advantages that it supplies the statement of the problem to be solved, which would otherwise be wanting, and that the same reading of the group gives good sense in No. 73, where again we need a statement of the problem.

The method is as follows. The container is assumed to have a base 10 cubits square, and indeed if the solution is to be unique it is obvious that two of the dimensions must be thus assumed. The 75 hundreds of quadruple-*hekat* are first multiplied by 20 to bring them to *khar*. This ought now to be reduced to cubic cubits by multiplying by $\frac{2}{3}$, for it is clear from the preceding examples that the *khar* is a capacity of $1\frac{1}{2}$ cubic cubits. This would have given 1000 cubic cubits, and by dividing this successively by 10 cubits and 10 cubits, the two assumed dimensions, the third dimension, also 10 cubits, would have been obtained. The reckoner curiously enough chooses an equally effective but less logical method. Instead of multiplying by $\frac{2}{3}$ to reduce *khar* to cubic cubits he divides *khar* at once by 10 cubits and again by 10 cubits, getting as his result 15, but of what unit it would not be easy to say. Only now does he multiply by his $\frac{2}{3}$ and obtain the correct answer, 10 cubits. The method is illogical because the units in which the working is done are confused, and we suspect that the reckoner is working not by the light of reason but by a mere practical rule.

There are two important points in the text. In the opening sentence *h:n* is the Relative *šdm.nf*-form and therefore must have perfect force, "A container into which corn has gone down to the extent of 75 hundreds of quadruple-*hekat*." This seems to tell against the proposal to take *h:i* in a purely metaphorical sense both here and in No. 35, for, if it had the meaning "to be contained in," or similar, we should expect here the Relative *šdm.f*-form, which has either imperfect or timeless meaning, in preference to the Relative *šdm.nf*-form which, being perfect, demands a concrete translation.

The group which I have here tentatively transcribed  has previously been read . Palaeographically, this is just possible, though the ligature in question is not used in the papyrus (except in the extraneous No. 87) for . Moreover, the meaning of the word, which can be no other than "content," and the parallel with the supposed synonym *rht* have undoubtedly influenced transcribers in favour of the reading . A compound in which the first part is the Neuter Absolute Pronoun *št* and the second is a part of the verb *rht*, or a noun derived from it, is almost unthinkable in Egyptian.

For these reasons another reading must be sought and *štwti* would seem to be the most probable. The word occurs twice in No. 46 and once in No. 60, where it is an obvious error for *škl*, "batter." For the final *w* in an abstract noun compare *šity* "proof."

No. 46. (Pl. N.)

No. 46.

"A container into which corn has gone to the extent of 25 hundreds of quadruple-hekat. What are its dimensions?"

You are to multiply 25 twenty times; it becomes 500. This is its content.

Operate on 500:

Take $\frac{1}{10}$ of it, namely 50

$\frac{1}{20}$ of it, namely 25

$\frac{1}{10}$ of $\frac{1}{10}$ of it, namely 5

$\frac{2}{3}$ of $\frac{1}{10}$ of $\frac{1}{10}$ of it, namely $3\frac{1}{3}$.

This container is 10 by 10 by $3\frac{1}{3}$.

Its working out:

1	25
10	250
20	500

This is its content.

1	500
$\frac{1}{10}$	50
$\frac{1}{10}$ of $\frac{1}{10}$ of it	5
$\frac{2}{3}$ of $\frac{1}{10}$ of $\frac{1}{10}$ of it	$3\frac{1}{3}$

This container proves to be of 10 cubits by 10 by $3\frac{1}{3}$.

That is the same."

This problem is precisely similar to the last, the only difference in wording being that we are asked to find the *rlt* in place of finding "How much is it by how much?" This being so, it is natural to translate *rlt* not by content but by numbers, i.e. dimensions, and this is supported by the fact that the content (in *khar*), which is mentioned twice in the working, is called not *rlt* but *stwti*, for which see under No. 45.

The method of this problem differs in no respect from that of the last. Once again the assumption is made that two of the required dimensions are 10 cubits and 10 cubits. Again too the multiplication by $\frac{2}{3}$, which ought to have been introduced to reduce *khar* to cubic cubits, is left over until the last step, where it has no logical meaning.

The working out is little more than a restatement of the method, with the sole difference that the steps in the multiplication of 25 by 20 are shown. The concluding words, *mitt pw* "This is equivalent (to the result obtained above)," are somewhat devoid of point, for they have only a real meaning when placed at the end of a rigorous proof.

No. 47. (Pl. O.)

No. 47.

"If the scribe says to you, Let me know (what) $\frac{1}{10}$ th becomes, in a rectangular container or a circular container:

$\frac{1}{10}$	becomes 10 quadruple-hekat of corn
$\frac{1}{20}$	" 5
$\frac{1}{30}$	" $(3\frac{1}{4} + \frac{1}{16} + \frac{1}{64})$ hekat + $1\frac{2}{3}$ ro
$\frac{1}{40}$	" $2\frac{1}{2}$ hekat
$\frac{1}{50}$	" 2 hekat
$\frac{1}{60}$	" $(1\frac{1}{2} + \frac{1}{8} + \frac{1}{32})$ hekat + $3\frac{1}{3}$ ro
$\frac{1}{70}$	" $(1\frac{1}{4} + \frac{1}{8} + \frac{1}{32} + \frac{1}{64})$ hekat + $(2\frac{1}{4} + \frac{1}{21} + \frac{1}{42})$ ro
$\frac{1}{80}$	" $1\frac{1}{4}$ hekat
$\frac{1}{90}$	" $(1\frac{1}{6} + \frac{1}{32} + \frac{1}{64})$ hekat + $(\frac{1}{2} + \frac{1}{18})$ ro
$\frac{1}{100}$	" 1 hekat."

No. 47. This example is nothing more than a statement of the values of the fractions $\frac{1}{10}$, $\frac{1}{20}$, $\frac{1}{30}$, etc. of a hundred quadruple-hekat, and the words "in a rectangular or a circular container" are unnecessary, having been perhaps added by a scribe who sought to connect this problem more closely with those which precede it.

The usual translation is "Let me know $\frac{1}{10}$, whether it be in a rectangular or a circular space." To this it may be objected that *hpr.f* could hardly take the place of the English verb "to be" in such a case as this, for *hpr* quite definitely means "to come into being," "to become" or "to happen," and in fact Egyptian would need no verb at all here, but would merely say *m š' dbn r pw*. It is therefore just possible that after *hpr.f* the words  have dropped out by haplography of the , and that the original reading was, "Let me know what $\frac{1}{10}$ becomes, in a rectangular or a circular container."

This is perhaps borne out by the sign which follows the fraction $\frac{1}{10}$ in the next line. If this line merely meant " $\frac{1}{10}$ th is 10 quadruple-hekat of corn," we should expect not the preposition  (supposing the sign to be an ) but , and so on throughout. The sign in question is perhaps that which is familiar to us from papyri of all periods as standing for "ditto," and the words for which it here stands are *hpr.f m*: this is corroborated by the position of the sign. A similar sign is used in Nos. 72 and 73 (see, however, notes to No. 72) to separate the *pfsw* of loaves from a following numeral.

The notation of the quadruple-hekat here used is perfectly regular (see Introduction, p. 26). Ten quadruple-hekat are expressed by a tall stroke after the quadruple-hekat sign, standing, as Griffith has shown,¹ for a vertical row of 10 dots. Five are expressed by a ligature possibly combining 5 dots, and two and one by 2 dots and 1 dot respectively.² The words *hekat* and *ro*, which it is necessary to fill in in order to produce an intelligible translation, refer in every case to quadruple-hekat and to quadruple-ro, i.e. to a *ro* which is the 320th part of the quadruple-hekat.

PART II. CALCULATION OF AREAS. Nos. 48-55.

No. 48. No. 48. (Pl. O.)

1	8	setat		1	9	setat					
2	1	thousand-of-land	6	setat	2	1	thousand-of-land	8	setat		
4	3	„	„	2	„	4	3	„	„	6	„ ³
8	6	„	„	4	„	8	7	„	„	2	„
						Total	8	„	„	1	„

This problem, which has no wording and consists merely of a figure with working out, is clearly the comparison of the area of a square of side 9 *khet* with that of a circle of diameter 9 *khet*. The area of the circle (on the left) is obtained as usual (cf. No. 41) by squaring $\frac{8}{9}$ of the diameter,⁴ viz. 8. The area of the square (on the right) is got of course by squaring the side 9. The nature of the measurements has been cleared up by Griffith. It is plain that the units placed under the sign  are each one-tenth of the units which stand free in front of it. Moreover, since the square of 9 *khet* is written 8 plus , it is further clear that the free standing units each represent ten *setat* or square *khet*, while the units under the  are each one square *khet*. Now the *khet* measures 100 cubits, and therefore a square *khet* (10,000 sq. cubits) may be regarded as the sum of 100 strips of land each one cubit broad and 100 long. Each of these strips was called a cubit-of-land, because it measured a cubit along one side of the square *khet*, and a thousand of these, known technically as    "a thousand-of-land," would make ten square *khet*, which is precisely what is

¹ P.S.B.A., XIV, 425.

² Cf. No. 69.

³ The text has 60 *setat*. Cf. No. 50.

⁴ For the value of π thus obtained see p. 81.

represented by the free standing units in our example. Each of these then represents a No. 48. thousand-of-land, *i.e.* a thousand of the narrow strips each 100 cubits by one cubit, and would be represented in early hieroglyphs by the sign for thousand.

The example seems curiously out of place here, and is perhaps an illustration to No. 50.

There is one point in this reckoning which has hardly received the attention it deserves. As a general rule the Egyptian does not trouble in his working out to insert the dimensions of the figures he uses. Thus he does not always openly distinguish between pure numbers and concrete lengths, weights, etc. So in finding the volume of a cylinder he will square $\frac{8}{9}$ ths of the diameter of the base and proceed to multiply this by the height without stopping to point out whether the figures used are cubits, square cubits, or cubic cubits until the conclusion of the reckoning.

For this reason it is interesting in No. 48 to find the dimensions inserted throughout. It is still more so to notice that in the first line of all, "1 8 *setat*," the unit is stated as *setat*. To our modern feeling this is wrong. The 8 in question is, strictly speaking, in units of long measure, *viz.* *khet*, and it is not until we multiply it by another unit of long measure, *viz.* 8 *khet*, that it can logically be expressed in square units. Similarly in the second half of the sum, the finding of the area of a field 9 *khet* square, the logical process is to multiply 9 *khet* by 9 *khet*; but the working actually written looks like the multiplication of 9 *setat* by the pure number 9. The explanation of this is that, if dimensions are to be stated at all in the working, the peculiar form of the Egyptian system of multiplication demands that the final dimension should be stated immediately in the first line. Thus it is obviously impossible to write 9 *khet* in the first line and 18, 36, and 72 *setat* in the following, for the first and last lines have next to be added together, which is impossible if one is in long and the other in square measure.

In other words, the difficulty lies in the very nature of the Egyptian multiplication process. It is strictly speaking not a multiplication but an addition or counting, and in reality it is impossible to get an area by adding together lengths. The Egyptian solved the logical difficulty in one of two ways. He either put in no dimensions until the working was complete, or he put in the final dimensions straight away. The latter method might be justified by considering the first line of the multiplication as giving the product of say 9 *khet* by 1 *khet*, *i.e.* 9 square *khet* or *setat*, and not as a mere statement that the multiplicand is 9 *khet*.

There is a somewhat similar example in No. 53. Here, despite the obscurity of detail and meaning, we are clearly dealing with the calculation of certain areas. In the *w:h tp* process the products (on the left in the original) are given in square measure from the first line to the last. The multipliers (on the right) ought, of course, to be in the pure arithmetical notation, and indeed the integers 1 and 2 are; but when we come to the fractional multiplier $\frac{1}{2}$ we are surprised to find it expressed in the form peculiar to square measure, the arm (half-*setat*) standing instead of the pure number $\frac{1}{2}$.

No. 49. (Pl. O.)

No. 49.

"Example of calculating (?) land. If it is said to thee, A rectangle of land of 10 *khet* by 2 *khet*. What is its acreage?"

The doing as it occurs:

1	1000
10	10,000
100	100,000
	$\frac{1}{10}$ th of 100,000 is 10,000
	$\frac{1}{10}$ th of $\frac{1}{10}$ of it is 1000

This is its content in land."

No. 49.

It is not easy to deal with the text of this and the following examples, for they teem with scribe's errors of the worst description. The problem is to determine the area of a rectangle whose sides are 10 and 2 *khet*. This is clear both from the setting out and from the figure. Yet the working is not consistent with this. The absence of the 2 from the working means either that the scribe was totally ignorant of the whole method or that there is an error in the setting and in the figure.¹ If we read 1 *khet* instead of 2 for the shorter side the working will be correct, though even then it is clumsy. The answer, as usual in these sums, is in cubits-of-land, *i.e.* in narrow strips each 100 cubits by 1 cubit, and the correct answer is 1000 of these. This is the answer given, but it should have been obtained directly from the two dimensions of the rectangle 10 and 1 *khet*. The multiplication of these gives 10 *setat* or square *khet*, and as each square *khet* contains 100 cubits-of-land the answer is 1000 cubits-of-land, or one thousand-of-land. The scribe, however, reduces the 10 *khet* to 1000 cubits and multiplies it by the 1 *khet*, which is 100 cubits.² This gives 100,000 square cubits, which he then reduces to cubits-of-land quite correctly by dividing by 100.

In the opening phrase *tp n ist*, *ist* can hardly be a noun—"Example of an *ist* of land"—for three reasons. In the first place this phrase would probably have been rendered by *tp n ist nt* (or *m*) *sh* (cf. Nos. 51 and 52), in the second place the sense would require a concrete meaning for *ist*, which is not borne out by its abstract determinative, and in the third place *tp n* is always followed in this papyrus by an infinitive. Thus, if the text is correct, *ist* ought to be the infinitive of a verb *isi*, which the sense would require to have a transitive meaning: unfortunately no such verb is known. Is it possible that the correct reading is *tp n nis*, "example of calling up," *i.e.* of reckoning out, as in Nos. 44 and 56?

ntf pw m sh. *ntf* might be used here in its normal pronominal sense, "This is it (*sc.* the *ifd*) reckoned in land-area," but is perhaps better taken in the pregnant sense first demonstrated by Erman,³ "belonging to it," or even nominally "its property," *i.e.* "its content."

No. 50. No. 50. (Pl. O.)

"Method of reckoning a circular piece of land of diameter 9 *khet*. What is its area in land?

You are to subtract one-ninth of it, namely 1; remainder 8.

You are to multiply 8 eight times; it becomes 64. This is its area in land, 6 thousands-of-land and 4 *setat*.

The doing as it occurs:

1	9
$\frac{1}{9}$ of it	1
subtract from it;	remainder 8

1	8
2	16
4	32
8	64

Its area in land is 6 thousands-of-land, 4 *setat*."

There is little to notice here. The sum is worked out correctly in square *khet* by the usual rule for calculating the area of a circle. The result is 64 square *khet*, which is reduced to 6 thousands-of-land (written like 60⁴) and 4 *setat* or square *khet*.

¹ Perhaps there is confusion with a right-angled triangle of base 2 *khet* and height 10.

² Cf. No. 52.

³ *A.Z.*, 34, 50 ff.; cf. *A.Z.*, 41, 135-6.

⁴ The sign is here as in No. 48 definitely 60, not the cursive form of 6 used above in No. 14 and in Eloquent Peasant, B 2, 136.

No. 51. (Pl. O.)

No. 51.

"Example of reckoning a triangle of land.¹ If it is said to thee, A triangle of 10 *khet* in its height and 4 *khet* in its base. What is its acreage?

The doing as it occurs:

You are to take half of 4, namely 2, in order to give its rectangle. You are to multiply 10 by 2. This is its acreage.

1	400	1	1000
$\frac{1}{2}$	200	2	2000
Its acreage is 2. ²			

Here we meet for the first time the triangle (*špdt*, "the pointed") and its area. Is this correctly determined? Eisenlohr thought not, and mathematicians have for the most part accepted his dictum. Yet the matter is hardly so simple as he supposed. There are in reality two interdependent problems to be solved. Firstly does the solution given apply to all triangles or only to those of a special type, and secondly is it correct? The evidence at our disposal consists of the following:—

- (1) The names of the triangle and its parts.
- (2) The position of the numbers marked in the figure.
- (3) The shape of the figure.
- (4) The striking phrase "This is its rectangle."

(1) The name *špdt* here given to the triangle gives no clue as to the shape of the figure. It is true that "the pointed figure" calls up most readily the idea of a triangle with short base and sharp vertex, *i.e.* with one angle definitely less than the others; but this hardly amounts to evidence, and even if this was the original meaning it is extremely likely that as a mathematical technical term it applied to the triangle in general.

The word *tp r* is a compound in which, as in so many Egyptian compounds, the *tp* adds little in meaning to the simple noun, and it is to be translated simply "mouth," Coptic **τᾱρρο**. It is beyond all doubt that it stands for the base of the triangle, and the very use of this word does lend some colour to the belief that the triangle dealt with is one with a sharp apex, the narrow base of which can be reasonably described as the "mouth," lying between the two long sides envisaged as jaws.

The word *myrt* is the crux of the problem. Its literal meaning is the "bank" or "edge" of a river or sea, more particularly a "harbour" or "quay." This being the case, the obvious rendering of the word as a mathematical term would be the "edge," *i.e.* the side of the triangle, a meaning which seems doubly suitable in the case of a triangle with narrow base, but which—and here lies the crux—presupposes that the two long sides are equal, for otherwise there would obviously be two different solutions for the area of a triangle, namely half the base multiplied by the two sides respectively. The only other possibility would seem to be to reject the tempting analogy of "side" or "edge" and to regard *myrt* as the vertical height, *i.e.* the length of the perpendicular from the apex to the base. This is just possibly the correct solution.

(2) With regard to the position of the numbers marked in the figure it is clear that the 4 *khet* written beside the base or *tp r* refers to that side, and by analogy it might be expected that the 10 *khet* written along the middle of the upper long side referred to the length of that side. It may be so, but it is far from certain. In Nos. 56–58, for instance, a measurement is written outside the left side of the pyramid figure which beyond all possible doubt gives the vertical height and not the slant height. This may, it is true, be partly due to the exigencies of arrangement in narrow horizontal bands, but it does at least show that the line on which a marked measurement is taken need not actually be drawn. But

¹ Or, "of finding the area of a triangle in land."

² *sc.* thousands-of-land.

No. 51. we may go farther than this. In No. 53, a difficult problem, but not totally devoid of sense, we have a picture of a triangle divided into three by lines presumably parallel to its base. The base of the apex portion is marked $2\frac{1}{4}$ in red, its area is filled in as $7\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$ in red, and along the lower side *almost at the apex* is a 7. In the working half the base, *viz.* $1\frac{1}{8}$ *khet*, is multiplied by this 7 *khet* and gives the area as marked in the figure. Clearly the 7 is the *mryt*, and yet it is here written not along the side, but exactly in the position occupied by the vertical height figure in the illustrations of the pyramids in Nos. 56-58. This shows that nothing must be argued from the position of the 10 *khet* in No. 51.

(3) It would be unwise to attribute too much importance to the shape of the triangles actually drawn in the papyrus. At the same time we should keep in mind the fact that the triangles illustrating these three problems, 51, 52 and 53, are as a matter of fact firstly isosceles and secondly erected on comparatively narrow bases.¹

(4) The words "This is its rectangle" make it probable that the solution was obtained graphically. It is of course just conceivable that the Egyptian, as Eisenlohr supposes, erected a rectangle on one of the long sides with half the base as the other dimension; but if this is the case, we must suppose that the solution only applied to tall isosceles triangles, isosceles because any other would yield two possible rectangles, tall because in the case of short triangles the solution is a manifest absurdity. It would, however, seem more logical to explain the words "This is its rectangle" by means of some such graphic solution as that shown in Figs. 3 and 4. If the triangle is isosceles and the *mryt* is its vertical height, we actually see in the drawing the rectangle contained by half the base and the height, Fig. 3. If the two sides of the triangle are unequal "its rectangle" is not actually shown, but a rectangle is seen (Fig. 4)

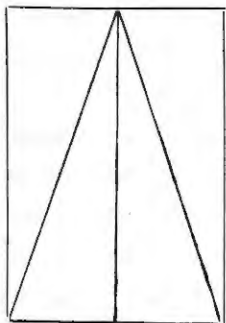


Fig. 3.

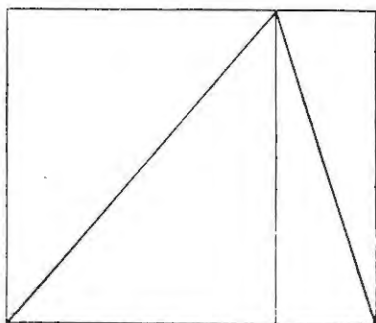


Fig. 4.

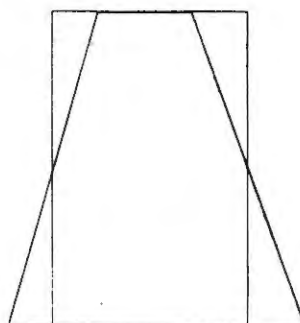


Fig. 5.

which by mere symmetry of triangles is clearly double the triangle in area. The same reasoning applies to No. 52, where the area of a truncated triangle is obtained by taking half the sum of the two parallel sides "in order to get its rectangle" and multiplying it by the *mryt*. Here it is possible to conceive the solution as having been obtained graphically as shown in Fig. 5. Did *mryt* here mean the slant side (with the involved assumption that the figure is regular), extreme cases would surely have shown the absurdity of the solution except in the case of very tall triangles.

The internal indications thus make it very difficult to draw any certain conclusion as to the exact meaning of *mryt* and the consequent correctness or otherwise of the Egyptian solution.

We have now to ask ourselves whether the triangle treated in this example is a general triangle or one with special properties. The possibilities are three: it may be scalene, *i.e.* general, isosceles, or right-angled.

(1) If it is scalene, the *mryt* must be the vertical height, for otherwise there would be two *mryt* and two possible solutions. In other words, if the triangle is scalene, then the Egyptian solved the problem correctly.

¹ The same is true of a similar but much damaged problem in the Moscow papyrus. A gap in the papyrus makes it impossible to say where the measurement of the *mryt* was written in the figure.

(2) If it be isosceles, the *mryt* might be either the vertical height or the length of one No. 51. of the equal sides. If it be the vertical height, it follows that the Egyptian had correctly determined the area of an isosceles triangle (probably graphically); but we cannot assume that he had also correctly solved the scalene triangle.

If the *mryt* is the length of the long sides, as Eisenlohr supposes, we can only say that the Egyptian had formed an approximation to the area of an isosceles triangle which was fairly accurate when the base was very small and became increasingly inaccurate as the base increased.

(3) If it be right-angled. This supposition, unlikely in the face of the definitely non-right-angled figures, which, however, are not final in themselves, must be ruled out in view of the fact that in the Moscow papyrus the right-angled triangle is clearly perfectly well understood and treated as half a rectangle: the two sides enclosing the right-angle are actually called the "length" and "breadth" respectively, terms manifestly taken from the terminology of a rectangle.

The matter may be summed up as follows. The *mryt* is either the vertical height or slant height of a triangle. In the first case the evidence is insufficient to show whether the solution given refers to a scalene triangle or only to the isosceles triangle, where a graphic solution may have been obtained. In the second, the triangle must obviously be isosceles, and, what is more, if the solution is to be even an approximation to the truth, the triangle must have a very small base.

It should be noted that whatever decision we here make must have its corollary in No. 52. If the *mryt* be the vertical height here it must be the same there also, and the solution will be correct whether the truncated triangle be isosceles or not. If on the other hand the *mryt* be the slant height, then the truncated triangle must be isosceles to avoid ambiguity and double solution.

A very similar difficulty also arises in the case of the problem of the truncated pyramid in the Moscow Papyrus published by Turaiev.¹ Here we have a frustrum of a pyramid. The top surface is 2 cubits (*a*) square and the bottom 4 cubits (*b*) square. The *sti* (*h*) is 6 cubits, and the volume is calculated by the formula

$$V = \frac{h}{3}(a^2 + ab + b^2)$$

If the *sti* is the vertical height of the frustrum the solution is correct, and in this case the Egyptian has here made a very notable achievement. Turaiev gives him this credit; but in view of the difficulty with regard to the *mryt* of a triangle, we must seriously ask ourselves whether the *sti* might not just be the slant height of the figure and not the vertical, assuming in this case that the pyramid was regular in the sense above described. On present evidence we have no means of deciding this, though it would be surprising that a people who arrived at the element $(a^2 + ab + b^2)$ correctly should have made so gross an error as to multiply it by a third of the slant instead of the vertical height. This however hardly amounts to argument.

So far we have judged the question of the solution of the triangle on purely internal evidence. There is one piece of external evidence which to Eisenlohr seemed so cogent that it led him to pronounce in favour of the *mryt* being the slant and not the vertical height of the triangle. In the great dedicatory inscription of the temple of Edfu, built by Ptolemy XI, mention is made of a large number of fields.² In each case four linear dimensions are given, which we may call *a*, *b*, *c* and *d*, and the area is determined by the formula

$$\text{Area} = \left(\frac{a+c}{2}\right) \left(\frac{b+d}{2}\right)$$

¹ *Ancient Egypt*, 1917, 100-102.

² BRUGSCH, *Thesaurus*, 531 ff.

- No. 51. where, presumably, a and c , b and d are pairs of opposite sides.¹ When the field is triangular the solution is obtained by making d equal to 0, *i.e.* regarding the triangle as a special case of a quadrilateral with one side zero.

This method of measuring land is by no means unique; there is good evidence for believing that in the Ptolemaic, Roman and Coptic periods it was the means by which land was measured in Egypt for purposes of taxation.² Now there are cases in these documents in which not only is one side zero, but the other pair of opposite sides is equal. The figure then becomes an isosceles triangle and its area is determined by the formula as $\left(\frac{a+0}{2}\right)\left(\frac{b+b}{2}\right)$ or $\frac{1}{2}ab$, *i.e.* half the base into one of the equal sides. Eisenlohr, who believes that the triangle in No. 51 is isosceles and that the *mryt* is one of the equal sides, maintains that the use of an identical formula in Egypt in later times bears out his claim. This is however hardly decisive. This method of field measuring was admittedly no more than an approximation for taxation purposes, and fractions less than $\frac{1}{84}$ of a square *khet*, sometimes even $\frac{1}{32}$ of a square *khet*, were omitted. Such a method necessitated only the measuring by rope of such lines as were already present, namely the four sides, whereas a correct determination would have involved measuring a diagonal and two perpendiculars dropped on to it, a very much more complicated process. In all probability large numbers of fields were approximately rectangular, in which case the error was small; and finally be it noted that the error was always in the tenant's favour, for the area of a quadrilateral field whose sides are a , b , c and d is $\frac{1}{2}\Sigma(ab \cdot \sin ab)$, where the angle ab is that contained by the two sides a and b . This is obviously a maximum when the sines of ab , bc , cd , and da are all unity, *i.e.* when the figure is a rectangle. Thus the formula $\text{Area} = \left(\frac{a+c}{2}\right)\left(\frac{b+d}{2}\right)$, *i.e.* $\frac{1}{4}\Sigma ab$, always gives a result smaller than the truth except in the case of a rectangle, when it is correct, or, in other words, the tenant never lost by this rough system of measurement.

In short we are hardly justified in arguing that, because the tax-gatherers of later Egypt reckoned quadrilaterals in a manner which disregarded the correct solution for the area of a triangle, the Egyptian mathematician of the Middle Kingdom was not acquainted with this correct solution. The question must be decided on other grounds than these.

- No. 52. No. 52. (Pl. P.)

"Example of reckoning a truncated triangle of land. If it is said to thee, A truncated triangle of land of 20 *khet* in its height, 6 *khet* in its base and 4 *khet* in the cut side. What is its acreage?

You are to combine its base with the cut side: result 10. You are to take a half of 10, namely 5, in order to give its rectangle. You are to multiply 20 five times, result 10 (*sic*). This is its area.

The doing as it occurs:

1	1000	— 1	2000
$\frac{1}{2}$	500	2	4000
		— 4	8000
		Total	10000, making in land 20 (read 10)

This is its area in land."

¹ Simon in his *Geschichte der Mathematik in Altertum*, 47-8, has rightly pointed out that in the later portion of this inscription, which was still unpublished when Eisenlohr wrote, this formula is in some cases not adhered to. He takes this to indicate that the Egyptians were themselves aware of its approximate nature, and in certain cases corrected the result. It would seem more likely that the variations from the formula are merely due to the inaccurate copying of a scribe or of a sculptor.

² See KENYON, *Catalogue of Greek Papyri in the Brit. Mus.*, II, 129 ff. (Pap. CCLXVII); GRENFELL, HUNT and SMYLY, *Tebtunis Papyri*, Pt. I, 385 ff.; CRUM, *Coptic Ostraca*, 42 ff., Ostrakon D.12; HALL, *Coptic and Greek Texts of the Christian Period in the Brit. Mus.*, 128, Coptic Ostrakon 29750.

The word *h:k* occurs in the Pyramid Texts, 673 c,¹ where it quite obviously means No. 52. "to cut off the tail," whence the determinative in our passage. The cutting line is clearly assumed to be parallel to the base.

The word *mryt* is here rendered by the ambiguous "height." In the light of the discussion on No. 51 it will be seen that it refers either to the vertical height of the figure, in which case the solution was obtained graphically, see Fig. 5, p. 92, and is correct, even if the triangle be not isosceles, or else to the slant height, in which case the triangle was clearly envisaged as isosceles, and the solution is wrong.

Two sets of working are given, the first without heading and the second under the title of *irt mi hpr*. In the first the unit is the *khet*. The two parallel sides are added and give 10 *khet*. This is now halved (5), and the result multiplied by the *mryt* in *khet*, namely 20. The result should be 100 square *khet*, but it is mentally divided by 10 in order to reduce it to thousands-of-land, which is the form in which the answer is expected in these sums.

In the second working the units are badly muddled. First 10 *khet*, the sum of the two parallel sides, is reduced to 1000 cubits and halved, giving 500 cubits. Then the 20 *khet* of the *mryt* are reduced to 2000 cubits and multiplied, not, as one would expect, by the 500 cubits, but by these turned back into 5 *khet*. This has the advantage of giving the result direct in cubits-of-land, i.e. in hundredths of square *khet*, and a division by 1000 reduces these to 10 thousands-of-land (wrongly written 20). The confusion of units here is doubtless entirely due to the desire to avoid using square *khet*, the Egyptian land-measurers preferring for practical use the cubit-of-land and the thousand-of-land.

No. 53. (Pl. P.)

No. 53.

— 1	$4\frac{1}{2}$ setat
— 2	9 setat
$\frac{1}{2}$	$2\frac{1}{4}$ setat
— $\frac{1}{4}$	$1\frac{1}{8}$ setat
Total $(5\frac{1}{2} + \frac{1}{8})$ setat	
$\frac{1}{10}$ of it is $(1\frac{1}{4} + \frac{1}{8})$ setat + 10 cubits-of-land ²	
$\frac{1}{10}$ of it subtracted, then this(?) is the area.	
1	7 setat
— 2	1 thousand-of-land and 4 setat
$\frac{1}{2}$	$3\frac{1}{2}$ setat
— $\frac{1}{4}$	$(1\frac{1}{2} + \frac{1}{4})$ setat
Total 1 thousand-of-land and $(5\frac{1}{2} + \frac{1}{4})$ setat	
$\frac{1}{2}$	$(7\frac{1}{2} + \frac{1}{4} + \frac{1}{8})$ setat.

It is hardly worth while to spend much time on a problem which is clearly incomplete and incorrect. All that is to be made out is that the second calculation is the finding of the area of the small triangle whose base and height³ are marked in the figure as $2\frac{1}{4}$ and 7 respectively. In this problem the multiplier $\frac{1}{2}$ is actually written as if it were $\frac{1}{2}$ setat (see p. 89), while there is continual confusion between thousands-of-land, which should be shown in free-standing units, and setat, which should be placed beneath the rectangle sign.

The first calculation is hopeless, and appears neither to have a meaning in itself nor to bear any relation to the figure. It begins by multiplying $4\frac{1}{2}$ by $1\frac{1}{4}$, correctly, despite inaccurate ticking and the introduction of an unnecessary step. The next lines are totally unconnected with this, and clearly come from some other reckoning. Possibly something has

¹ I owe the reference to Gunn.

² The sign, which exactly resembles the hieratic for 30, clearly means 10 cubits-of-land from No. 54. It is perhaps very cursively made in both cases.

³ For the ambiguity in this term see Nos. 51 and 52.

No. 53. been omitted by haplography. There may even have been confusion with No. 54, where 7 *setat* and the number 10 also occur.

For the dimidiated parts of the *setat* here used see Introduction, pp. 24-5.

No. 54. No. 54. (Pl. P.)

"To divide <7 *setat*> of land into 10 fields.

	1	10
—	$\frac{1}{2}$	5
—	$\frac{1}{5}$	2
1	$(\frac{1}{2} + \frac{1}{5}) \text{ setat} + 7\frac{1}{2} \text{ cubits-of-land}$	
—2	$(1\frac{1}{4} + \frac{1}{5}) \text{ setat} + 2\frac{1}{2} \text{ cubits-of-land}$	
4	$(2\frac{1}{2} + [\frac{1}{4}]) \text{ setat} + 5 \text{ cubits-of-land}$	
—8	$5\frac{1}{2} \text{ setat} + 10 \text{ cubits-of-land.}$	

The problem, as is clear from the working, is to divide 7 *setat* of land (unfortunately omitted by the scribe in line 1) into 10 fields, for "fields" must be the meaning of the second *ht*. The method is to divide 7 by 10 (see, however, notes on No. 55), result $\frac{1}{2} + \frac{1}{5}$. This quantity is then expressed in *setat* and cubits-of-land, multiplied by 10, and shown (though the actual addition is missing) to yield 7 *setat*.

Note that in the first step, division of 7 by 10, no units are mentioned; contrast No. 55, and see commentary there.

The notation of the *setat* here used is perfectly regular, see pp. 24-5. The special hieratic signs are used for the $\frac{1}{2}$, the $\frac{1}{4}$ and the $\frac{1}{5}$ of the *setat*, which, with $\frac{1}{10}$ and $\frac{1}{32}$, are the only fractions permitted, odd amounts being entered in cubits-of-land, expressed by placing the number under the arm or cubit-sign. Ten cubits-of-land has, however, a special sign, precisely like the hieratic for 30 (cf. No. 53), but perhaps in reality a cursive writing of a 10 under an arm: half a cubit-of-land is shown in hieratic by a sign equivalent to that used for the pure fraction $\frac{1}{2}$.

No. 55. No. 55. (Pl. P.).

"To divide 3 *setat* of land into 5 fields. You are to operate on 5 *setat* (*sic*) to find three *setat* of land.

	1	5
<i>sic</i>	$\frac{1}{2}$	$2\frac{1}{2}$
<i>sic</i>	$\frac{1}{10}$	$\frac{1}{2}$
	$\frac{1}{2} + \frac{1}{10} \text{ results.}$	

You are to multiply $\frac{1}{2} + \frac{1}{10}$ five times:

—1	$\frac{1}{2} \text{ setat} + 10 \text{ cubits-of-land}$
2	$1\frac{1}{5} \text{ setat} + 7\frac{1}{2} \text{ cubits-of-land}$
—4	$2\frac{1}{4} + \frac{1}{5} \text{ setat} + 2\frac{1}{2} \text{ cubits-of-land}$

Thus you find the acreage to be 3 *setat*."

The problem is exactly similar to the last. In the setting out the scribe has written "1 *setat*" instead of the simple numeral 5, which it remotely resembles. There is considerable confusion of units and dimensions. A modern worker would divide 3 *setat* by 5 and get his answer direct in *setat*. The Egyptian here quite illogically divides 3 *setat* by 5 *setat*, and gets his answer as a pure fraction. Yet there is a reason for this. The very nature of Egyptian division makes it impossible to obtain the quotient otherwise than in the form of pure number, for it is obtained by adding together certain of the trial multipliers on the left, which can only be pure numbers, not weights or lengths. Thus if we wish to divide 3 square

miles by 5 we can do so directly and get the answer in acres, square poles, square yards, No. 55. square feet, etc. An Egyptian has no direct process for doing this; he can only divide 3 by 5, giving $\frac{1}{2} + \frac{1}{10}$, and turn this afterwards into acres, square poles, square yards, etc. Thus in the present case an Egyptian could not mark $\frac{1}{2} + \frac{1}{10}$ as *setat*, because the *setat*-notation does not recognize such a quantity; it must be expressed as $\frac{1}{2}$ *setat* plus 10 cubits-of-land. Similarly when in No. 54 $\frac{1}{2} + \frac{1}{5}$, obtained as a pure number, comes to be converted into *setat* it must be turned into the acknowledged notation, just as, if expressed in *hekat*, it would have to be shown in the Horus-eye notation. The $\frac{1}{2}$ is allowed to stand, being one of the fractions for which a sign exists, but $\frac{1}{5}$, i.e. 20 cubits-of-land, must be broken up into $\frac{1}{8}$ *setat*, which is $12\frac{1}{2}$ cubits-of-land, plus $7\frac{1}{2}$ cubits-of-land. Egyptian cannot write $\frac{1}{5}$ *setat* any more than it can write $\frac{1}{3}$ *hekat*.

The use of the verb *lbi* here and in No. 54 is unusual. This verb followed by the preposition *lnt* generally means "to take away" or "subtract from" (Nos. 43, 50 and 64). Yet there seems no way of avoiding the conclusion that it here refers to division.

PART III. ANGLE OF SLOPE OF PYRAMIDS, &c. Nos. 56-60.

No. 56. (Pl. Q.)

No. 56.

“Example of reckoning out a pyramid 360 in length of side and 250 in its vertical height. Let me know its batter.

You are to take half of 360: it becomes 180.

You are to reckon with 250 to find 180.

Result $\frac{1}{2} + \frac{1}{5} + \frac{1}{50}$ of a cubit.

A cubit being 7 palms, you are to multiply by 7:

1	7
$\frac{1}{2}$	$3\frac{1}{2}$
$\frac{1}{5}$	$1\frac{1}{3} + \frac{1}{15}$
$\frac{1}{50}$	$\frac{1}{10} + \frac{1}{25}$

Its batter is $5\frac{1}{5}$ palms."

The meaning of this and the following examples depends entirely on the interpretation given to the three terms *whs:-tbt*, *pr-m-ws* and *skd*. It is fairly obvious from the figure that the *whs:-tbt* is a ground measurement, and if so it can hardly be other than the diagonal or side of the base. Similarly *pr-m-ws* is clearly a measurement of height, not necessarily vertical, and as such it might be either the vertical height, the slant height from a corner to the apex, or the slant height from the mid-point of a side to the apex. The required *skd* is clearly the relation of the *pr-m-ws* to half the *whs:-tbt*. See Fig. 6.

The diagram illustrates a triangle with apex *D* and base line *Q*. A vertical line segment connects *D* to the base. Two slanted lines also connect *D* to the base. A horizontal line segment is drawn across the triangle, intersecting the vertical line and one of the slanted lines.

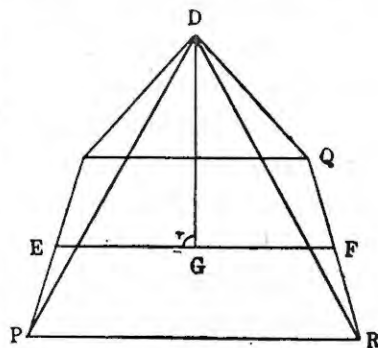


Fig. 6.

Eisenlohr took the *wh₃-tbt* to be the diagonal of the base, PQ, and the *pr-m-wš* to be the slant height from a corner to the apex, DP; the *škd* would then be the cosine of the angle DPQ made by the edge with the diagonal of the base. In support of this interpretation he argued that it gives a batter which agrees well with that of certain existing pyramids and secondly that it is unlikely that *wh₃-tbt* and *pr-m-wš* should mean the same as *snti* and *k₃i n hrw* of No. 60, which he held to be beyond all doubt the length of the side of the base and the vertical height.

This view is certainly erroneous.¹ In the first place the monument dealt with in No. 60 is not a pyramid but an *urn*, and there is no reason at all why similar measurements in this and a pyramid should not have had totally different technical names. In

¹ It was first attacked by Borchardt in *Ä.Z.*, 31, 9 ff.

- No. 56. the second place Eisenlohr's interpretation gives for the *škd* a measurement for which it is impossible to see any practical use. The Egyptians always thought concretely, even when doing mathematics, and the clue to the understanding of this problem certainly lies in seizing the true significance of the reduction of the answer to palms per cubit. The object is to provide the mason with a simple practical rule for dressing at the required angle the stones of the outer facing of the pyramid. All he has to do in this case, when confronted with a solid block of stone, parallelipedal in shape, is to measure one cubit upwards on the outer edge, and then $5\frac{1}{2}\frac{1}{5}$ palms inwards at right-angles. The line joining the point thus reached to the point from which he started gives the correct angle of dressing. Of the outer blocks of a pyramid only a very small minority lie on the slant edges, while the vast majority lie in the sloping sides. The figure needed by a mason would thus be not the angle of the slant edges given by Eisenlohr's interpretation, but that of the sides. What is more, the slant edges of a pyramid are in actual building merely determined secondarily as the intersections of the sides.

We can make the *škd* correspond with this ratio if we take the *pr-m-ws* to be the vertical height DG, and the *wh₃-tbt* to be a side of the base, for the angle of slope (DĜG) of a side QDR is one whose cotangent is half a side of the base divided by the vertical height, i.e. $\frac{GF}{GD}$.

No one who has grasped the essentially practical nature of all Egyptian mathematics will doubt the accuracy of the above solution. Unfortunately no corroboration can be obtained from the names of the measurements themselves. The word *wh₃-tbt*, a compound formed of the verb *wh₃* "to seek" and the noun *tbt* "a sandal," should refer to a ground measurement, but that is as far as we can go. The compound *pr-m-ws*, perhaps the origin of *πυραμῖς*, has by some been taken to mean "that which comes forth from the saw," i.e. a measurement which can only be seen in section. If this were the case the word *ws* could hardly have been written without the saw or knife-determinative. The house-sign after *ws* seems to be needed as a determinative to *ws* itself and can hardly apply to the whole compound *pr-m-ws*. *ws* therefore should be some kind of building, but I can find no examples of its use.

The word used for batter is equally obscure. If the *š* is causative it may be formed from *kd* "to build" or "form," and it would be one of those cases where the addition of the causative prefix barely alters the meaning. *škd* would then mean "that which forms," i.e. the measurement which builds up the pyramid, as indeed it is, for, the base once marked out, the laying of outer blocks accurately cut to the *škd* determines the structure. Yet in this case we should rather expect with *škd* the sign of the man building and not merely the abstract determinative.

The method of this problem needs little comment. Note that the dimensions 360 and 250 are given in no particular unit. The Egyptian was doubtless aware that the measurement he proposed to find, being a ratio and not a length, was independent of the unit of the original dimensions.

The 360 is halved, result 180, and this last is then divided by 250. The result is then illogically stated to be $\frac{1}{2} + \frac{1}{5} + \frac{1}{50}$ of one cubit, instead of the pure number $\frac{1}{2} + \frac{1}{5} + \frac{1}{50}$. Here the Egyptian really means to say that 180 is to 250 in the same proportion that $\frac{1}{2} + \frac{1}{5} + \frac{1}{50}$ of a cubit is to a cubit, or in other words that the angle determined by the base 180 and perpendicular 250 can also be determined by base $(\frac{1}{2} + \frac{1}{5} + \frac{1}{50})$ cubit and perpendicular 1 cubit. He has introduced the 1 cubit simply in order to reduce his result to a practical form for the use of the mason.¹ The sum is made still more practical by the reduction of the $\frac{1}{2} + \frac{1}{5} + \frac{1}{50}$ cubit to palms, namely $5\frac{1}{2}\frac{1}{5}$.

¹ For a most interesting practical application of this *škd*-value in the building of a mastaba see PETRIE, *Medum*, Pl. VIII, and text thereto. There is some admirable material relevant to these problems in BORCHARDT, *Gegen die Zahlenmystik an der grossen Pyramide bei Gise*; Berlin, 1922.

No. 57. (Pl. Q.)

No. 57.

"A pyramid 140 in length of side, and 5 palms and a finger in its batter. What is the vertical height thereof?

You are to divide one cubit by the batter doubled, which amounts to $10\frac{1}{2}$.¹ You are to reckon with $10\frac{1}{2}$ to find 7, for this is one cubit.

Reckon with $10\frac{1}{2}$: two-thirds of $10\frac{1}{2}$ is 7.

You are now to reckon with 140, for this is the length of the side:

Make two-thirds of 140, namely $93\frac{1}{3}$. This is the vertical height thereof."

In this example we are given the length of the side and the amount of the batter in palms and fingers (a finger being one-fourth of a palm) per vertical cubit. We are asked to find the vertical height.

The working, though correct, is slightly obscured to our modern way of thinking by the fact that, instead of finding EG (see Fig. 6, p. 97) from the datum EF by halving, and then determining DG by means of the batter, the batter is doubled (twice $5\frac{1}{4}$ palms = $10\frac{1}{2}$) and the proportion used is:—

$$DG : EF :: 7 : 10\frac{1}{2}$$

There is an admirable parallel to this illogical style of working in No. 45.

No. 58. (Pl. Q.)

No. 58.

"A pyramid whose vertical height is $93\frac{1}{3}$. Let me know its batter, 140 being the length of its side.

You are to take half of 140, namely 70. You are now to reckon with $93\frac{1}{3}$ to find 70.

Reckon with $93\frac{1}{3}$: its half is $46\frac{2}{3}$,

its quarter is $23\frac{1}{3}$.

You are to make $\frac{1}{2} + \frac{1}{4}$ of a cubit. Reckon with 7; its half is $3\frac{1}{2}$; its quarter is $1\frac{1}{2} + \frac{1}{4}$; total 5 palms 1 finger. This is its batter.

Working out:

1	$93\frac{1}{3}$
$\frac{1}{2}$	$46\frac{2}{3}$
$\frac{1}{4}$	$23\frac{1}{3}$

You are to make $\frac{1}{2} + \frac{1}{4}$ of a cubit.

Now a cubit is seven palms.

1	7
$\frac{1}{2}$	$3\frac{1}{2}$
$\frac{1}{4}$	$1\frac{1}{2}$ (read $1\frac{1}{2} + \frac{1}{4}$)
Total	5 palms 1 finger.

This is the batter."

This example is concerned with the same numbers as the last, but here we are given the length of the side and the height and are asked to find the batter. The method is logical and consists simply in halving the side and dividing the resulting 70 by the height $93\frac{1}{3}$. The numbers are suitably chosen, for 70 is precisely $\frac{3}{4}$ or $(\frac{1}{2} + \frac{1}{4})$ of $93\frac{1}{3}$. It then only remains to reduce $\frac{1}{2} + \frac{1}{4}$ of a cubit to palms and fingers, which is done by multiplying 7 by it. Result $5\frac{1}{4}$ palms or 5 palms 1 finger.

iw ir mh 1 šsp 4 pw. The position of *iw* in front of *ir* is interesting syntactically. Cf. *iw ir didit hr nb dbn* in No. 62. Gunn quotes also *Urk.*, IV, 366, 13.

The traces after the numeral do not suit  (for its use in the Nominal Sentence with *pw* cf. Nos. 64, 70 and 71 and notes), nor yet  read by Borchardt.

¹ Literally " $10\frac{1}{2}$ results," i.e. from the doubling, not the division. The translation given avoids the ambiguity.

No. 59. No. 59. (Pl. Q.)

"A pyramid the vertical height (*sic*) whereof is 12 and the side (*sic*) 8. <Find its batter.>
You are to reckon with 8 to find 6, for this is half the height.

$$\begin{array}{r} 1 \quad 8 \\ \diagdown \frac{1}{2} \quad 4 \\ \diagdown \frac{1}{4} \quad 2 \end{array}$$

You are to take a half and a quarter of 7, for this is 1 cubit.

$$\begin{array}{r} 1 \quad 7 \\ \diagdown \frac{1}{2} \quad 3\frac{1}{2} \\ \diagdown \frac{1}{4} \quad 1\frac{1}{2} + \frac{1}{4} \end{array}$$

It comes to 5 palms 1 finger. Behold this is its batter.
What ?"

No. 59b. No. 59B. (Pl. Q.)

"You are to reckon a pyramid of 12 (*sic*), whose batter is 5 palms 1 finger. Let <me> know the height thereof.

You are to reckon with $5\frac{1}{4}$ doubled to find 1 cubit, which is 7 spans: it comes to $10\frac{1}{2}$. Two-thirds of it is 7.

Reckon with 12: two-thirds of it is 4 (read 8). Behold this is the height."

Nos. 59 and 59B are two quite distinct problems, the second of which is the reverse of the first. Unfortunately the scribe did not realize this, and has made No. 59B appear as part of the working of No. 59 by introducing it by the word *hr·hr·k* and failing to use red ink for the opening word or words. The verbal form *sdm·hr·k* is never used in this papyrus to introduce a problem. In the original from which our scribe copied the problem was probably, like its fellows, introduced directly by *mr* "a pyramid." It is possible that this was in black instead of being, as it should be, in red. We know enough of our scribe's intelligence to assert that this would be quite sufficient to conceal from him the fact that a fresh problem had begun.

In No. 59 there is an unfortunate error in the setting out, for the side and the height have been transposed. In order to give a batter of 5 palms 1 finger it is the height which must be 8 and the side 12. Otherwise the batter would be only $2\frac{1}{2}$ spans.

The phrase which ends the problem is a mystery. It may be corrupt. Is it connected with the scribe's error in running the next problem on to this?

In No. 59B we are given the side and batter and asked to find the height. After the words "a pyramid of 12" we expect *m whi·tbt·f* "in its side"; but it is quite possible that the Egyptian is sufficient as it stands, and that "a pyramid of 12" was the technical expression for "a pyramid built on a base 12 square."

No. 60. No. 60. (Pl. R.)

"A cone (?) of 15 cubits in its base and 30 in its height. Let me know its batter.

Reckon with 15: its half is $7\frac{1}{2}$.

Multiply $7\frac{1}{2}$ four times (*sic*) to find 30.

The result is 4. This is the batter thereof.

Working:

$$\begin{array}{r} 1 \quad 15 \\ \diagdown \frac{1}{2} \quad 7\frac{1}{2} \\ 1 \quad 7\frac{1}{2} \\ 2 \quad 15 \\ \diagdown 4 \quad 30 \end{array}$$

No. 60. from one of the examples in which it occurs. There can be no doubt that this view is correct. I should, however, be inclined not to delete the $\searrow$ from the text, for though the form hpr m X "it becomes X " occurs in the Berlin Papyrus 6619¹ and is grammatically possible, hpr being a $\underline{s}dm.f$ -form without ending, the form $hpr.f$ m X is usual in our papyrus. A further argument against the retaining of the text as it stands is the fact that if rhy or even $\underline{s}trhy$ meant the vertical height in which the sloping side diverges from the vertical by a cubit, it would be nonsense to add, as the papyrus does, "This is its $\underline{s}kd$," for the $\underline{s}kd$ is the reverse of this measurement, namely the divergence from the vertical in a vertical height of one cubit.

The fact is that it is useless to attempt to defend the text as it stands. The $\underline{s}twi$ has no place here and must have come from some other problem, and the amount of contamination cannot be determined.² It was at least sufficient to mislead the scribe into dividing 30 by $7\frac{1}{2}$ instead of *vice versa* and to prevent his attempting to express his result in palms, as should be done in the case of a batter. The problem affords no case for the belief that the batter was in some cases measured by the tangent instead of the cotangent of the base angle.

¹ And perhaps below in No. 62, see p. 14, note 5.

² There may be confusion between the $\underline{s}twy$ of Nos. 45 and 46, which clearly means "content," and the somewhat similar word used for the height of the truncated pyramid in the Moscow problem (*Ancient Egypt*, 1917, 100-102).

BOOK III. MISCELLANEOUS PROBLEMS.

No. 61. (Pl. R.)

No. 61.

Line 1	$\frac{2}{3}$ of $\frac{2}{3}$ is $\frac{1}{3} + \frac{1}{9}$	
2	$\frac{1}{3}$ of $\frac{2}{3}$ is $\frac{1}{6} + \frac{1}{18}$	
3	$\frac{2}{3}$ of $\frac{1}{3}$ is $\frac{1}{6} + \frac{1}{18}$	
4	$\frac{2}{3}$ of $\frac{1}{6}$ is $\frac{1}{12} + \frac{1}{36}$	
5	$\frac{2}{3}$ of $\frac{1}{2}$ is $\frac{1}{3}$	
6	$\frac{1}{3}$ of $\frac{1}{2}$ is $\frac{1}{6}$	
7	$\frac{1}{6}$ of $\frac{1}{2}$ is $\frac{1}{12}$	
8	$\frac{1}{12}$ of $\frac{1}{2}$ is $\frac{1}{24}$	
9	$\frac{1}{9}$ of $\frac{2}{3}$ is $\frac{1}{18} + \frac{1}{54}$	$\frac{1}{9}, \frac{2}{3}$ of it is $\frac{1}{18} [+ \frac{1}{54}]$
10	$[\frac{1}{9} ?]$	
11	$[\frac{1}{9} ?]$	
12	$[\frac{1}{5}, \frac{2}{3} \text{ of it is } \frac{1}{10} + \frac{1}{30}] ?$	
13	$[\frac{1}{5}, \frac{1}{3} \text{ of it is } \frac{1}{15}] ?$	
14	$[\frac{1}{5}, \frac{1}{2} \text{ of it is } \frac{1}{10}] ?$	
15	$[\frac{1}{5}, \frac{1}{4} \text{ of it is } \frac{1}{20}]$	
16	$\frac{1}{7}, \frac{2}{3} \text{ (of it) is } \frac{1}{14} + \frac{1}{42}$	$[\frac{1}{3} \text{ of it is } \frac{1}{21}] ?$
17	$\frac{1}{7}, \frac{1}{2} \text{ of it is } \frac{1}{14}$	$[\frac{1}{4} \text{ of it is } \frac{1}{28}] ?$
18	$\frac{1}{11}, \frac{2}{3} \text{ of it is } \frac{1}{22} + \frac{1}{66}$	$\frac{1}{3} \text{ of it is } \frac{1}{33}$
19	$\frac{1}{11}, \frac{1}{2} \text{ of it is } \frac{1}{22}$	$\frac{1}{4} \text{ of it is } \frac{1}{44}$

No. 61 brings us over on to the verso of Pap. 10058. It is a table of multiplication of fractions. That it is not part of the original plan of the treatise is evident from the fact that it lies between Book II, Mensuration, and Book III, Miscellaneous Problems, that it lies outside the double vertical ruling, in a margin obviously intended to be left blank, and that it is very carelessly written. It has in fact been placed here by the scribe in order that it might be in an accessible spot for reference when needed.

Its main interest lies in the fact that, though a single table, it contains two different forms of statement. Thus in lines 1-4 we find the form $\frac{2}{3}$ of $\frac{2}{3}$ is $\frac{1}{3} + \frac{1}{9}$, while in lines 15-19 we find the form $\frac{1}{7}, \frac{1}{2}$ of it is $\frac{1}{14}$. In line 9 the statement is made twice, once in the second form and once in the margin in the first form, while in lines 5-8 the second form seems originally to have stood but to have been altered afterwards to the first.

These variations have an interesting significance which has not been insisted on by the commentators. An Egyptian cannot take one-ninth of $\frac{2}{3}$: he can take one-third of any quantity by simply taking two-thirds and halving it, but he cannot obtain one-ninth direct from one-third, for he cannot divide by 3, only by 2. The consequence is that to speak of taking one-ninth is technically incorrect, and the Egyptian should avoid the use of the phrase even in a table of results. Thus in line 9 we find in the column the correct form of statement: "One-ninth, $\frac{2}{3}$ of it is $\frac{1}{18} + \frac{1}{54}$." The less correct "One-ninth of $\frac{2}{3}$ is $\frac{1}{18} + \frac{1}{54}$ " being added in the margin owing to an error explained below. It would seem that the succeeding lines of the table were written in the correct form, but without the marginal addition.

In lines 5-8 the position is very curious. The scribe would seem to have written originally $\frac{2}{3}, \frac{1}{2}$ of it is $\frac{1}{3}$, and so on, though the other form of statement, that used in lines 1-4, would have been quite unexceptionable, the only multipliers involved being $\frac{2}{3}$ and $\frac{1}{2}$, both of which are legitimate. Perceiving this he seems to have altered these four lines by deleting, very perfunctorily, the $\frac{2}{3}$'s and adding $\frac{1}{2}$'s. In the excess of his zeal he has altered

- No. 61. line 9 in a similar manner, writing it, however, afresh in the margin, though this line should really have been left as it was. The point may seem a small one, but it has a real significance for the study of Egyptian mathematics.

The gap in the middle of the table probably contained five lines. Of these the first two or three doubtless gave fractions of one-ninth and the remainder began the fractions of one-fifth.

With this table should be compared the very similar but more complete table of fractions of Byzantine date published by Thompson.¹ The tablet which remains gives in fractional form the fifteenth part of all whole numbers from 1 to 15, and the sixteenth part of all whole numbers from 1 to 16.

No. 61b. No. 61B. (Pl. R.)

"To make two-thirds of an aliquot part. If it is said to thee, What is two-thirds of $\frac{1}{3}$, you are to make its double and its six times: that is two-thirds of it. Behold it is done likewise in the case of any aliquot part which may occur."

The translation of *tît gbt* or *tîst gbt* is fixed by the working. To find two-thirds of x we take twice x and six times x and presumably add them. Clearly this is a short statement of a practical rule for multiplication by $\frac{2}{3}$, for since $\frac{2}{3} = \frac{1}{2} + \frac{1}{6}$ all we have to do is to multiply 5, the denominator of our fraction, by 2 and then by 6, invert the results and add. Thus

$$\frac{2}{3} \text{ of } \frac{1}{5} = (\frac{1}{2} + \frac{1}{6}) \text{ of } \frac{1}{5} = \frac{1}{10} + \frac{1}{30}$$

A *tît gbt* is thus an aliquot part.² The word *tît* means a "sign" or "figure" of something. The verb *gbt* means "to be weak," and the compound must be "a weak sign." Why this should be the technical term for an aliquot part it is not easy to see, unless a weak sign is one which is placed beneath the fractional sign, or "inverted" as we now say. The word *tîst* also occurs in Pap. Kahun, Plate VIII, l. 50, in a mathematical technical sense.³ "One *tîst* is subtracted, remainder 11." Here the number from which it is subtracted is apparently 12, though in the obscurity of the passage this is not quite certain, and in this case it is hard to see why the writer did not merely say "Subtract 1." According to Maspero's interpretation⁴ of the problem the 12 are the 12 months of a year, in which case *tîst* would actually stand for a month! In fact, so uncertain is the meaning of the passage that it is impossible to draw from it any conclusion as to the meaning of *tîst*.

The existence of this rule in the papyrus is not without interest. We have seen that the Egyptian regarded $\frac{2}{3}$ as an aliquot part and was apparently able to take two-thirds of an integral number by a single process. Moreover, his method of finding $\frac{1}{3}$ and $\frac{1}{6}$ was by halving and re-halving $\frac{2}{3}$. Even in the treatment of fractional quantities the same was the case, $\frac{2}{3}$ always being found as a step towards $\frac{1}{3}$ and $\frac{1}{6}$, as for instance in the table of this example (No. 61). It is for this very reason that in the table of the division of 2, $\frac{2}{3}$ was never treated, though it might at once have been resolved into $\frac{1}{2} + \frac{1}{6}$. It is therefore interesting to find that when a fraction had to be dealt with the equation $\frac{2}{3} = \frac{1}{2} + \frac{1}{6}$ was brought into use.

Note that this is the sole instance in our papyrus of a general rule, except perhaps No. 66.

No. 62. No. 62. (Pl. R.)

"Example of reckoning a bag containing various precious metals. If it is said to thee, A bag in which are gold, silver and lead. This bag has been bought for 84 rings: what is assignable to⁵ each precious metal?"

¹ See above, p. 8.

² In No. 70 *tît* by itself seems to have the same meaning. Since only aliquot parts could be written, the word ought to mean any written fraction.

³ GRIFFITH, *P.K.*, Text, 18.

⁴ *Op. cit.*, 101.

⁵ Or, with Gardiner (see below), "What is the amount of."

Now what is given for a *deben* of gold is 12 rings, for silver 6 rings, and for a *deben* No. 62. of lead 3 rings. You are to add together that which is given for a ring (*sic*, read *deben*) of each precious metal; result 21. You are to reckon with this 21 to find 84 rings, for that is what has been bought in this bag. It comes to 4, which you assign to each metal.

The doing as it actually occurs:

4	is multiplied (?)	twelve times; the gold turns out to be 48.	This is its amount.
„	„	six times; the silver turns out to be 24.	
„	„	three times; the lead turns out to be 12.	
„	„	twenty-one times	Total 84.”

There is little difficulty in getting the correct mathematical drift of this problem. Eisenlohr did it in 1877. But it may be doubted whether even in 1923 it is possible to give a final and absolutely certain translation.

The problem is that a bag, or similar object, contains equal weights (though this is never stated in so many words) of gold, silver and lead. The total value of the bag is 84 rings, and we are also given the price per *deben* of each metal. Problem, find the weight of each metal in the bag. Answer, 4 *deben*.

The working explains itself and is perfectly modern in type. The values in rings of a *deben* of each metal are added together and come to 21. Thus if the bag contained 1 *deben* of each it would be worth 21 rings. But in reality it is worth 84. Therefore the number of *deben* of each metal must be 4.

The clue to correct translation lies in realizing, as Gardiner was the first to do,¹ that, in the phrase “Add together that which is given for a ring of each metal,” the word ring is a mistake for *deben*. To keep “ring” and translate “Add up that which is given in rings for each metal” involves two errors. In the first place, the Egyptian for “in rings” is not *hr s'ty* but *m s'ty*, and, in the second, the *n* written in hieratic without a dot over it is the Genitive Exponent “of” or “belonging to,” not the preposition “for,” which has the dot. It is true that the papyrus is not fully consistent in the matter (see Nos. 39 and 40), but in the present example the distinction is clearly made. Moreover, in the phrase *hpr-hr m 4 didi-k n 'st nbt* it is to be noted that *didi-k* is Masculine, not Neuter like *didi-t* above: it must therefore refer to the numeral 4 and be the Relative Form in its rather uncommon continuative sense,² “The result is 4, and this is what you are to attribute to each metal.” Were the meaning of this last phrase more concrete, “what you are to put in (the bag) of each metal,” we should expect *m 'st nbt* rather than *n*. There is an exact parallel in the Moscow Papyrus, where the former translation gives good sense and the latter no sense at all.

Coming now to points of detail, the word *krft* is found again in Pap. Ebers, 53, 12-14, where it is clearly some kind of cloth bag in which some portion of the date fruit is placed in order to be soaked and boiled. The meaning “bag” seems required in our passage. The masculine word *krf* quoted by Brugsch, *Wörterbuch*, 1467, from DÜMICHEN, *Kalenderische Inschriften*, 35, cols. 33 and 36, though obviously from the same root, need not have the same meaning.

For *in* meaning “to buy” see the Old Kingdom sale of a house quoted below; also Gardiner's note *Ä.Z.*, 43, 34.

For the position of *Q*@ in *iw ir didit hr nb dbn* compare No. 58 and note there.

In the working-out portion the papyrus is much damaged, and has been patched, in ancient times, by someone who either knew roughly what ought to stand in the gaps or who had actually the broken fragments before him. The former is the more likely alternative, for the mender has failed to supply the opening words of No. 63, and, what is more, he has been unable to complete the beginning of line 9 in the present example. This as it stands

¹ *Ä.Z.*, 43, 46-7.

² See, however, p. 14, note 5.

No. 62. is a puzzle. We expect *ir·hr·k w:h tp m* "You are to multiply 4 twelve times . . ." Yet this is not what stood there, for under the  is the top of a sign which from its shape and position can only be . The next group is a clear . This is followed by a blank due to a patch extending vertically into the lines above: the small black trace shown here in the facsimile is non-existent in the original. In the blank space we can hardly read anything but . We thus get the phrase *irt pw* which we have already met in No. 38, and in both cases the context is the same, *irt pw X r spw Y*. At the same time the explanation attempted of the phrase in No. 38 will hardly apply here, for there the words were used in excuse or justification of an unusually bold step, whereas here they introduce the *irt m:l hpr* (here practically constituting a proof) and take the place of the more usual *ir·hr·k X r spw Y*. Possibly the explanation of *irt pw* as a passive form of the well-known *sdm·f pw* suggested on p. 77 note 1 might hold here, "It (namely the result 4 obtained above) means that 4 must be multiplied by 12, 6 and 3 respectively to get the required values." This, however, is merely conjecture.

The main interest of the problem lies in the evidence it gives of the existence of a system of exchange based on the value of "rings" made of various metals. The word for "rings" is here written   or  . Its phonetic reading is almost certainly given by the writing    in line 3. This was decomposed by Griffith¹ into two separate words   and  . He supposed that *s'ty* denoted generally the goods to be bought, or that it might be a real or imaginary substance used as a common measure for the *debens* of all the metals. His translation was "84 pieces of *shati*." This separation of the signs would leave *s'ty* without a determinative, which, as it is an uncommon word, is improbable, and unless there is an error in the papyrus it is impossible to escape the conclusion that   is the phonetic reading of  and that the whole group is one word.² This word fortunately occurs once again in Egyptian literature, in an Old Kingdom inscription³ recording the sale of a house and some of its fittings or effects. There it is written  , and followed by a sign which is either a form of the old determinative of metal or an actual pictogram of a ring.⁴ Sethe took this word to be *s't*, the well-known word for loaves or cakes, but as a house would hardly be sold for 10 cakes he had to assume that 10 measures of cakes were intended, which is not probable.

Whatever view we may adopt with regard to the reading of the word it is clear that the  was a unit of some kind whereby the values of various objects could be compared and exchanges made. In this case it is probable that the word-sign represents the actual unit, and if so it may originally have been not the picture of the seal-stone but a ring of metal of some kind. This is doubtless what was in Möller's mind when he stated that the sign used in this problem for  is in reality a mistake for .

Now a unit written  , determined by the weight sign, has long been known from account papyri of the New Empire. Thus Griffith gives instances from Pap. Bulaq 11 of "rings" of gold and of silver, in which the values of certain commodities are expressed. "Half-rings" are also used, as well as "rings" simply, which comparisons show to have been silver.⁶

Gardiner has further illustrated the use of the "ring" from papyri of the XVIIIth Dynasty from Kahun, now in the Berlin Museum, and shown that the "ring" as used in

¹ F.S.B.A., XIV, 436-9.

² Suggested by Gardiner, A.Z., 43, 47.

³ SETHE, *Aegyptische Inschrift auf den Verkauf eines Hauses*; SOTTAS, *Étude critique sur un acte de vente immobilière*; cf. Chassinat in *Recueil de Travaux*, 39, 79-88; VON BISSING, *Ein Hauskauf im IV Jahrtausend vor Chr.* (*Sitzungsb. der Bayerischen Akad. d. Wiss., Philos.-philolog.-hist. Kl.*, 1920).

⁴ It differs, however, from the seal-stone sign which occurs elsewhere in the inscription.

⁵ *Hieratische Paläographie*, I, 40, note 1.

⁶ See SPIEGELBERG, *Rechnungen aus der Zeit Setis I*, 89 ff.

these papyri was a weight equivalent to one-twelfth of a *deben*. Comparing this with the No. 62. Rhind example, where "what is given for a *deben* of gold is 12 rings," he concluded that the rings referred to in the Berlin papyri were of gold.

Thus it is clear that in the New Kingdom a regular currency in "rings" of silver and of gold, more particularly the latter, had become usual in Egypt. Since, however, the "ring" was clearly a weight we must not assume that payment was made in actual ring-shaped objects of gold, which would almost have constituted a coinage even if not inscribed. The Rhind papyrus doubtless takes us a stage farther back towards the origin of this system, to a time when the "ring" was purely a weight and had not associated itself with any particular metal.

No. 63. (Pl. S.)

No. 63.

"[Example of dividing] 700 loaves among 4 men, $\frac{2}{3}$ to one, $\frac{1}{2}$ to another, [$\frac{1}{3}$ to another, and $\frac{1}{4}$ to another]. Let me know the share of each of them.

You are to add together $\frac{2}{3}$, ($\frac{1}{2}$), $\frac{1}{3}$ and $\frac{1}{4}$; result $1\frac{1}{2} + \frac{1}{4}$. You are to divide 1 by $1\frac{1}{2} + \frac{1}{4}$; result $\frac{1}{2} + \frac{1}{14}$. You are to take $\frac{1}{2} + \frac{1}{14}$ of 700, namely 400. You are to take $\frac{2}{3}$ of 400, which is $266\frac{2}{3}$; $\frac{1}{2}$ of 400, which is 200; $\frac{1}{3}$ of 400, which is $133\frac{1}{3}$; and $\frac{1}{4}$ of 400, which is 100. These are the shares of each man among them.

The doing as it occurs:

Number, 700
$\frac{1}{2} + \frac{1}{14}$ is 400
$\frac{2}{3}$ of 400 to one: 266 $\frac{2}{3}$
$\frac{1}{2}$ of 400 to another: 200
$\frac{1}{3}$ of 400 to another: 133 $\frac{1}{3}$
$\frac{1}{4}$ of 400 to another: 100
Total 700 "

At the beginning of this problem and the end of the last the papyrus has been torn and a patch placed over the gap. Parts of two lines are lost, and the mender seems also to have copied on to the new piece as best he could the signs of a third line, which was still present but which he was forced to cover up in order to get an overlap for his patch. The first line should probably be restored *tp n psš*: Eisenlohr's *tp n irt* is not long enough to fill the space. In the second line we must restore $\frac{1}{3}$ n *kī* $\frac{1}{4}$ n *kī*, or something similar.

The problem is curiously stated, it appearing at first sight that $\frac{2}{3}$ of the 700 loaves are to go to the first man, half to the second and so on. This of course is impossible, and the fractions are really only proportionate estimates of the shares of each.

The solution is on modern lines. The four fractions are added and give $1\frac{1}{2} + \frac{1}{4}$. The first man then receives $\frac{\frac{2}{3}}{1\frac{1}{2} + \frac{1}{4}}$ of 700, the second $\frac{\frac{1}{2}}{1\frac{1}{2} + \frac{1}{4}}$ and so on. In Egyptian this working is expressed differently. The sum $1\frac{1}{2} + \frac{1}{4}$ of the fractions is turned upside down, i.e. in Egyptian 1 is divided by it. The result is $\frac{1}{2} + \frac{1}{14}$. The total 700 is then multiplied by this $\frac{1}{2} + \frac{1}{14}$, which is the same thing as dividing it by $1\frac{1}{2} + \frac{1}{4}$. The result is 400, and we have now only to multiply 400 successively by $\frac{2}{3}$, $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ to get the shares.

No. 64. (Pl. S.)

No. 64.

"Example of distributing differences. If it is said to thee, 10 *hekat* of barley¹ to 10 men, the difference of each man over his neighbour being $\frac{1}{8}$ of a *hekat* of barley.

The mean share is $\frac{1}{2}$ *hekat* (read 1 *hekat*). Take 1 from 10; the remainder is 9. A half of the common difference is taken, namely $\frac{1}{16}$ *hekat*. Multiply it 9 times, result ($\frac{1}{2} + \frac{1}{16}$)

¹ Plural strokes omitted in plate.

No. 64. *hekat*. Add to the mean share. You are now to subtract $\frac{1}{8}$ *hekat* for each man down to the last.

The doing as it occurs:

$$\begin{array}{ccccc}
 1\frac{1}{2} + \frac{1}{16} & 1\frac{1}{4} + \frac{1}{8} + \frac{1}{16} & 1\frac{1}{4} + \frac{1}{16} & 1\frac{1}{8} + \frac{1}{16} & 1\frac{1}{16} \\
 \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} & \frac{1}{2} + \frac{1}{4} + \frac{1}{16} & \frac{1}{2} + \frac{1}{8} + \frac{1}{16} & \frac{1}{2} + \frac{1}{16} & \frac{1}{4} + \frac{1}{8} + \frac{1}{16} \\
 & & & & \text{Total } 10.1''
 \end{array}$$

Put into modern language the problem is to form a series of 10 terms in arithmetical progression, their sum being 10 *hekat* and the common difference $\frac{1}{8}$ *hekat*. The Egyptian method is to find the last (*i.e.* highest) term. This is done by taking the mean share, *i.e.* the share which each would get if the division were made equally. Unfortunately the scribe has here written $\frac{1}{2}$ *hekat* instead of 1. To this is next added half the common difference, *i.e.* $\frac{1}{2}$ of $\frac{1}{8}$, which is $\frac{1}{16}$, multiplied by the number of terms less one, *i.e.* 9. This gives the last term.

This rule was doubtless obtained empirically. If an arithmetical series be written down it will be seen at once that, supposing the number of terms to be odd, the middle term of the series is the mean share, *i.e.* the whole sum divided by the number of terms (n), and each share above this adds on the common difference, so that the last term consists of the mean share (m) plus the common difference (d) multiplied by the number of terms on either side of the middle term, *i.e.* $m + d \cdot \frac{n-1}{2}$, or as the Egyptian rather illogically put it $m + \frac{d}{2}(n-1)$. When the number of terms in the series is even the mean share lies midway between the two terms $m - \frac{d}{2}$, $m + \frac{d}{2}$. But the rule still holds, for the first term above the mean share is $m + \frac{d}{2}$, the next $m + \frac{3d}{2}$, the next $m + \frac{5d}{2}$, and the last will be $m + d \cdot \frac{n-1}{2}$. The last and highest term having been found, it is only necessary to keep subtracting the common difference $\frac{1}{8}$ from it to get all the previous terms.

In modern arithmetic we do not use quite the same method, for we avoid the use of the mean share m . Thus if a be the first term and l the last, we have the equation

$$l = a + (n-1)d$$

Now the Egyptian mean share m is clearly $\frac{a+l}{2}$ $\therefore a = 2m - l$. Substituting for a in the above equation we get $l = 2m - l + (n-1)d$ or $l = m + \frac{n-1}{2} \cdot d$, which is the Egyptian equation.

Unless we are prepared to give *prw* two different meanings in the same example, which is almost impossible, we must translate it throughout in the sense demanded by the phrase *prw n s nb r snw.f*, which can from the context only mean "The excess² (or difference) of each man over his fellow," cf. *twnw* in No. 40. The opening words, "Example of dividing differences," now seem a little harsh, since strictly speaking it is the loaves and not the differences that are divided. We need, however, only suppose a slightly pregnant use of *psš* which we may best express in English by "distribute."

prw . . . m it hkt 1/8 pw. This form of nominal sentence seems redundant, either m or *pw* being unnecessary. Yet we have the same form again in Nos. 70 and 71.

šht. I can find no other examples of the figurative use of the verb in this sense. The metaphor must be either "to weave in" the men one after the other, or "to catch" them as in a net. The verb occurs with the abstract determinative in difficult passages, Prisse 6, 7 and 6, 9, and B.M. 10509, 2, 10.³

hry phwi. Literally "he who has the end," *i.e.* "the last." Cf. *Urk.*, IV, 1104, where, however, a sense of inferiority in rank is also implied.

For the sign 𓆎 at the end of line 1 see under No. 70.

¹ Strictly speaking the *hkt*-sign should precede if the stroke is to be read 10 and not 1.

² For *prw* = "excess," "surplus," see GARDINER, *J.E.A.*, IX, 19, n. 5; SETHE, *Einsetzung des Verzierr*, note 130.

³ Also Pap. Petrograd 1116 A, recto, 111 (Gunn).

No. 65. (Pl. S.)

No. 65.

"Example of reckoning out 100 loaves for 10 men, a sailor, a foreman and a watchman with double.

Its working:

You are to add up the crew, result 13. Reckon with 13 to find the hundred loaves: result $7\frac{2}{3} + \frac{1}{39}$.

Then shall you say, This is the ration of the 7 men, and of the sailor, the foreman and the watchman with double.

	$7\frac{2}{3} + \frac{1}{39}$
	$7\frac{2}{3} + \frac{1}{39}$
	$7\frac{2}{3} + \frac{1}{39}$
	$7\frac{2}{3} + \frac{1}{39}$
	$7\frac{2}{3} + \frac{1}{39}$
	$7\frac{2}{3} + \frac{1}{39}$
	$7\frac{2}{3} + \frac{1}{39}$
Sailor	$15\frac{1}{3} + \frac{1}{26} + \frac{1}{78}$
Foreman	$15\frac{1}{3} + \frac{1}{26} + \frac{1}{78}$
Watchman	$15\frac{1}{3} + \frac{1}{26} + \frac{1}{78}$
Total	100."

The problem is a simple one. A hundred loaves are to be divided among 10 men, three of whom are to receive double portions. Three doubled is 6, and $6 + 7$ is 13. Thus all we have to do is to divide the 100 loaves into 13 portions, giving two portions to each of the favoured men and one each to the rest.

Note the resolution of $\frac{2}{39}$ by table into $\frac{1}{26} + \frac{1}{78}$.

The reading of the second line has given some trouble. The word $\overline{\text{𓂏𓂏𓂏}}$ is certain, despite the curious form of the 𓂏 , and the word which follows it can hardly be other than $\overline{\text{𓂏𓂏}}$. We thus get the compound *rmṯt* 'pr, which on the analogy of *rmṯt* ṯst must mean "men of the crew or gang," or "the crew or gang" simply. Sethe has pointed out that 'pr is used of a group of men working on land as well as of the crew of a ship.¹ In this case we have to do with a ship's crew if the reading *nfw*, sailor, is correct, and judging by the form of the hieratic it is much more probable than the only other possibility, which is $\text{𓂏} \text{𓂏}$, a follower.

In *t*; *t*: 100 the article is feminine to agree with the feminine numeral *st*. See SETHE, V.Z.Z., 50, and contrast *p*; *t*: 1000 in No. 74, where the numeral *h*: (1000) is masculine.

No. 66. (Pl. S.)

No. 66.

"Ten *hekat* of fat has been issued for a year. What is the daily portion thereof?

Its working out:

You are to turn the 10 *hekat* of fat into *ro*, making 3200. Now turn a year into days, result 365. You are to divide 3200 by 365. Result $8\frac{2}{3} + \frac{1}{10} + \frac{1}{2100}$, making in *ro* (sic) $\frac{1}{64}$ *hekat* and $(3\frac{2}{3} + \frac{1}{10} + \frac{1}{2100})$ *ro*. This is the daily portion.

The doing as it occurs:

1	365
2	730
4	1460
$\frac{2}{3}$	$243\frac{1}{3}$
$\frac{1}{10}$	$36\frac{1}{2}$
$\frac{1}{2100}$	$\frac{1}{6}$
Total	$8\frac{2}{3} + \frac{1}{10} + \frac{1}{2100}$

You may do similarly for any problem put to you resembling this example."

¹ In BORCHARDT, *Das Grabdenkmal des Sahure*. II, 85, note 6.

No. 66. The method needs no explanation, except that in the working 8 2920 has been omitted after 4 1460. It is worthy of notice that in the contracts of Hapzefa the daily portion is obtained from the yearly by dividing not by 365 but, as he expressly states, by 360, the five epagomenal days being there neglected.

In line 1 *pri* is clearly used in its technical sense of "to be issued or delivered" from a storehouse or government department, for which see the Siut contracts and Pap. Bulaq 18 *passim*. The form here must be the Pseudo-participle, indicating a state, best translated by a Perfect in English, "has been issued." The problem is, as usual, concrete. An official has received a year's supply of fat, and is asking himself how much he can afford to use daily.

The phrase *mī tp pn* is our authority for translating *tp* as "example" in the heading of the problems throughout the papyrus. Note here the formulation of a general rule and see on 61B, p. 104.

No. 67. No. 67. (Pl. T.)

"Example of reckoning the produce of a herdsman. Behold now this herdsman came to the numbering of cattle¹ with 70 oxen: said this accountant of cattle to this herdsman, How few are the head of oxen which thou hast brought! Where then are thy numerous head of oxen? This herdsman said to him, What I have brought thee is two-thirds of one-third of the cattle which thou didst entrust to me. Count for me and thou wilt find me complete.

The doing as it occurs:

1	1	1	$\frac{1}{6} + \frac{1}{18}$	Multiply 70 by $4\frac{1}{2}$
$\frac{2}{3}$	$\frac{2}{3}$	2	$\frac{1}{3} + \frac{1}{9}$	Result 315: these are what
$\frac{1}{3}$	$\frac{1}{3}$	4	$\frac{2}{3} + \frac{1}{6} + \frac{1}{18}$	were entrusted to him
$\frac{2}{3}$ of $\frac{1}{3}$ of it is $\frac{1}{6} + \frac{1}{18}$		$\frac{1}{2}$	$\frac{1}{9}$	1 315
Divide 1 by $\frac{1}{6} + \frac{1}{18}$		Total	1	$\frac{2}{3}$ 210
				$\frac{1}{3}$ 105
				$\frac{2}{3}$ of $\frac{1}{3}$ of it is 70: these
				are what he brought."

The mathematics of the problem are simple. The question is, If two-thirds of one-third of a number is 70, find the number. Now in Egyptian $\frac{2}{3}$ of $\frac{1}{3}$ is $\frac{1}{6} + \frac{1}{18}$ (see No. 61B). This is inverted, as we should say, or, as the Egyptian has it, 1 is divided by it. The result is $4\frac{1}{2}$, and this has only to be multiplied by 70 to give the answer 315.

The translation is less easy. The situation seems to be that the accountant of cattle has entrusted a certain number of cattle to a herdsman to rear, with instructions to produce two-ninths of them on the day of cattle-numbering. That this system of letting out cattle was practised in Egypt is very clear from the accounts preserved among the Kahun papyri.² Both there and here the cattle produced at the numbering are termed *b:kw*, which Griffith renders "produce" and Maspero "taxes,"³ both meanings being equally common in Egyptian. In the light of the present example "produce" would seem the better translation, for the herdsman is simply delivering over a certain number not of his own cattle but of some which have been entrusted (*šip*) to him by another.

The crux of the passage lies in line 3, in the hieratic group which follows the  of *tn*. Eisenlohr failed to transliterate it. Griffith read  and translated the clause "very few are the heads of oxen you are contributing: what is the whole number of your heads of oxen of various kinds?" There are four objections to this. In the first place, there is no word for "what," *tr* being merely an interrogative particle meaning "pray." In the second

¹ Cf. *Beni Hasan*, I, Pl. VIII, l. 17, and Pl. XIII; *L.D.*, II, 31; *Urk.*, IV, 75, 14.

² See GRIFFITH, *K.P.*, Text, 43 and 45-47, and the interesting inscription *Beni Hasan*, I, Pl. VIII.

³ GRIFFITH, *op. cit.*, 101.

place, *tnw nb* could not mean "the whole number," but only "every number." In the third No. 67. place, *tnw* "a number" has the *w* written out in M.K. texts, and therefore cannot be the word we have here. In the fourth place, the group after the bird cannot be read  for it would give us a form for  which is quite foreign to this and any other papyrus of the period, and the sign below it would be very short for  and lacks the final turn downwards which this sign almost always has in the Rhind.

Surely what we have here is not *tnw* "number" but *tni* "where," and the group following the bird is nothing but the sign  which frequently determines *tni*. The form is curious, it is true, but its relation to those of papyri of about the same period, such as Ebers and Westcar, is not hard to see. Unfortunately no other instance of the sign occurs in Rhind.

The translation is now clear: "Pray where are your many head of cattle?" or, in other words, "What has become of all the cattle I entrusted to you?" which is a perfectly natural question to follow the statement "How few cattle you have brought!" The accountant doubts that the herdsman has brought the whole two-ninths. The herdsman replies, Here are 70, work it out, and you will find that it is two-ninths of 315, which, as you can verify from your roll, is what you delivered to me.

 in line 3. It is difficult to see what else could be read, since the sign for  is exactly similar to that used in No. 62, where however it may just be for  and not  (see p. 106 and note 5). It is clear, too, that the ancient scribe who patched up the torn papyrus was of the same opinion, for in mending line 2 he has written this sign almost like the hieroglyphic . I have followed Griffith in translating "head of cattle," though I am far from convinced that we have reached the correct solution.

hsb ni gm-k wi km-kwi. Griffith divided the words *hsb-ni gm-kwi km-kwi*, and translated "I have reckoned and I found that I had completed my contribution." This is improbable, since the pseudo-participle *gm-kwi*, unless an archaism, could only be passive in meaning in a papyrus of this date. Moreover, the verb *gm* should be followed by a complete pseudo-nominal sentence: thus for "I found that I was complete" we expect *gm-ni wi km-kwi*.¹ For *km* "to be complete in one's payment" see Pap. Bulaq 18 *passim*.

No. 68. (Pl. T.)

No. 68.

"If a scribe says to thee, Four gangers; they have drawn 100 great quadruple-hekat of corn. The gang of the first ganger consists of 12 men, that of the second 8, that of the third 6, and that of the fourth 4, total 30.

You are to divide 100 by 30; result $3\frac{1}{3}$, making in corn $(3\frac{1}{4} + \frac{1}{16} + \frac{1}{64})$ hekat and $1\frac{2}{3}$ ro. Multiply by 12 for the first, 8 for the second, 6 for the third, and 4 for the fourth.

1	$(3\frac{1}{4} + \frac{1}{16} + \frac{1}{64})$ hekat + $1\frac{2}{3}$ ro
2	$(6\frac{1}{2} + \frac{1}{8} + \frac{1}{32})$ „ + $3\frac{1}{3}$ „
4	$(13\frac{1}{4} + \frac{1}{16} + \frac{1}{64})$ „ + $1\frac{2}{3}$ „
8	$(26\frac{1}{2} + \frac{1}{8} + \frac{1}{32})$ „ + $3\frac{1}{3}$ „
Total, the first, 40 hekat.	

1	$(3\frac{1}{4} + \frac{1}{16} + \frac{1}{64})$ hekat + $1\frac{2}{3}$ ro
2	$(6\frac{1}{2} + \frac{1}{8} + \frac{1}{32})$ „ + $3\frac{1}{3}$ „
4	$(13\frac{1}{4} + \frac{1}{16} + \frac{1}{64})$ „ + $1\frac{2}{3}$ „
8	$(26\frac{1}{2} + \frac{1}{8} + \frac{1}{32})$ „ + $3\frac{1}{3}$ „
Total, $(26\frac{1}{2} + \frac{1}{8} + \frac{1}{32})$ hekat + $3\frac{1}{3}$ ro, the second.	

¹ At the same time the Pseudo-participle can in independent sentences be used in the 1st Person Singular without a preceding pronoun; also after *h'-n* (see Shipwrecked Sailor, 109, 157, 169, 174, 177, where, however, contrast 39, 131, 155).

No. 68.

1	$(3\frac{1}{4} + \frac{1}{16} + \frac{1}{64})$	hekat	+ $1\frac{2}{3}$ ro
2	$(6\frac{1}{2} + \frac{1}{8} + \frac{1}{32})$	„	+ $3\frac{1}{3}$ „
sic 4	$(13\frac{1}{4} + \frac{1}{16} + \frac{1}{64})$	„	+ $(1)\frac{2}{3}$ „
Total, the third, 20 hekat.			
1	$(3\frac{1}{4} + \frac{1}{16} + \frac{1}{64})$	hekat	+ $[1\frac{2}{3}$ ro]
2	$(6\frac{1}{2} + \frac{1}{8} + \frac{1}{32})$	„	+ $3\frac{1}{3}$ „
4	$(13\frac{1}{4} + \frac{1}{16} + \frac{1}{64})$	„	+ $1\frac{2}{3}$ „
Total, the fourth, $(13\frac{1}{4} + \frac{1}{16} + \frac{1}{64})$ hekat [+ $1\frac{2}{3}$ ro]			

List of these:

gangers:		great quadruple-hekat of corn:	
The first	12	25 + 10 + 5	40
The second	8	$(26\frac{1}{2} + \frac{1}{8} + \frac{1}{32})$ hekat	+ $3\frac{1}{3}$ ro
The third	6	20	20
The fourth	4	$(13\frac{1}{4} + \frac{1}{16} + \frac{1}{64})$ hekat	+ $1\frac{2}{3}$ ro
Total	30	100 hekat	100 "

The problem is that the 100 hekat of corn has been earned by 4 gangs together and is given to the four gangers to divide into four portions proportionate to the size of their gangs. The further division within each gang does not enter in here at all.

The working has a very complicated appearance, partly from the fact that it contains a great deal of repetition, and still more from the fact that the scribe, copying probably from a tabulated original on to a papyrus divided into narrow horizontal strips, was forced to destroy the tabular arrangement, and thus several pieces of the work have got out of place.

The total number of men is found to be 30. The hundred quadruple-hekat of corn are divided by this, and the result is $3\frac{1}{3}$ quadruple-hekat per man. To get the shares of the gangs we have to multiply this by 12, 8, 6 and 4. The $3\frac{1}{3}$ is first turned into the Horus-eye notation, giving $(3\frac{1}{4} + \frac{1}{16} + \frac{1}{64})$ hekat and $1\frac{2}{3}$ ro. The four multiplications are all worked out separately, despite the fact that since the multipliers in the first are 2, 4, and 8 all the results could have been obtained from this one piece of work. The sum ends with a table giving each gang, the number of men composing it, its share in quadruple-hekat expressed first in correct hekat notation, and second in ordinary pure integers and fractions. The last line gave the totals of each column. The 𓏏 under the sign @ for 100 in the last column should of course be omitted, since the column is not in hekat notation. This is a curious reminiscence of the converse mistake in the setting of the problem, where 100 quadruple-hekat is written 𓏏 instead of 𓏏 .

shn, "to embrace," must be used figuratively here of "drawing" wages collectively. I can find no other instances.

Nos.
69-78.

Nos. 69 to 78. EXCHANGE OF BREAD AND BEER. Plates U—W.

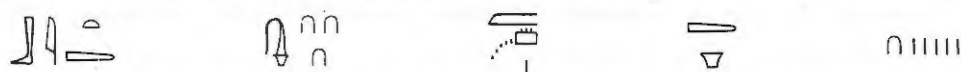
These examples deal with the strength, $\text{𓏏} \text{ 𓏏} \text{ 𓏏}$, of bread and of beer, with the exchange of loaves of various sizes, and with the exchange of bread for beer. The word *pfsw* (read *pšw* or *fšw*? See SETHE, *Verbum*, I, 216) must mean literally the "cooking" or "cooking-value."¹ The *pfsw* of a loaf of bread is simply the number of such loaves which can be made out of a hekat of corn. Thus if the *pfsw* of a loaf is 12 (per hekat) the loaf must contain one-twelfth of a hekat of corn. Similarly the *pfsw* of a jug of beer of a certain size is the number of such jugs which can be made out of a hekat of corn.²

¹ First explained by Dümichen, *A.Z.*, 1870, 41 ff.² It seems impossible to find an English word to cover both meanings.

The importance of the *pfšw* clearly rested on the fact that it formed a basis on which Nos. 69-78. loaves of bread and jugs of beer could be exchanged for one another, or for loaves or jugs of beer of different sizes, or even for other commodities whose value in relation to the *hekat* of corn was known or could be found.¹

It should be noticed that there is a rather important difference between the *pfšw* of bread and that of beer, due to the natural difference between a solid and a fluid. The *pfšw* of a loaf, giving as it does the amount of corn in the loaf, practically determines its size, apart from small variations due to cooking. The *pfšw* of a *ds*-measure of beer cannot of course alter the size of the measure, but, as it gives the amount of corn used to produce the beer, it determines the strength. In other words, the *pfšw* determines the size of loaves of bread and the strength of beer.

In the present papyrus the *pfšw* are all reckoned per simple *hekat*, while in the calendar inscription of Medinet Habu they are reckoned per quadruple-*hekat*. In the Rhind the *pfšw* of loaves runs from 5 to 45. At Medinet Habu we read of *bit*-cakes of *pfšw* up to 100, and *pršn*-cakes of *pfšw* up to 30, in both cases per quadruple-*hekat*. These sacrificial cakes were clearly much smaller than the ordinary loaves and doubtless made of a finer quality of flour. The usual type of *pfšw* entry in the Medinet Habu inscription² is as follows:—



“*Bit*-bread of *pfšw* 30; amount of corn used $\frac{1}{2}$ *hekat*, yielding 15 *bit*-cakes.

The *pfšw* of beer, as Griffith has shown, tended to increase in course of time, that is to say the beer became less strong. In the Middle Kingdom Papyrus Bulaq 18 the *pfšw* is invariably 2 *ds* per simple *hekat*. Here in the Rhind it is 2, $2\frac{1}{2}$ and 5 to the *hekat*. In an inscription of Tuthmosis IV at Karnak beer of *pfšw* 4 is mentioned. Finally, in the calendar of Medinet Habu the *pfšw* of beer is 5 (*špnt*-jugs), 10 and even 20 (*ds*-jugs in the last two cases) per quadruple-*hekat*, which gives $1\frac{1}{4}$, $2\frac{1}{2}$ and 5 for the simple *hekat*.

Quite distinct from the *pfšw* in the mind of the Egyptian is another term, which is in reality nothing but its inverse. This is the *hrt*,³ which here means the content of a loaf in grain. Thus if the *pfšw* of bread per *hekat* is 4, then each loaf has a content of $\frac{1}{4}$ *hekat*.

In these examples various kinds of grain and flour are mentioned, and it will be best to discuss at once their translation into English.

The first of these is *šš*. This is certainly nothing more than a general word for several kinds, probably for all kinds, of grain, and is well rendered in English by “corn” or “grain.” Its general meaning is clear from such passages as DÜMICHEN, *Opferfeste*, Pl. V, where columns of offerings are added up to give 4 *khar*(?) of *it mḥty* and 1 *khar*(?) of *it šm*^c, and these two are combined in the words “Total, *šš* 5 *khar*(?).” In the Annals of Tuthmosis III this general meaning is also clear. Thus in one passage (*Urk.*, IV, 694) the harvest of the land of Retenu is said to consist of *šš šš it šwt bdt*, “various kinds of corn (including) *it*, *šwt* and *bdt*.” That this is the correct rendering, and that the *šš* must be a general term including the three species *it*, *šwt* and *bdt*, is clear from a comparison of the two passages quoted. The uses of *šš* in Rhind fully bear out this conclusion. Thus in Nos. 35 and 37 it is used of a quantity of grain in a case where the particular species is immaterial and the amount is all that matters. It is employed in the granary sums Nos. 42 ff. under precisely similar circumstances, and so too in No. 68. In No. 82 it seems to include both *šwt* and *bdt* (if this be the right reading), but the example is too obscure in meaning to allow of certainty.

¹ In the tomb of Amenemhab a certain official is labelled “Overseer of the granary of the king, who reckons the *pfšw* of bread and beer.” The scene shows the provision of supplies for an army. *Urk.*, IV, 912.

² DÜMICHEN, *Kalender-Inschriften*, Pl. I.

³ Called *rht* in No. 69, perhaps less correctly. Contrast No. 74, where *rht* = the number of loaves. Pap. Moscow has *hrt* throughout.

Nos.
69-78.

The cereals specifically mentioned in the papyrus are four, *šwt*, *it*, *bdt* and *bš*.¹ The first of these, *šwt*, has been identified with wheat, *Triticum turgidum* or *Triticum durum*, the Coptic *coyo*. It occurs but once in Rhind, in the very unsatisfactory No. 82, where it is used to prepare bread for feeding geese.

bdt, Coptic *βωτε*, is a kind of spelt, *Triticum dicoccum*, much cultivated in Europe and known in Germany as *Emmer*. It is mentioned three times in Rhind. In No. 79 no clue is given as to its use: in No. 82 it is made into bread for geese, and in No. 84 it forms the food of oxen.

it includes two types of barley, *Hordeum hexastichum* and *Hordeum vulgare*, Coptic *ειωτ*. The Egyptians distinguished two kinds, that of Upper and that of Lower Egypt, written in later times simply "Upper Egyptian" and "Lower Egyptian" with the word barley omitted.² The former occurs in No. 74.

The fourth kind of grain, *bš*, is of infrequent occurrence. In Rhind it is referred to only in No. 71, where it is made into beer. There is in this passage a scribe's error which led Eisenlohr to take *bš* for the reading of the measure known to have been called *hkt*. It occurs three times in the Kahun Papyri, Pls. XV, 66, XVIII, 3, and XX, 3. Griffith in his note (*K.P.*, Text, 44) quotes examples from the Ebers Papyrus and Pap. Bulaq 18.³ In the Kahun papyri it occurs on each occasion in lists between Upper Egyptian barley and dates, and in the *pfšw* problems of the Moscow papyrus it is always associated with *bnr*, dates, in the brewing of beer, in a puzzling connection the exact drift of which I cannot at present perceive. Stern in his *Glossar zum Papyrus Ebers* identifies it, wrongly, with the grain now known as *dura*, *Sorghum vulgare*, which appears to be a late importation into Egypt.

nd (Nos. 69 and 70) has generally been identified with the Coptic *noeit*, and in consequence translated "flour." The equation may be correct, but it is phonetically far from satisfactory. In Rhind as elsewhere this substance is used in the making of bread. In *Urk.*, IV, 688, it occurs with *šš* and *šwt* in a list of tribute from Syria. In SPIEGELBERG, *Rechnungen aus der Zeit Setis I*, Pl. IVb, it appears to be obtained from *bdt*, despite the author's remarks on pp. 39-40 of the Text. The Rhind examples give no clue to its exact nature.

The most puzzling of all these types of grain or flour is undoubtedly that called *wdyt*. It occurs in Nos. 72 to 78 and 82. In most of these cases it is made into bread, but in 77 it is used for beer, and in 78 for both bread and beer. In No. 74 the amount of Upper Egyptian barley contained in a certain number of loaves is found; the next words are "Then shalt thou say, this is (the amount of) *wdyt*," from which it would at first sight appear that *wdyt* is the meal obtained by grinding Upper Egyptian barley. In No. 82, however, bread for the feeding of geese is made from *wdyt*, and later in the sum we find a rather unintelligible calculation of the amount of *šwt*⁴ and *bdt* which must be ground in order to produce this *wdyt*. If any conclusion can be drawn from these not very satisfactory indications, it would appear that *wdyt* is a term for flour or meal in general, whether made from barley, wheat or spelt. But we have no clue whatsoever as to the relation of *wdyt* to *nd*. I can find no other instances of *wdyt* in Egyptian.

No. 69. No. 69. (Pl. U.)

"Three and a half *hekat* of flour made into 80 loaves. Let me know the content of a single loaf in flour. Let me know their strength.

¹ See on these cereals SCHULZ, *Die Getreide der alten Aegypter*, and *Beiträge zur Kenntniss der Geschichte der Spelzweizen im Altertum*, in *Abhandlungen der Naturforschenden Gesellschaft zu Halle*, N.F., Nos. 5 and 6, 1916 and 1918; also his articles in *Berichte der deutschen botanischen Gesellschaft*, XXXIV, on the same subject; compare HROZNY, *Das Getreide im alten Babylonien*, Vienna, 1914.

² Sethe in *Ä.Z.*, 44, 19.

³ Add Pap. Leyden 349, verso 2, 8, along with *nd*.

⁴ Reading not quite certain, see p. 124.

You are to reckon with $3\frac{1}{2}$ to find 80:

No. 69.

1	$3\frac{1}{2}$
10	35
— 20	70
— 2	7
— $\frac{2}{3}$	$2\frac{1}{3}$
— $\frac{1}{21}$	$\frac{1}{6}$
— $\frac{1}{7}$	$\frac{1}{2}$

The strength is $22\frac{2}{3} + \frac{1}{7} + \frac{1}{21}$

— 1	$22\frac{2}{3} + \frac{1}{7} + \frac{1}{21}$
— 2	$45\frac{1}{3} + \frac{1}{4} + \frac{1}{14} + \frac{1}{28} + \frac{1}{42}$
— $\frac{1}{2}$	$11\frac{1}{3} + \frac{1}{14} + \frac{1}{42}$
— 1	320
— 2	640
— $\frac{1}{2}$	160
Total 1120 in ro	

You are to reckon with 80 to find 1120.

The doing as it occurs:

1	80
— 10	800
2	160
— 4	320
Total 1120	

The content of a single loaf in flour is $\frac{1}{32}$ hekat + 4 ro.

1	$\frac{1}{32}$ hekat + 4 ro
2	$(\frac{1}{16} + \frac{1}{64})$ hekat + 3 ro
4	$(\frac{1}{8} + \frac{1}{32} + \frac{1}{64})$ hekat + 1 ro
8	$(\frac{1}{4} + \frac{1}{16} + \frac{1}{32})$ „ + 2 „
— 16	$(\frac{1}{2} + \frac{1}{8} + \frac{1}{16})$ „ + 4 „
32	$(1\frac{1}{4} + \frac{1}{8} + \frac{1}{64})$ „ + 3 „
— 64	$(2\frac{1}{2} + \frac{1}{4} + \frac{1}{32} + \frac{1}{64})$ hekat + 1 ro

Result $3\frac{1}{2}$ hekat of flour.”

The working is simpler than it looks: it is in great disorder owing to the efforts of the scribe to fit it into the narrow zones into which the papyrus was divided. It consists of two parts, (1) the finding of the *pfsw* and its proof, and (2) the finding of the *rht* and its proof. The *pfsw* is obtained by multiplying $3\frac{1}{2}$ to get 80, or, as we put it, dividing 80 by $3\frac{1}{2}$. The result is $22\frac{2}{3} + \frac{1}{7} + \frac{1}{21}$, and by way of proof this number is multiplied by $3\frac{1}{2}$ and shown to give 80, though the actual addition is not inserted.

The reckoning of the content begins with the reduction of $3\frac{1}{2}$ hekat to ro, viz. 1120 ro. Next 80 is multiplied to find 1120, i.e. 1120 is divided by 80. This gives the content of a loaf as 14 ro, or, in the Horus-eye notation, $\frac{1}{32}$ hekat and 4 ro. This result is finally proved by being multiplied by 80 and shown to give $3\frac{1}{2}$ hekat.

For $\overline{\text{ⲓ}} \text{ⲟ} \text{ⲙ} \text{ⲙ}$ see above, p. 114.

wʿt nt t, which occurs again in No. 70, is interesting grammatically. Since the word *t* is masculine *wʿt* cannot be the New Egyptian indefinite article, “a loaf,” which would require *wʿ nṯ t*, and it must therefore be the abstract noun of number, whose existence was first

No. 69. demonstrated by Sethe¹ and which corresponds to the Coptic *oye*. The literal translation would be "A unit of loaves." The use is suitable here because we are finding the content not of any particular loaf but of the unit loaf.

No. 70. No. 70. (Pl. U.)

" $7\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$ hekat of flour made into 100 loaves. What is the content of a single loaf in flour? What is their strength?"

You are to reckon with $7\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$ to find 100:

1	$7\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$
2	$15\frac{1}{2} + \frac{1}{4}$
4	$31\frac{1}{2}$
8	63
$\frac{2}{3}$	$5\frac{1}{4}$
Total	$99\frac{1}{2} + \frac{1}{4}$; remainder $\frac{1}{4}$
	$\frac{1}{63}$ is $\frac{1}{8}$ th. For $\frac{1}{4}$ double the fraction.

$\frac{1}{42} + \frac{1}{126}$ is $\frac{1}{4}$

The strength is $12\frac{2}{3} + \frac{1}{42} + \frac{1}{126}$

1	$12\frac{2}{3} + \frac{1}{42} + \frac{1}{126}$
2	$25\frac{1}{3} + \frac{1}{21} + \frac{1}{63}$
4	$50\frac{2}{3} + \frac{1}{14} + \frac{1}{21} + \frac{1}{126}$
$\frac{1}{2}$	$6\frac{1}{3} + \frac{1}{84} + \frac{1}{252}$
$\frac{1}{4}$	$3\frac{1}{6} + \frac{1}{168} + \frac{1}{504}$
$\frac{1}{8}$	$1\frac{1}{2} + \frac{1}{12} + \frac{1}{336} + \frac{1}{1008}$
Total	2520 (read 100).

You are to reckon with 100 to find 2520:

1	100
10	1000
20	2000
5	500
$\frac{1}{5}$	20

The content of a single loaf is $(\frac{1}{16} + \frac{1}{64})$ hekat + $\frac{1}{5}$ ro of flour.

1	$(\frac{1}{16} + \frac{1}{64})$ hekat + $\frac{1}{5}$ ro
10	$(\frac{1}{2} + \frac{1}{4} + \frac{1}{32})$ hekat + 2 ro
100	$(7\frac{1}{2} + \frac{1}{4} + \frac{1}{8})$ hekat of flour."

The method is exactly that of No. 69. The strength is found first by dividing 100 by $7\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$, answer $12\frac{2}{3} + \frac{1}{42} + \frac{1}{126}$. This result is then proved by multiplication. The total of products in this should be 100, but we find instead of it 2520. This is the correct answer to the next step, namely the reduction of $7\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$ hekat to ro, which is the first part of the finding of the *hrt*, but which the scribe has omitted. The case is one of simply haplography of ro.

In front of the first word of this example stands in black the sign  (see Pl. S, No. 64). Griffith is probably right in suggesting that it is here used in its not uncommon meaning of "stand" or "stop" (cf. p. 68), and refers to the irregularity of the page at this point. It is quite impossible to read it into the structure of the first line of No. 64. For its use, perhaps quite differently, in account papyri see SPIEGELBERG, *Rechnungen aus der Zeit Setis I*, Text, 48 and 58, Plates VIII and XIII. Compare too the *hrt* of Pap. Harris A, 1, 13 and 2, 4.

¹ A.Z., 47, 7-16, and V.Z.Z., 42-44.

Note here again, in line 1 and later, the unusual hieratic form of the numeral 7. It is No. 70. used in No. 53 of 7 *setat*: here it is used of 7 *hekat*, as also in No. 75 and No. 84.

$k\bar{b} \text{ tit } r \frac{1}{4}$. Either "double the fraction up to $\frac{1}{4}$," or passive, "the fraction is doubled up to $\frac{1}{4}$." The *tit* referred to is clearly $\frac{1}{8}$, i.e. an aliquot part, which confirms the view taken as to the meaning of this word in No. 61B.

In the last line but three we have the curious redundant form of nominal sentence with both *m* and *pw* used in No. 64 and again in No. 71.

No. 71.

No. 71.

"One *des*-measure of beer, a quarter of which has been poured off. It has then been made up with water and tasted with regard to what the strength is. You are to convert the 1 *des* into *besha*-grain: result half (a *hekat*) of *besha*-grain. You are to subtract a quarter of it, namely $\frac{1}{8}$ *hekat*: the remainder is $\frac{1}{4} + \frac{1}{8}$ *hekat*. You are to operate on $\frac{1}{4} + \frac{1}{8}$ to find 1. Result $2\frac{2}{3}$. This is the strength, namely $2\frac{2}{3}$."

With this example we come to the *pfšw* of beer. At the time when this papyrus was written the *pfšw* of a *des* of beer was 2, i.e. each *des* contained $\frac{1}{2}$ *hekat* of grain, or in other words each *hekat* produced 2 *des* of beer.

It is important to notice that the vertical stroke which follows the determinative of *ds* both in line 1 and in line 2 is the numeral 1 and not merely a stroke accompanying the determinative $\overline{\text{Q}}$, which would be an improbable writing for this period. We must therefore translate not "A *des*-jug" but "One *des*-measure." In other words, though *ds* may originally have been the name of a jug of a certain shape used principally or wholly for beer, it had by this time become quite definitely a measure of liquid capacity. It has not been sufficiently recognized that the Egyptians had a series of measures of liquid as well as of solid capacity, each measure being used for some particular liquid or liquids and for those alone. Thus in Pap. Kahun, Pl. XXVI, 1-33, we have an account of various kinds of vessels to be made by a potter. The text is not easy, but it seems clear that the capacity of the various vessels was given; and if this is correct the *ds* was certainly a measure, for the *tnft* vessel is to be made with a capacity of 2 *ds*.¹ Similarly in the Siut contracts Hapzefa contracts for a *st*: of beer for every quarter-*ds* which is offered. Here it is impossible to deny that the *ds* is a definite measure, and indeed its use throughout these very formal contracts makes it obvious. Unfortunately we have no evidence for determining the relations of these various measures to one another or to the *hekat*. An exception exists in the case of the *pg*:, a honey measure, which, as is clear from Harris I, 39, 6, is equivalent to 4 *hnw*, i.e. $\frac{2}{3}$ of a *hekat*.

In the present problem we are given a *des*-measure of beer, one quarter of whose contents has been poured away and the jug filled up with water. Required to find the strength of the mixture now in the jug. The original beer contained $\frac{1}{2}$ *hekat* of corn, and what is left after the pouring out will contain $\frac{1}{2}$ less ($\frac{1}{4} \times \frac{1}{2}$), i.e. $\frac{1}{4} + \frac{1}{8}$ *hekat*. The filling up with water does not alter the amount of corn now represented in the jug, so that the corn content of the mixture is still ($\frac{1}{4} + \frac{1}{8}$) *hekat*. To get the *pfšw* we have to invert this, which gives $\frac{8}{3}$ or $2\frac{2}{3}$. This is the *pfšw* of the mixture, i.e. it is the number of *ds*-measures of beer of this strength which can be made from a *hekat* of corn.

There is a scribe's error in the second line which led Eisenlohr to read *bš*: as the name of the unit of capacity which we now know to be *hk:t*. After *hpr-hr bš*: the writer has first missed out the *hekat* sign $\overline{\text{Q}}$ which is needed to tell us what the unit here is, having confused it by haplography with the determinative $\overline{\text{Q}}$ of *bš*:, and secondly he wrote the ordinary pure fraction $\frac{1}{2}$ instead of the sign for $\frac{1}{2}$ *hekat* in the Horus-eye notation. This error led Eisenlohr to read "Result $\frac{1}{2}$ *besha*" instead of "Result *besha*-corn $\frac{1}{2}$ *hekat*." Notice that in

¹ Similar evidence is to be found in Pap. Bulaq 18. See for example Pl. XXIV, where a *mns*:vase of beer seems to contain 3 *ds* and a *kby*-vase 2 *ds*. See *J.Z.*, 57, 56, note 13.

- No. 71. the next line the *hekat* sign is inserted before the $\frac{1}{8}$ *hekat* but is omitted thenceforward, the unit being already clearly enough marked.

sty mw is the usual Egyptian for to "pour water." An early example Pyr. 230 b.

dn is unknown, and the order of its determinatives, the more specific  after the more general  is suspicious.

dp has a strange determinative for which I can find no parallel.

The phrase *dp·ntwf r pfsw m* "It has been tasted regarding strength what" is impossibly elliptical, and can hardly be right as it stands. We expect something like *dp·ntwf r rh pfsw·f m m* "It has been tasted in order to know what is its strength."

For *bš*; corn see above, p. 114.

In the last words we remark the same redundant form of nominal sentence, *pfsw m 2 $\frac{2}{3}$ pw*, that we saw in Nos. 64 and 70. In the plate the *m* has been omitted.

No. 72. (Pl. V.)

"Example of exchanging loaves for loaves. If it is said to thee, 100 loaves of strength 10 exchanged for a number of loaves of strength 45.

You are to make the excess of 45 over¹ 10, namely, 35. You are to reckon with 10 to find 35; result $3\frac{1}{2}$. You are to multiply 100 by $3\frac{1}{2}$; result 350; add 100 to it; result 450.

Then shalt thou say, This means that the 100 loaves of strength 10 are exchanged for 450 loaves of strength 45, making in *wdyt*-flour 10 *hekat*."

The method is curiously roundabout. The obvious method is to divide 45 by 10, result $4\frac{1}{2}$, and to multiply this by 100, result 450. For some reason the Egyptian prefers to deal with the excess of the *pfsw* 45 over the 10, which is 35. He then finds the number of loaves corresponding to this *pfsw* to be 350, to which he adds the 100 to get the answer 450. This is really an astounding procedure, for the working out of the number of loaves corresponding to the imaginary *pfsw* 35 involves all the knowledge necessary to work out directly the number corresponding to 45. The problem was to solve the proportion

$$a : b :: c : x, \text{ where } a \text{ is } 10, b \text{ } 45, \text{ and } c \text{ } 100.$$

Instead of multiplying *b* by *c* and dividing by *a* the Egyptian works out the proportion

$$a : b - a :: c : x - c$$

Finally he adds *a* to the second term and *c* to the fourth, which of course does not affect the proportion and gives the value of *x*.

When both the number of loaves and their *pfsw* are to be given the *pfsw* comes immediately after the word for loaves and is followed by the number preceded by a sign resembling . This sign degenerates into a mere dot. In Pap. Kahun, Pl. XXVIa, this sign stands before the *pfsw* as well as before the number of loaves, and the same is true of Pap. Bulaq 18 (see for example Pl. XXIV of that papyrus).² That it is not the preposition *r* "to the amount of," or similar, is proved by *t; t;  100* (No. 73, line 2), where the article is made feminine to agree with the numeral *st*, 100, showing that the Egyptian said "The 100 loaves" and not "the loaves up to 100," or in other words that he did not pronounce the . A similar sign perhaps in No. 47, where see the notes.

For *sw*, "excess," compare No. 22 and note there.

db; pw t; t; etc. It is possible to take this as a nominal sentence with *db* as a noun, but the sense produced is not good, "This is the exchange of the 100 loaves of strength 10 for 450 loaves of strength 45." Here *pw* should refer to the answer 450, found just above, and in this case we should not expect to find the 450 mentioned in the sentence. It is therefore

¹ Delete in the plate the reference letter *a* over the .

² In Rhind the *pfsw* may be preceded by a dot (Nos. 77 and 78), but not by the -like sign.

possible that we have here a passive use of the $\text{\textit{sdm.f pw}}$ formula of the Ebers Papyrus, "This No. 72. means that the 100 loaves of strength 10 are exchanged for 450 loaves of strength 45." I can, however, quote no instances of such a construction, except possibly the $\text{\textit{irt pw}}$ of Nos. 38 and 62, where, however, the form, if it is indeed passive, is $\text{\textit{sdm.twf}}$ and not $\text{\textit{sdm.wf}}$.

For $\text{\textit{wdyt}}$ see above, p. 114. The determinative $\text{\text{𓏏}}$ has been omitted through haplography with the following $\text{\text{𓏏}}$, which is the sign for $\text{\textit{hkt}}$, the stroke which follows indicating 10 according to the regular notation. This 10 *hekat* is the amount of flour contained in the 100 large or 450 small loaves: it is not actually used in the working.

No. 73. (Pl. V.)

No. 73.

"If it is said to thee, 100 loaves of strength 10 exchanged for strength 15, how many is that in exchange for them?"

You are to find the amount of the 100 loaves in $\text{\textit{wdyt}}$ -flour [namely 10 *hekat*]. You are to multiply 10 by 15; result 150. Then shall you say, This is their exchange.

The doing as it occurs: 100 loaves of strength 10 exchanged for 150 loaves of strength 15: 10 *hekat*."

Here in place of the absurd method of No. 72 we find a straightforward reduction of the 100 loaves of $\text{\textit{pfsu}}$ 10 to the amount of flour used in making them, which is clearly 10 *hekat*. This 10 *hekat* when turned into loaves of $\text{\textit{pfsu}}$ 15 will obviously yield 150.

An unfortunate error of copying has obscured this simple working. After the words $\text{\textit{ir-hr-k hrt t: t: 100 m wdyt}}$ the original must have read $\text{\text{𓏏}}$, "namely 10 *hekat*," but the scribe's eye, misled perhaps by the similarity of $\text{\text{𓏏}}$ (especially when written without its cross-stroke) and $\text{\text{𓏏}}$, has wandered to the opening words of No. 76, $\text{\textit{ki t: 10}}$, which here make nonsense. Perhaps in his prototype the first line of No. 73 ended at $\text{\textit{wdyt}}$, and the scribe instead of dropping his eyes to the line below went straight on to the left into the opening words of No. 76.

$\text{\textit{wr pw r db:s}}$. Here we have again the late Egyptian $\text{\textit{wr}}$ meaning "how many" or "how much" which we met in No. 45. The sentence seems clumsy and the $\text{\textit{r}}$ could well be omitted, "How much is their exchange?" We must remember, however, that $\text{\textit{r db:s}}$ is the usual Egyptian for "in exchange for" (Coptic $\epsilon\tau\beta\epsilon$), so that the construction is less unnatural than it appears at first sight. Note the Pronominal Suffix $\text{\textit{s}}$, Feminine Singular to agree with the feminine numeral $\text{\textit{st}}$ 100. Cf. No. 65 and note, and contrast $\text{\textit{db:f}}$ in No. 74.

The stroke at the end of the sum stands for the 10 *hekat* of flour involved in the calculation, as is clear from No. 72.

No. 74. (Pl. V.)

No. 74.

"Another. A thousand loaves of strength 5 exchanged for (loaves of) strength 10 and 20. What is their exchange?"

You are to reduce to corn(?) the thousand loaves of strength 5; result 200 *hekat* of Upper Egyptian barley. Then shall you say, This is (the amount of) $\text{\textit{wdyt}}$ -flour.

You are to take half of 200 *hekat*, namely 100 *hekat*. You are to multiply 100 *hekat* by 10; result 1000: this is the number of strength 10. You are to multiply the 100 *hekat* by 20; result 2000. This is the number of strength 20.

The doing as it occurs:

A thousand loaves of strength 5, making in $\text{\textit{wdyt}}$ -flour	200 <i>hekat</i> .
Exchange, 1000 of strength 10,	" " 100 "
Exchange, 2000 of strength 20,	" " 100 <i>hekat</i> ."

Here we have a similar example with a complication introduced, for the exchange is to be reckoned in bread of two different strengths, 10 and 20. The 1000 loaves of strength 5

- No. 74. are reduced to *hekat*, giving 200 *hekat*. This is then divided into two equal halves of 100 *hekat* each, and one half is turned into 1000 loaves of strength 10 and the other into 2000 loaves of strength 20. That the exchange is to consist of equal amounts of bread of strength 10 and of strength 20 is an assumption of which no hint is given in the setting out, but one which the working shows to have been made.

Here again the scribe has not been happy with the text. In the first line he writes $\underline{db} : m \ 10 \ \circlearrowleft \ 20$, which would mean "exchanged for 20 loaves of strength 10." He has evidently failed to see that the 10 and 20 are both *pfsw*, and that therefore the $\circlearrowleft$ is not needed.¹

$\underline{pf} \dot{s} : \underline{hr} : k \ p : t : 5 \cdot 1000$. The first sign of this phrase can hardly be transcribed otherwise than $\cap$. The meaning must be "Make the *pfsw*-reckoning with the 1000 loaves," in other words, find the amount of corn in them. Note the Masculine $p : t : 1000$, since the numeral 1000 (*h*;) is Masculine, and contrast the Feminine used with 100 (*st*) in Nos. 70 and 73. The Masculine suffix $\underline{db} : f$ also agrees with 1000.

- No. 75. No. 75. (Pl. V.)

"Another. 155 loaves of strength 20 exchanged for loaves of strength 30.

You are to express the 155 loaves of strength 20 in *wdyt*-flour; that is, $(7\frac{1}{2} + \frac{1}{4})$ *hekat*. Multiply by 30; result $232\frac{1}{2}$.

The doing as it occurs:

155 loaves, strength 20, making in *wdyt*-flour $(7\frac{1}{2} + \frac{1}{4})$ *hekat*
exchanged for $232\frac{1}{2}$ " strength 30, " " $(7\frac{1}{2} + \frac{1}{4})$ *hekat*."

No comment is here necessary, except that we again have the unusual form of the numeral 7 that we have found in Nos. 53 and 70.

- No. 76. No. 76. (Pl. V.)

"Another. A thousand loaves of strength 10 exchanged for a number of loaves of strength 20 and 30.

Let him hear:

$$\begin{array}{r} \frac{1}{20} \quad \frac{1}{30} \\ 1\frac{1}{2} \quad 1 \\ \hline \text{Total} \quad 2\frac{1}{2} \end{array}$$

Multiply it to get 30:

$$\begin{array}{r} 1 \quad 2\frac{1}{2} \\ -10 \quad 25 \\ -2 \quad 5 \\ \hline \text{Total} \quad 12 \end{array}$$

Find the content of the 1000 loaves in *wdyt*-flour, namely 100 *hekat*.

Multiply by 12; the result thereof is 1200, their exchange in loaves of 20 and of 30.

1000 loaves of strength 10, making in *wdyt* 100 *hekat*.
1200 " " 20, " " 60 "
1200 " " 30, " " 40 *hekat*."

The problem seems at first sight similar to No. 74, but from the working we perceive that there is a difference in the conditions, though it is never stated in words. In No. 74 the total amount of corn in the loaves of the two different strengths was to be the same, whereas here the number of loaves of the two strengths is to be the same.

¹ Unless he used it as a mere separating mark. See note on No. 72.

We solve the problem by means of the equation $\frac{1}{20}x + \frac{1}{30}x = 100$, where x is the number of loaves of either kind, and the Egyptian does what is in effect the same thing. He adds $\frac{1}{20}$ and $\frac{1}{30}$, using as common denominator 30. The sum is then $\frac{1+1\frac{1}{2}}{30}$ or $\frac{2\frac{1}{2}}{30}$, which by the multiplication process is shown to be $\frac{1}{12}$. Then the number of *hekat* (100) multiplied by the 12 gives the number of loaves of each kind.

sdm.f is out of place here, as in No. 37. It occurs in examples (e.g. No. 30) where the question is put by a scribe, "If a scribe says to you . . . let him hear . . ." It is just possible that what stood here in the original was *ssmt.f* "its working."

hprt im pw. A very unusual way of expressing result in our papyrus. *hprt* is of course a Neuter Participle. Cf. GRIFFITH, *P.K.*, Pl. VIII, *Siut*, Pl. 7, line 300, and Pap. Moscow.

No. 77. (Pl. V.)

No. 77.

"Example of exchanging beer for bread. If it is said to you, 10 *des* of beer exchanged for (bread of) strength 5.

You are to turn the 10 *des* of beer into *wdyt*-flour, that is 5 *hekat*. You are to multiply the 5 *hekat* by 5, result 25. Then shall you say, This is their exchange.

The doing as it occurs:

Ten *des* of beer; 5 *hekat* of *wdyt*-flour
exchanged for 25 loaves of strength 5: 5 *hekat* of *wdyt*-flour."

This needs no comment. The *pfsw* of the beer is, as throughout the papyrus, 2 *des* per *hekat*, and the exchange in bread is obtained by means of a reduction to *hekat* of *wdyt*.

No. 78. (Pl. W.)

No. 78.

"Example of exchanging bread for beer. If it is said to you, A hundred loaves of strength 10 exchanged for a quantity of beer of strength 2.

You are to make the 100 loaves of strength 10 into *wdyt*-flour, that is 10 *hekat*. Multiply by 2; the result thereof is 20. Then shall you say, This is their exchange."

This offers nothing new except that the *irt m' hpr* of the previous examples, i.e. the tabulation of the working, is omitted.

No. 79. (Pl. W.)

No. 79.

"An inventory of a household(?).

1	2801	7	houses
2	5602	49	cats
4	11204	343	mice
Total	19607	2301 (<i>sic</i>)	spelt
		16807	<i>hekat</i>
		Total	19607."

The meaning of this table was first explained by Rodet (*Journal Asiatique*, 1881, 450 ff.).¹ It is evidently based on a nursery problem of the following nature:—

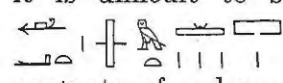
Seven houses; in each are 7 cats; each cat kills 7 mice; each mouse would have eaten 7 ears (or grains) of spelt; each ear of spelt will produce 7 *hekat*. What is the total of all these?

¹ Rodet gives a good parallel from the *Liber Abaci* of Leonard of Pisa, 1202 A.D., edit. Boncompagni, Rome 1857, I, 311, which deserves quoting again: "Septem vetulae vadunt Romam; quarum quaelibet habet burdone 7; et in quolibet burdone sunt sacculi 7; et in quolibet sacco panes 7; et quilibet panis habet cultellos 7; et quilibet cultellus habet vaginas 7. Quaeritur summa omnium praedictorum." The nursery-rhyme beginning "As I was going to St. Ives" is perhaps its lineal descendant.

No. 79.

We thus get in effect a geometrical progression whose first term is 7 and whose common ratio is 7, the number of terms being 5. The Egyptian solves this in two ways, firstly he adds up the terms, which is not difficult in a series with so few terms, and secondly he gets the answer by multiplying 2801 by 7. But unfortunately he does not make it clear whence came the number 2801. The modern expression for the sum of a geometrical series is $a \frac{r^n - 1}{r - 1}$, where a is the first term, r the common ratio and n the number of terms. Applying this to the present case we get s (the sum) $= 7 \cdot \frac{16807 - 1}{7 - 1} = 7 \times \frac{16806}{6} = 7 \times 2801$, which shows exactly what are the figures used by the Egyptian.

It is further to be noticed that when $a = r$, i.e. when the series is the sum of the powers of any number, r^n is the last term, and the equation then becomes $s = r \frac{l-1}{r-1}$, where l is the last term, a formula which the Egyptians may have obtained empirically. If this is the case, it is possible that the only geometric series with which they dealt were of this nature, viz. sums of the powers of a number. In any case the solution of even this limited type of geometric series is very flattering to their mathematical intelligence.

The signs forming the heading of this sum are mainly unintelligible as they stand, and it is difficult to suggest any restoration which will fit or account for the traces.¹ Perhaps  is as likely as anything. *imt-pr*² means originally "the inventory of the contents of a house," and hence "a deed of conveyance" or even "a will." Here we should have the word in its literal sense.

No. 80. No. 80. (Pl. W.)

"As for a vessel in which are corn-measures for the clerks of the slave-prison:

Expressed in *henu*

1 <i>hekat</i>	10
$\frac{1}{2}$ "	5
$\frac{1}{4}$ "	$2\frac{1}{2}$
$\frac{1}{8}$ "	$1\frac{1}{4}$
$\frac{1}{16}$ "	$\frac{1}{2} + \frac{1}{8}$
$\frac{1}{32}$ "	$\frac{1}{4} + \frac{1}{16}$
$\frac{1}{64}$ "	$\frac{1}{8} + \frac{1}{32}$

This is nothing more than a table for expressing the *hekat* and its Horus-eye parts, $\frac{1}{2}$, $\frac{1}{4}$, etc., in terms of *henu*, of which there are 10 to a *hekat*. It is not easy to see why this should need the imposing title which is here given to it, and since this table is repeated at the beginning of the next example it is possible that there has been an error of some kind in the copying, and that we have lost the table which originally stood under this heading.

The *dbh*, as Griffith points out, must be the wooden vessel with which labourers are seen measuring out grain in the tomb representations. Determined with the grain-sign it occurs in the Protestation of Innocence in Chapter 125 of the Book of the Dead, "I have not increased or reduced the measuring-vessel." Among the gifts dedicated by Tuthmosis III to Amūn are figured seven *dbh* marked "Measuring-vessels of gold for measuring the divine property."³

*h*₃ determined by the grain-sign seems to be unknown as a measure; determined by the rope it is the common word for a measuring-tape or the plumb-line of a balance. In this passage it is probably the same word as *h*₃ determined by the grain-sign which occurs in GRIFFITH, *Siut*, Pl. XV, line 9, where Khety, speaking of his benefits to his city, says, "I was abundant in Lower Egyptian barley . . . making the city to live by the *h*₃ and by the

¹ The *B.M. Facs.*, despite its suspect appearance, is almost accurate.

² See GRIFFITH, *K.P.*, Text, 29.

³ *Urk.*, IV, 635.

hekat." From this, as well as from the determinative, we may assume that the h_3 was an No. 80. actual measure like the *hekat*, and not a vessel of size not necessarily fixed like the *dbh*.

This does not throw very much light on the relation of the title of this example to the table which stands under it. What, moreover, is the reason for the introduction of the clerks of the slave-prison ($\dot{s}n^c$), since surely the use of the *hekat* and the *henu* was common to all transactions whether governmental or otherwise at this period.

$\dot{s}n^c$ is the factory or complex of factories where all sorts of provisions etc. were made. The prisoners taken by the king are set to work in the $\dot{s}n^c$ of Amun. We might render *ergastulum*.

No. 81. (Pl. W.)

No. 81.

"Another reckoning of the *henu*.

Now $\frac{1}{2}$ *hekat* is 5 (*henu*)

$$\begin{array}{lcl} \frac{1}{4} & \text{,,} & 2\frac{1}{2} \\ \frac{1}{8} & \text{,,} & 1\frac{1}{4} \\ \frac{1}{16} & \text{,,} & \frac{1}{2} + \frac{1}{8} \\ \frac{1}{32} & \text{,,} & \frac{1}{4} + \frac{1}{16}^1 \\ \frac{1}{64} & \text{,,} & \frac{1}{8} + \frac{1}{32} \end{array}$$

Now $(\frac{1}{2} + \frac{1}{4} + \frac{1}{8})$ *hekat* in *henu* is $8\frac{1}{2} + \frac{1}{4}$

$$\begin{array}{llll} (\frac{1}{2} + \frac{1}{4}) & \text{,,} & \text{,,} & 7\frac{1}{2} \\ (\frac{1}{2} + \frac{1}{8} + \frac{1}{32}) & \text{,,} & + 3\frac{1}{3} \text{ ro} & \text{,,} \text{,,} 6\frac{1}{2} + \frac{1}{16} \text{ (sic)} \quad \text{That is } \frac{2}{3} \text{ of a } \textit{hekat} \\ (\frac{1}{2} + \frac{1}{8}) & \text{,,} & \text{,,} & 6\frac{1}{4} \quad \text{That is } \frac{1}{5} \text{ (? } \frac{5}{8} \text{) } \text{,,} \text{,,} \\ (\frac{1}{4} + \frac{1}{8}) & \text{,,} & \text{,,} & 3\frac{1}{2} + \frac{1}{4} \quad \text{That is } 3 \text{ (? } \frac{3}{8} \text{) } \text{,,} \text{,,} \\ (\frac{1}{4} + \frac{1}{32} + \frac{1}{64}) & \text{,,} & + 1\frac{2}{3} \text{ ro} & \text{,,} \text{,,} (3\frac{1}{4} + \frac{1}{8}) + \frac{2}{3} \text{ (sic)} \quad \text{That is } \frac{1}{7} \text{ (sic)} \text{,,} \text{,,} \\ \frac{1}{4} & \text{,,} & \text{,,} & 2\frac{1}{2} \quad \text{That is } \frac{1}{4} \text{,,} \text{,,} \\ (\frac{1}{8} + \frac{1}{16}) & \text{,,} & + 4 \text{ ro} & \text{,,} \text{,,} 2 \quad \text{That is } \frac{1}{5} \text{,,} \text{,,} \\ ([\frac{1}{8}] + \frac{1}{32}) & \text{,,} & + 3\frac{1}{3} \text{ ro} & \text{,,} \text{,,} 1 + [\frac{2}{3}] \quad \text{That is } [\frac{1}{6}] \text{,,} \text{,,} \end{array}$$

Now $(\frac{1}{8} + \frac{1}{16})$ *hekat* + 4 ro is 2 *henu*

$$\begin{array}{llll} (\frac{1}{16} + \frac{1}{32}) & \text{,,} & + 2 \text{ ro} & \text{,,} 1 \text{,,} \quad \text{That is } \frac{1}{10} \text{,,} \text{,,} \\ (\frac{1}{32} + \frac{1}{64}) & \text{,,} & + 1 \text{ ro} & \text{,,} \frac{1}{2} \text{,,} \quad \text{That is } \frac{1}{20} \text{,,} \text{,,} \\ \frac{1}{64} & \text{,,} & + 3 \text{ ro} & \text{,,} \frac{1}{4} \text{,,} \quad \text{That is } \frac{1}{40} \text{,,} \text{,,} \\ \frac{1}{16} & \text{,,} & + 1\frac{1}{3} \text{ ro}^2 & \text{,,} \frac{2}{3} \text{,,} \quad \text{That is } \frac{1}{30} \text{ (? sic) of a } \textit{hekat} \\ \frac{1}{32} \text{ hekat} + 1\frac{2}{3} \text{ ro (sic)}^3 & \text{,,} & \frac{1}{3} & \text{,,} \quad \text{That is } \frac{1}{60} \text{ (sic) of a } \textit{hekat} \\ \frac{1}{64} \text{,,} + 1\frac{1}{3} \text{ ro (sic)}^4 & \text{,,} & \frac{1}{5} & \text{,,} \quad \text{That is } \frac{1}{50} \text{,,} \text{,,} \\ \frac{1}{2} & \text{,,} & 5 & \text{,,} \quad \text{That is } \frac{1}{2} \text{,,} \text{,,} \\ \frac{1}{4} & \text{,,} & 2\frac{1}{2} & \text{,,} \quad \text{That is } \frac{1}{4} \text{,,} \text{,,} \\ (\frac{1}{2} + \frac{1}{4}) & \text{hekat} & 7\frac{1}{2} & \text{,,} \quad \text{That is } (\frac{1}{2} + \frac{1}{4}) \text{,,} \text{,,} \\ (\frac{1}{2} + \frac{1}{4} + \frac{1}{8}) & \text{,,} & 8\frac{1}{2} \text{ (sic) henu} & \text{,,} \quad \text{That is } (\frac{1}{2} + \frac{1}{4} + \frac{1}{8}) \text{ of a } \textit{hekat} \\ (\frac{1}{2} + \frac{1}{8}) & \text{,,} & 6\frac{1}{4} \text{ henu} & \text{,,} \quad \text{That is } (\frac{1}{2} + \frac{1}{8}) \textit{ hekat} \\ (\frac{1}{4} + \frac{1}{8}) & \text{,,} & (\frac{1}{2} + \frac{1}{4}) \text{ (sic) henu} & \text{,,} \quad \text{That is } (\frac{1}{4} + \frac{1}{8}) \text{,,} \\ (\frac{1}{2} + \frac{1}{8} + \frac{1}{32}) & \text{,,} & + 3\frac{1}{3} \text{ ro} & \text{,,} 6\frac{2}{3} \text{ henu} \quad \text{That is } \frac{2}{3} \textit{ hekat} \\ (\frac{1}{4} + \frac{1}{16} + \frac{1}{64}) & \text{,,} & + 1\frac{2}{3} \text{ ro} & \text{,,} 3\frac{1}{3} \text{,,} \quad \text{That is } \frac{1}{3} \text{,,} \\ \frac{1}{8} \textit{ hekat} & \text{,,} & 1\frac{1}{4} & \text{,,} \quad \text{That is } \frac{1}{8} \text{,,} \\ \frac{1}{16} & \text{,,} & (\frac{1}{2} + \frac{1}{8}) \text{ henu} & \text{,,} \quad \text{That is } \frac{1}{16} \text{,,} \\ \frac{1}{32} & \text{,,} & (\frac{1}{4} + \frac{1}{16}) & \text{,,} \quad \text{That is } \frac{1}{32} \text{,,} \\ \frac{1}{64} & \text{,,} & (\frac{1}{8} + \frac{1}{32}) & \text{,,} \quad \text{That is } \frac{1}{64} \text{,,} \end{array}$$

¹ The *B.M. Facs.* gives a vertical stroke instead of the ligature for 6.

² Below this, on the bottom edge of the papyrus, the tops of the figures $\frac{1}{16}$ *hekat* (?) and $\frac{1}{3}$. Not shown in *B.M. Facs.*

³ The 1 is certain, though wrong.

⁴ The scribe wrote $1\frac{1}{3}$ and then crossed it out. Read $1\frac{2}{3}$.

No. 81. This is simply a table for expressing the various more complicated fractions of the *hekat* in *henu*. The text is very incorrect.

It begins with a repetition of the table of No. 80, on which see note above. Then succeeds a more elaborate table, of which the following is a typical line:—

$$\frac{1}{64} \text{ hekat} + 3 \text{ ro is } \frac{1}{4} \text{ henu; that is, } \frac{1}{40} \text{ of a hekat.}$$

In the first column is the required fraction of the *hekat*, expressed correctly in the Horus-eye notation with smaller fractions added in *ro*. In the central column is the corresponding number of *henu*, and in the last the quantity expressed as a pure fraction of a *hekat*. This last column is added in red, except in the first group of nine quantities, where it is placed before in black, and is hopelessly incorrect and incomplete. It is possible that the scribe in writing in the black portions of the text forgot to leave room for this first red section in the proper place and had to crowd it in as best he could. For the relation of Column 3 to Column 1 see Introduction, p. 25.

No. 82. No. 82. (Pl. X.)

“Estimate of the food of a poultry farm.

Reckoned in bread per day:	<i>wdyt</i> -flour.
Fatted geese: that which 10 birds eat is	$2\frac{1}{2}$ <i>hekat</i> .
making in 10 days	25 <i>hekat</i> .
making in 40 days	100 <i>hekat</i> .
That which must be ground in order to produce (?) it:	
spelt (?)	$(166\frac{1}{2} + \frac{1}{8} + \frac{1}{32})$ <i>hekat</i> and $3\frac{1}{3}$ <i>ro</i> .
wheat	$(66\frac{1}{3} + \frac{1}{4} + \frac{1}{16} + \frac{1}{64})$ <i>hekat</i> ¹ and $1\frac{2}{3}$ <i>ro</i> .
That which is to be subtracted at the rate of one-tenth,	
	$(6\frac{1}{2} + \frac{1}{8} + \frac{1}{32})$ <i>hekat</i> and $3\frac{1}{3}$ <i>ro</i> .
Remainder to be given $(93\frac{1}{4} + \frac{1}{16} + \frac{1}{64})$ <i>hekat</i> and $1\frac{2}{3}$ <i>ro</i> .	
making in grain in <i>hekat</i>	$(93\frac{1}{4} + \frac{1}{16} + \frac{1}{64})$ and $1\frac{2}{3}$ <i>ro</i> .
making in double- <i>hekat</i>	$(47\frac{1}{2} + \frac{1}{4} + \frac{1}{64})$ <i>hekat</i> and $3\frac{1}{3}$ <i>ro</i> .”

The general sense of the problem would appear to be to find the amount of grain (*ss*) needed to make the bread eaten by certain poultry in a certain time. Nevertheless, there are many obscurities of detail.

It is clear from line 5 that the bread needs 100 *hekat* of *wdyt*-flour. But in line 6 we meet with a crux. The first signs are clearly $\overline{\text{Ⲁ}} \text{Ⲁ} \text{Ⲁ} \text{Ⲁ} \text{Ⲁ}$, and in view of the parallel *ntt r lbt* below it is difficult to avoid the conclusion that these words mean “That which must be ground.” Then follows a ligature,² and next a sign which I cannot transliterate (it looks like a bird) and beneath it a Ⲁ . Griffith read this last sign as Ⲁ , the numeral 100, referring to the 100 *hekat* of flour: this is unlikely, for 100 *hekat* is written Ⲁ . The next sign Griffith reads Ⲁ , probably rightly, though it has this form nowhere else in the papyrus, nor indeed in any papyrus, and though the ligature for Ⲁ which ought to follow it is missing: it is exactly like Ⲁ . The probable meaning of the line is “That which must be ground to produce it (i.e. the 100 *hekat* of *wdyt*) is $166\frac{2}{3}$ *hekat* of spelt. The arrangement of the hieratic shows that in the next line the words “That which must be ground to produce it” must be repeated, and we thus have for line 7 “That which must be ground to produce it is $66\frac{2}{3}$ *hekat* of

¹ Note the unique use here of Ⲁ to represent $33\frac{1}{3}$ *hekat* and the retention of the resulting $\frac{1}{3}$ *hekat* in defiance of the rule of correct notation. Cf. p. 26, note 2.

² Perhaps an incorrect determinative Ⲁ to *nd*.

No. 83.	making for 10 <i>ro</i> -geese	1 <i>hekat</i> of Lower Egyptian barley
	for 10 days	10 <i>hekat</i>
	for a month	30 <i>hekat</i>

Daily allowance of food for a fatted

<i>ro</i> -goose, what (it) eats is	$(\frac{1}{8} + \frac{1}{32})$ <i>hekat</i> + $3\frac{1}{3}$ <i>ro</i> , per bird
<i>terp</i> -goose	$(\frac{1}{8} + \frac{1}{32})$ <i>hekat</i> + $3\frac{1}{3}$ <i>ro</i> „
crane ¹	$(\frac{1}{8} + \frac{1}{32})$ <i>hekat</i> + $3\frac{1}{3}$ <i>ro</i> „
<i>set</i> -bird ²	$(\frac{1}{32} + \frac{1}{64})$ <i>hekat</i> + 1 <i>ro</i> „
<i>ser</i> -bird	$\frac{1}{64}$ <i>hekat</i> + 3 <i>ro</i>
turtle-dove ³	3 <i>ro</i>
quail ⁴	3 <i>ro</i>

Total” (*sic*)

No. 83 consists of three separate sections, the mathematical content of which is of very small value.

The meaning of the first, which occupies the first three lines, is clear. If 4 birds eat one *henu* of food, *i.e.* 32 *ro*, then the share of one bird will be 8 *ro*, or $\frac{1}{8}$ *hekat* (which is 5 *ro*) plus 3 *ro*. The wording, however, is not quite so straightforward, owing to the fact that line 2 has been lost and incorrectly restored on a fresh strip by the ancient mender of the papyrus. Thus the two signs after $\frac{1}{8}$ are meaningless and unnecessary. Moreover, the word *hnw* is incomplete, and where the signs $\square$ $\frac{1}{8}$ stand we need $\frac{1}{8}$: in any case, a nominal sentence with *pw* after *ir* meaning “if” is suspect. The restorer would seem to have worked on the analogy of the second part of the example (line 5 below), not noticing that the method of statement was slightly different. We could of course save *pw* by taking *ir* as the particle, “As for the food of,” but this gives a much poorer sense to the whole, which, as being the statement of a problem, demands a conditional clause.

The distinction between the geese who are “cooped up” and those who are allowed to enter the pond was first perceived by Griffith. The feminine form *htmyt* must be wrong, for the *r*-goose is masculine. The restorer is again at fault. Possibly the whole word is wrong, for he certainly restored it as a word giving a suitable contrast with *h.f r ss* in line 4.

The second portion of the example, lines 4–8, is little more than a statement of the fact that $(\frac{1}{16} + \frac{1}{32})$ *hekat* + 2 *ro* is equivalent to 32 *ro* or 1 *henu*. This allowance is then worked out for ten geese during a month.

The third portion of the section (lines 9–end) would seem to concern the poultry farmer more than the mathematician. It is a list of various kinds of domesticated birds, showing the amount which they eat. The word “total” at the end suggests that a simple addition in dimidiated parts of the *hekat* was intended, but it was never carried out, or at least was not transferred to our copy of the papyrus.

The various types of bird mentioned are all well known from the monuments, and some have been identified with comparative certainty. They mostly occur in the more complete offering lists such as *Beni Hasan* I, Pl. XVII, and Cairo sarcophagus 28001. For good illustrations consult DÜMICHEN, *Resultate*, Pl. IX (Ptahhotep); *El Bersheh*, I, Pl. XXII; BORCHARDT, *Grabdenkmal des Sahure*, II, Bl. 55 and 56, and text thereto. Compare also Pap. Kahun, Pl. VIII, where the values of some of these birds are compared with that of the *st*-duck.

¹ L.D., II, 61, 150; B.M. Stela 134 (1164).

² The long-tailed duck, *Dafla acuta*, Linn. See *Rec. Trav.*, 33, 59–64.

³ Figured in the tomb of Rekhmara. (cf. SETHE, *Urk.*, IV, 1124; cf. DAVIES, *Mustaba of Ptahhetep*, I, 20, and Pl. VIII, 113; GRIFFITH, *Hieroglyphs*, 20.

⁴ A.Z., 30, 25.

No. 84. (Pl. X.)

No. 84.

" Estimate of the food of a stall of oxen.

	... food	ordinary ... food (?)
	hekat	hekat
Bull; that which 4 eat, good bulls of Upper Egypt	24 hekat	2 hekat
" " 2 " good bulls of Upper Egypt	22 hekat	6 hekat
" " 3 " ordinary . . . cattle	20 hekat	2 hekat
" " 1 " . . . cattle	20 hekat	
Total of this food	8 (sic)	6 hekat
Making in spelt	9 (?) "	7½ "
Amounting in 10 days to	90 "	75 "
Amounting in a month to	200 "	90 "
Making in double-hekat (?)	(61½ + ½) hekat	30 hekat."
	+ 3 ro	

With this problem the papyrus reaches its limit of unintelligibility and inaccuracy. The question is to calculate the food of certain cattle, which are apparently to be fed on two different kinds of food the names of which it is not easy to transcribe. Both the foods are reckoned in *hekat*. From this it is to be expected that in the table which follows we should have three figures in each line, viz. the number of cattle of a particular kind, the quantity of the first food and the quantity of the second. This actually appears to be the case. Thus in line 2 we seem to have four dots (not strokes, oddly enough) to represent the four bulls: under the heading of the first kind of food we find two strokes followed at once by 4 dots, which in the *hekat* notation means 24 *hekat*: under the other type of food stand two dots, meaning 2 *hekat*.

So far there is no serious difficulty. When the addition comes to be made the numbers of bulls are not added at all, being unessential to the result. The *hekat* in the last column are correctly added up to 10, rightly expressed by a stroke. In the centre column, however, the strokes, of which there are eight, each standing for 10 *hekat*, are mistaken for units and added up to 8, while the dots are added separately to give, quite rightly, 6 *hekat*.

In the next line, where this food is reduced to terms of spelt, the 8 is calmly neglected, the 6 *hekat* of the one food give 9 *hekat* of spelt, while the 10 *hekat* of the other food give 7½ *hekat* of spelt. As we know nothing of the foods in question we cannot judge of the accuracy of this. This is evidently a daily allowance, for it is now correctly multiplied by 10 to get the ration for 10 days, 90 and 75 *hekat* respectively. Next the allowance for a month is found, but, sad to relate, three times 90 is given as 200 and three times 75 as 90. A final resolution into double-*hekat* gives opportunity for two more errors, half of 200 *hekat* being given as (61½ + ½) *hekat* + 3 *ro*, and half of 90 *hekat* as 30. Griffith was tempted by these last figures to suggest the existence of a triple-*hekat*.¹ Ninety *hekat* turned into triple-*hekat* would become 30 and 200 would become 66⅔, to which the figure given, 61½, is at least a better approximation than it is to 100. Unfortunately the sign which stands between the } and the 𐍌 in the name of the unit is very uncertain, the reading 𐍌 being far from convincing, and we may well re-echo Griffith's own words, "The treble *hekat* is extremely doubtful."

Such is the calculation contained in this example. It is clearly full of mathematical errors, and there are obviously other mistakes so serious that any attempt to guess at what originally stood in the text would be the merest waste of time.

¹ P.S.B.A., XIV, 428.

No. 85. No. 85. (Pl. X, bottom, left.)

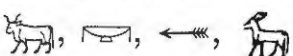
The Mathematical Papyrus proper ends with No. 84, which takes us well into the fourth leaf of the verso of Papyrus 10058. In the blank which remains between this and the end of the roll in its present state is written, upside down, the curious group of signs numbered 85 by Eisenlohr and found on Pl. XXI of the British Museum facsimile, on the right-hand side. The signs are rather cursive hieroglyphs written in black in two vertical columns. It is possible that this writing may be a cryptogram with a meaning of some kind, but it is wiser not to follow Eisenlohr's example of attempting a translation. It seems equally possible that it is a mere random group written in order to try a pen or to practise the formation of the signs.

No. 86. No. 86. (Pl. Y.)

This number is given by Eisenlohr to certain fragments of a papyrus of accounts used in ancient times to mend a weak place in the roll. They now lie near the right-hand end of 10057 verso (Pl. XXI, left, in *B.M. Facs.*). A rapid examination will show that there are here three separate fragments or strips placed vertically, but not in position relative to one another. They may easily be correctly arranged. The narrow strip on the left should be cut out and placed to the immediate left of the broad strip at the top. The central strip in the lower half must be turned upside down and placed so that the double horizontal line becomes continuous with that on the main strip to the right.

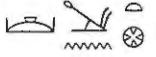
This re-arrangement has now been carried out in the original papyrus, presumably since the making of the facsimile, but the work has not been quite accurately done, and the lower strip, with the left-hand portions of lines 13 to 18, has been set a few millimetres too far to the left. In the centre of line 9 there is a tiny patch which, as the double black line and red traces show, should be turned upside down and placed between the two main strips in line 14 below.

The resulting text is a mere fragment from a papyrus of accounts. If the transcription  at the beginning of line 1 is correct, and there is not much doubt about it, it must have been preceded by a date and a king's name, and therefore a considerable portion of each line is probably lost on the right. In a text so mutilated as this translation is difficult and almost worthless, and even transcription is very far from certain. The rendering here given is presented mainly for the sake of completeness. Any attempt at comment would be a waste of time and space.

The writing of these accounts is considerably more cursive than that of the mathematical papyrus, but little if at all later in date; compare the forms of the signs  with those of other papyri of the Hyksos Period. Griffith even attributes it to the scribe of Rhind.¹

1. "...living for ever. List of the food in Hebenti (?).²
2.[his] brother, the steward Kamōse....
3.of his year, 50 pieces of silver twice a year (??)....
4.2...., i.e. 3 pieces of silver, in the year....
5.one (?), twice, i.e. one sixth and one sixth. Now as for one....
6.12 *henu*, i.e. $\frac{1}{4}$ piece of silver; one.....
7.5 pieces of [silver]; their price therefor;³ 120 fish (?), twice;....
8.year: 90 quadruple-*hekat* of barley, 100 + x of spelt....
9.90 of barley, 192 of spelt....

¹ P.S.B.A., XVI, 248.

² BRUGSCH, *Dictionnaire Géographique*, 469, records a place called  in the 14th nome of Upper Egypt.

³ For *swnt iri*. Cf. Griffith, *P.K.*, Pl. XXXIX, ll. 3, 9, etc. Or read perhaps "Their price 120 *iri*-fish."

10.146½(?); 185(?) of barley, 300 + x of spelt.
11.½; there was delivered one donkey-load(?) of wine....
12.½ piece of [silver]....4(?),¹ i.e....of silver....
13.1¼ Thirty-six(?) *henu* of fat, i.e....of silver....
14.hundred and ninety-six....of barley, 410 quadruple-*hekat* of spelt."

No. 86.

The last line is repeated five times.

No. 87. (Pl. Y.)

No. 87.

This is written about the centre of the blank verso of 10057, at the top of the sheet. The translation is as follows:—

"Year 11, second month of the harvest season, Heliopolis was entered.

First month of the inundation season, day 23, the Army Commander(??) attacked(?) Zaru.² Day 25, information was received that Zaru had been entered.³

Year 11, first month of the inundation season, day 3. Birth of Set: the majesty of this god caused his voice to be [heard].⁴ Birth of Isis, heaven rained.⁵"

These notes contain entries of two kinds, the one referring apparently to military events and the other to weather conditions. They are dated in the eleventh year of a king whose name unfortunately is not given.

Möller describes the entries as of somewhat later date than the mathematical portion of the papyrus, possibly emanating from the same hand as No. 86. This is supported by the fact that endorsements on the back of a papyrus are not usually of much later date than the papyrus itself. Can this dating be supported by the evidence of the other entries, those which concern the weather? These were first translated and their import shown by Erman.⁶ The voice of Set is doubtless the thunder, and the facts described are that it thundered on the birthday of Set and rained on the following day, the birthday of Isis.

In a leaflet appended to his *Historical Studies* Petrie has discussed these entries and attempted to fix the date of the Hyksos period from them by the use of the Sothic Cycle. His argument is as follows. 1. The entries are of "about the same age" as the mathematical papyrus, which, it will be remembered, is dated in the 33rd year of Apophis I, the Hyksos king.⁷ 2. The birthdays of Set and Isis, which of course fall on the third and fourth intercalary days, are here given as falling on the third and fourth of the first month of the year, because the scribe forgot the intercalary days and went straight on from the last month of one year to the first of the next. 3. If we accept the length of the period between the XIIth and XVIIIth Dynasties as only about 200 years, the date of the 33rd year of Apophis will be somewhere between 1600 and 1750 B.C. At those dates the third and fourth intercalary days would have fallen early in September, when rain is practically unknown in Egypt. On the other hand, if we accept

¹ For the noun here cf. perhaps Pap. Turin, P. R. Pl. IX, line 6, and Pap. Leyden 343, recto, 5. 8.

² Reading very uncertain. For *tw* see p. 77 above, and *A.Z.*, 57, 38. The sign after the ox can hardly be , however. It is tempting to read    at the beginning of line 3, but so uncertain that it is not worth while to speculate on the possible identity of this general with the King *Mr-mš* whose statues, usurped by an Apophis, were found at Tanis (PETRIE, *Tanis* I, 8-9 and Pl. III, 17).

³ For *ḥ* in the passive with the place entered as subject compare Pap. Petrograd 1116B, recto 34.

⁴ The sign  must have been omitted here after   , for to utter the voice is not *rdt hrw* but *wdt hrw* (see Sinuhe, B, 140, and Gardiner's note thereto). In any case we cannot read    as passive, for there is a superfluous .

⁵ Here *irt* must be read as a *sdmf*, despite its incorrect form. Perhaps the scribe started out with an infinitive *irt*, parallel to *rdt* above, and then changed his mind.

⁶ *A.Z.*, 29, 59.

⁷ Petrie actually takes as a working hypothesis for the date the 11th year of Iamnis, the successor of Apophis.

- No. 87. Petrie's long dating, by which the date of the calendrical entries would be about 2370 B.C., the rainy weather would occur on Feb. 14 (or March 16 if we adopt the Thoth-year). These dates fall into the rainy season, and therefore the higher dating should be correct.

There are three assumptions in this reasoning: firstly that the notes are not much later in date than the mathematical part of the papyrus, secondly that the scribe forgot the intercalary days, and thirdly that the thunder and rain must fall into the usual rainy season in Egypt.

The first of these assumptions has the support of Möller and is quite possibly correct. This is, however, a matter of small importance, for the third assumption is totally unjustified. Here we have two entries concerning the weather which are thought worthy to be set side by side with important military events, and which must therefore in all probability have been something out of the ordinary. No one in England troubles to record, or even to remember, the occasions on which it snowed in December, but few forget the rare occasions when it snows in June. When therefore Petrie finds that on the short dating hypothesis these entries demand rain and thunder in the middle of September, a very unusual time, he is by no means at liberty to argue that therefore the short dating is incorrect, nor is it in the least a matter for self-congratulation that the long dating would bring this wet weather into the usual rainy season in Egypt.

Equally unjustified is Petrie's second assumption, namely that the scribe forgot the intercalary days on which the gods' birthdays fell and attributed the birthdays to the first five days of the new year. It is conceivable that in certain complicated cases a scribe should inadvertently omit the intercalary days, as he seems to have done in the Ebers calendar, but that he should wrongly attribute the birthdays of the gods to any other days is practically unthinkable.

Under these circumstances it would appear that to attempt to draw any chronological conclusions from these entries is but to ask for failure. They are not to be exploited either for or against the short dating.

At the same time they do contain a problem which we are not entitled to shirk. How are we to explain the attribution of the two birthdays to the third and fourth of the first month instead of to the third and fourth intercalary days. Two possibilities may be suggested. It may be that here we have a comparison of civil and Sothic years similar to that given in the Ebers calendar. Thus the birthdays of Set and Isis fell on the third and fourth intercalaries according to the civil calendar, but on the third and fourth of the first month according to the Sothic year, which, imaginary as its dates were to the people in general, was always kept in mind by the priests, and was furnished, as the Ebers calendar shows, with its months and days. In this case it will readily be seen that the civil year was at this time five days out of coincidence with the Sothic year, and this would have happened about 20 years before a Sothic Period, which Period could manifestly only be that of 1321 B.C. In this case we get a date of about 1341 for the entries. This is completely at variance with Möller's opinion with regard to the date of the script, and on this ground may be rejected.¹

The other possibility is that we have here a trace of that remarkable event in the Egyptian calendar first pointed out by Gardiner,² namely the change by which the first month of the year ceased to be Mesore and became Thoth. This change took place probably some time before the late XIXth Dynasty. How long before this we cannot say, but Gardiner has shown that in the

¹ It is perhaps just worth while to remember that cursive hieratic material for the late Intermediate Period and XVIIIth Dynasty is almost wanting, and comparisons are consequently impossible. Should this subsequently be discovered in any quantity it might force us to modify our opinion concerning these very rapidly scrawled entries. They contain two or three forms which seem to be rather XVIIIth Dynasty than Hyksos, but these may well be due to the fact that the cursive script was always ahead of the official copy-hand in innovations. The simple form of the word-sign for *hwi* "to strike" here used hardly survived the beginning of the XVIIIth Dynasty. That the entries should be as late as 1341 B.C. is almost out of the question. For the little that is known concerning XVIIIth Dynasty hieratic hands see Möller in *A.Z.*, 56, 34 ff.

² *A.Z.*, 43, 166 ff.

XIXth Dynasty itself there existed some confusion between the new and the old systems. The No. 87. way in which this change took place is not actually known, but we do know that the order of the months, *i.e.* their relative position with regard to one another, remained unaltered. The most likely supposition seems to be that at some period the intercalary days were moved from their position immediately before Mesore and placed immediately after it. Let us suppose this done in some particular year. Then after Epiphi 30 would follow the new Mesore 1, and the new Mesore 30 would fall on the old Mesore 25. The intercalary days now fall on the old Mesore 26-30, but a scribe having in mind the old system in which Thoth 1 followed on Mesore 30 might well make the mistake of attributing the five divine birthdays to Thoth 1-5 (Mesore 26-30 old reckoning).

This explanation is but a suggestion. Should it be judged fanciful, one may reply that it is more likely to be correct than the theory that a scribe under normal circumstances forgot that the five divine birthdays fell on the intercalary days. It affords of course no basis at all for fixing the date of the entry.

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EXPLANATION OF THE PLATES.

IN order to avoid possible confusion the plates have been lettered from A to Y (omitting I). Under each plate are noted the numbers of the plates of the *British Museum Facsimile* on which the sums transcribed on that plate will be found.

Signs here underlined in black are written in red ink in the original. In the case of signs written in vertical columns the line indicating red ink is drawn to the left of the column.

In the transcription the arrangement of the original, chaotic though it sometimes is, has been retained, except in cases where its retention would have involved great waste of space and consequent expense.

It should be remembered that the transcription is made not from the *B.M. Facs.* but from the original papyrus, and is therefore in some cases at variance with the *Facsimile*. The more important of these cases are noted in the critical notes. To note all the minor instances would have rendered the notes far too bulky.

Square brackets [], very sparingly used, denote restorations of lost or damaged signs which may be regarded as practically certain.

ERRATA.

- Plate A. Division of 2 by 5.—Last sign in top line should be  not .
- Div. by 17.—The multipliers 3 and 4 in the left column should be ticked.
- Div. by 23.—The multiplier $\frac{1}{2}$ in the central column needs the tick.
- Plate D. Div. by 91.—Insert a tick after the first bird.
- Plate H. No. 23.—For the underlined $5\frac{1}{2} + \frac{1}{4}$ in the left half read $5\frac{1}{2} + \frac{1}{8}$.
- No. 24.—Insert a tick before the multiplier $\frac{1}{4}$ in third column.
- Plate J. No. 30.—Insert a dot (the multiplier 1) before $13\frac{1}{2}$ in the fourth line.
- Plate M. No. 37.—Insert ticks before the multiplier $\frac{1}{3}$ in third column (fourth line from top); also before the multipliers $\frac{1}{4}$ and $\frac{1}{3}$ in the bottom line of the left half of the sum.
- Plate P. No. 53.—For $\times$ marked *sic* we should perhaps substitute  in both cases.
- Plate Q. No. 58, line 4.—For $\times$ read .
- No. 59, line 3.—For $\times$ read .
- Plate S. No. 64.—In line 1 insert the plural strokes under the first .
- Plate U. No. 69.—In the multiplication on the left insert ticks before the multipliers 16 and 64.
- No. 70.—In line 7 of multiplication on left the fraction $\frac{1}{2}$ should be set back close to $\frac{1}{2}$.
- No. 71.—In the last line insert  before the numeral 2 near the end.
- Plate V. No. 72, line 2.—Delete the reference to note *a*.
- Underline first six signs in Nos. 72 and 73, the word *kī* in Nos. 74–76, and the words *tp n* in No. 77.

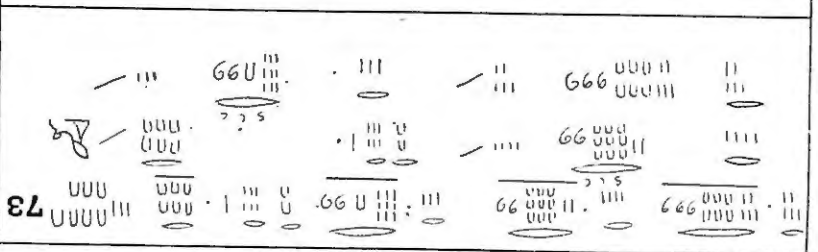
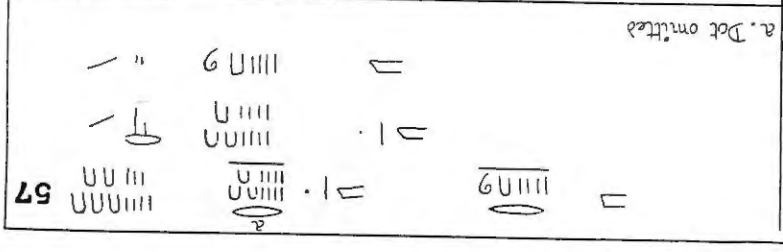
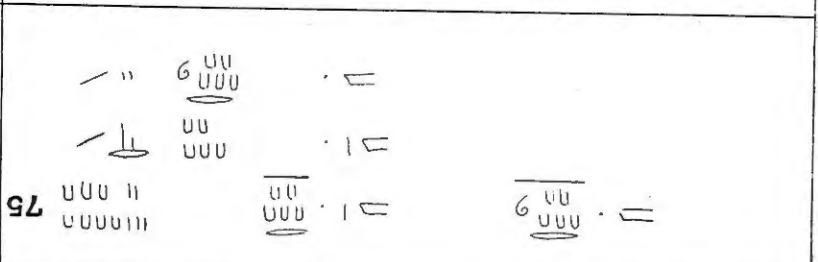
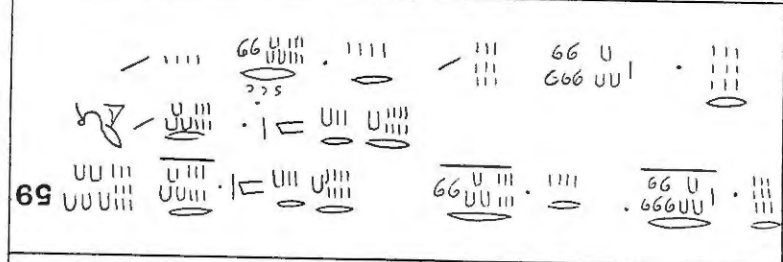
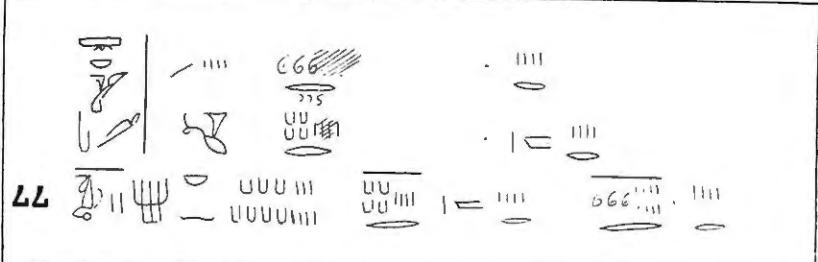
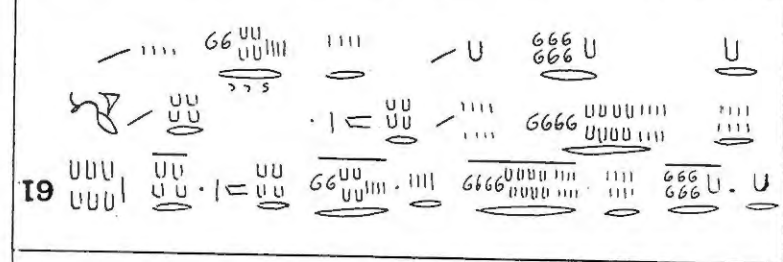
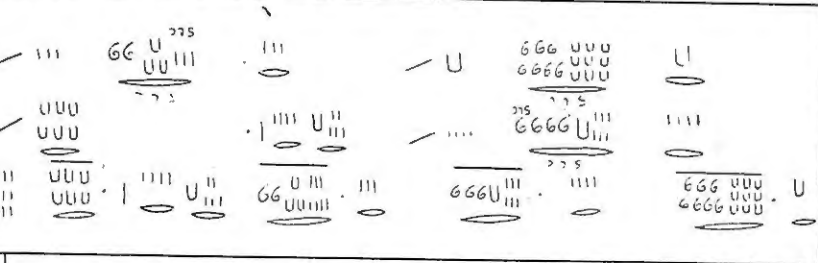
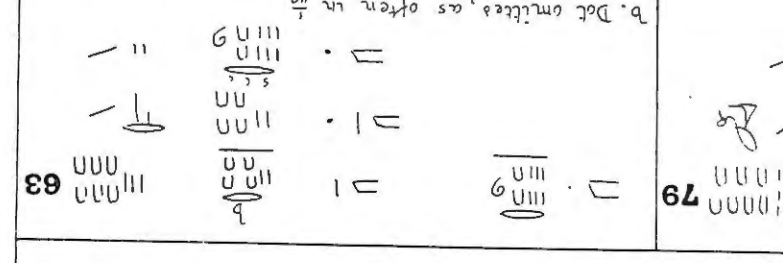
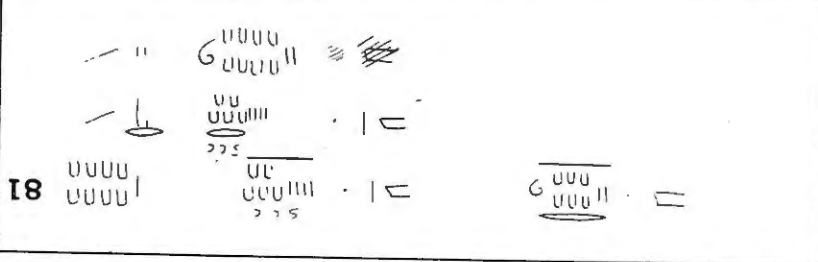
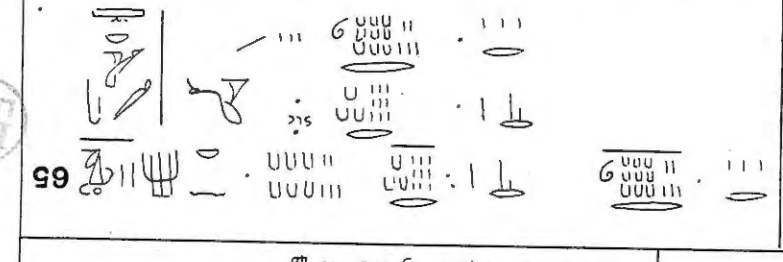
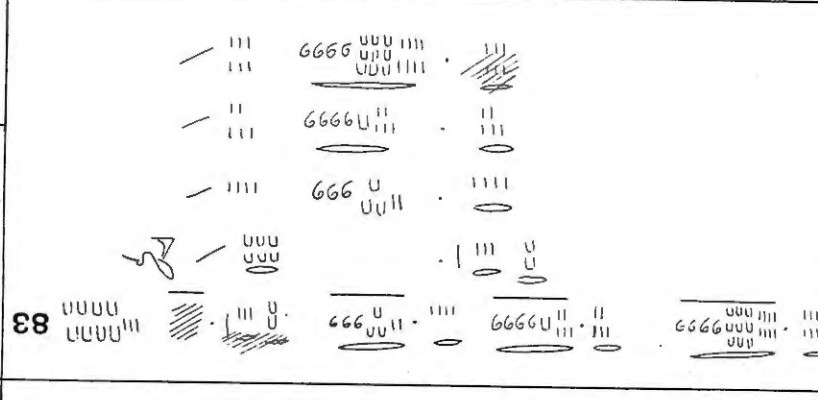
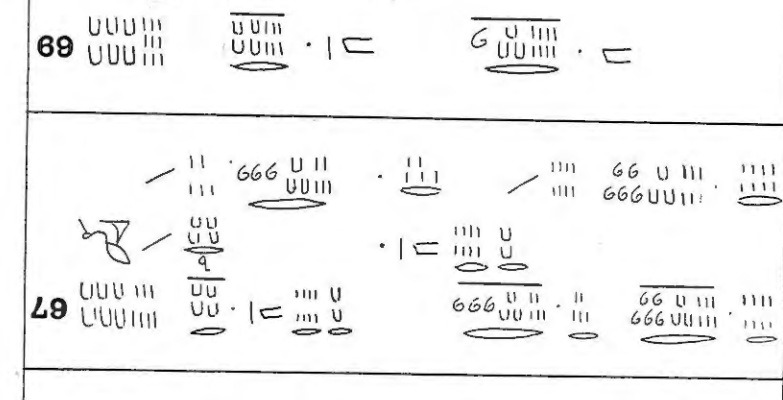
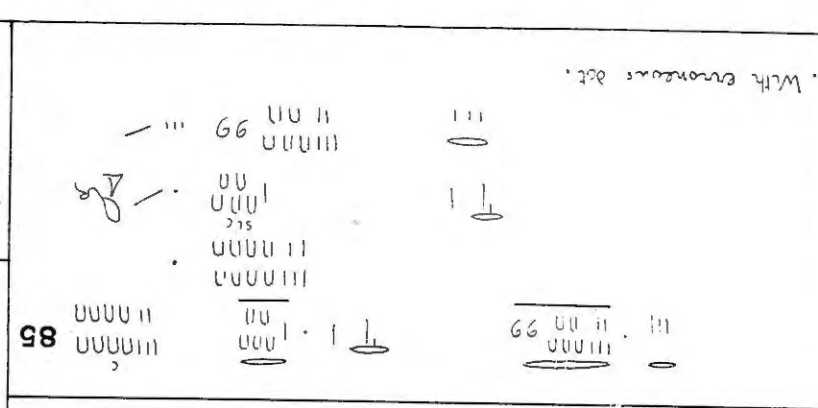
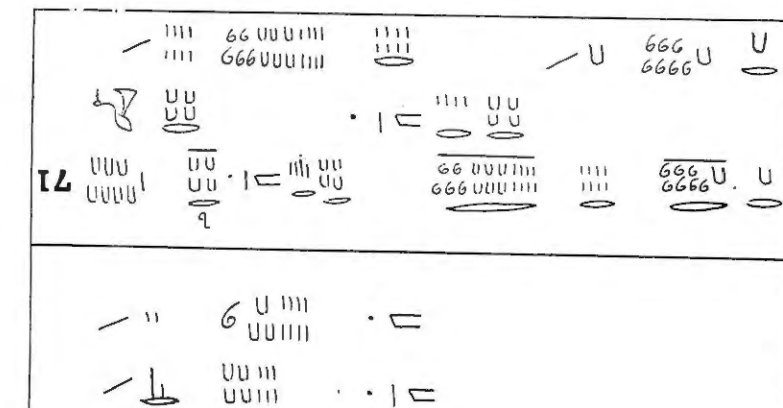
Handwritten text in a cuneiform script, arranged in several lines. The text appears to be a list or a series of entries, possibly related to the plates shown on the right.

BU
FILE

3	Handwritten cuneiform script, including a large symbol at the top left.
5	Handwritten cuneiform script.
7	Handwritten cuneiform script.
9	Handwritten cuneiform script.
11	Handwritten cuneiform script, including a large symbol at the top left.
13	Handwritten cuneiform script.
15	Handwritten cuneiform script.
17	Handwritten cuneiform script.
19	Handwritten cuneiform script.
21	Handwritten cuneiform script.
23	Handwritten cuneiform script.

a. Fragment too high in facsimile.
b. With fractional dot, hardly to be called erroneous.

41	43	27	45	29	47	31	49	33	51	35	53	37	55	39
-----------	-----------	-----------	-----------	-----------	-----------	-----------	-----------	-----------	-----------	-----------	-----------	-----------	-----------	-----------

73 	57 a. Dot omitted 
75 	59 
77 	61 
79 	63 b. Dot omitted, as often in 60 
81 	65 
83 	67 
85 	69 
c. With erroneous dot.	

B.M.Facs. Plates IV to VI.

a. Dot omitted. 87	89	91	b. so written as often 93	c. Very uncertain See text. 95	97	99	Brackets [] denote restorations. 101
----------------------------------	-----------	-----------	---	--	-----------	-----------	---

B.M.Facs. Plates VI and VII.

(See also Plate E.)



[illegible]

B. M. Facs. Plates VII and VIII.
(See also Plate E.)



	21
	22
	23
	24
	25
	26
a. Possibly mere prolongation of . b. With dash above, like .	

31

30

82

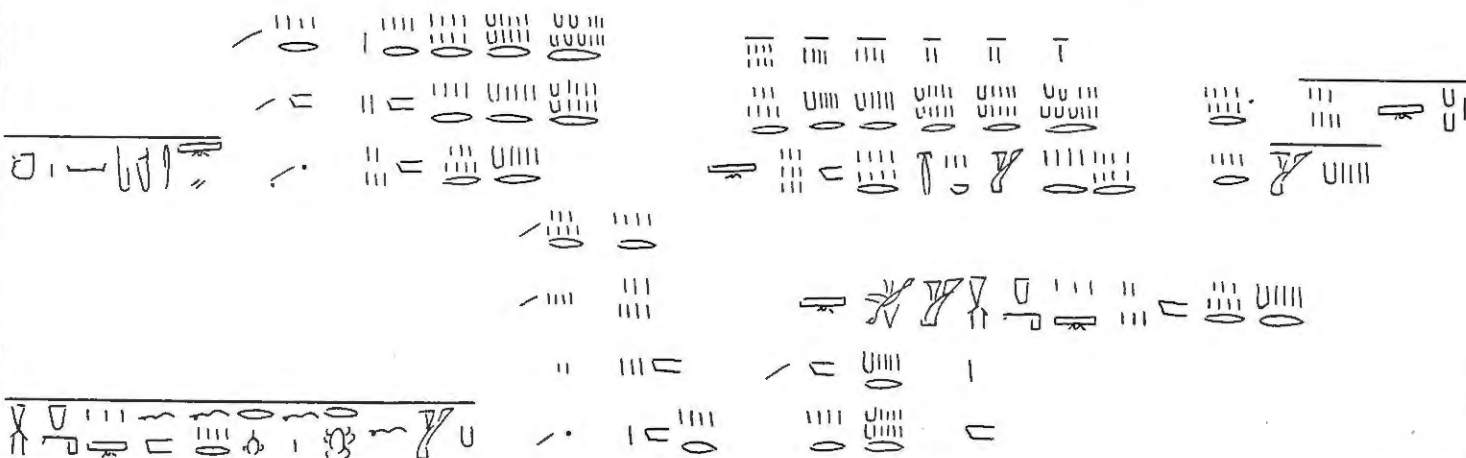
27

67

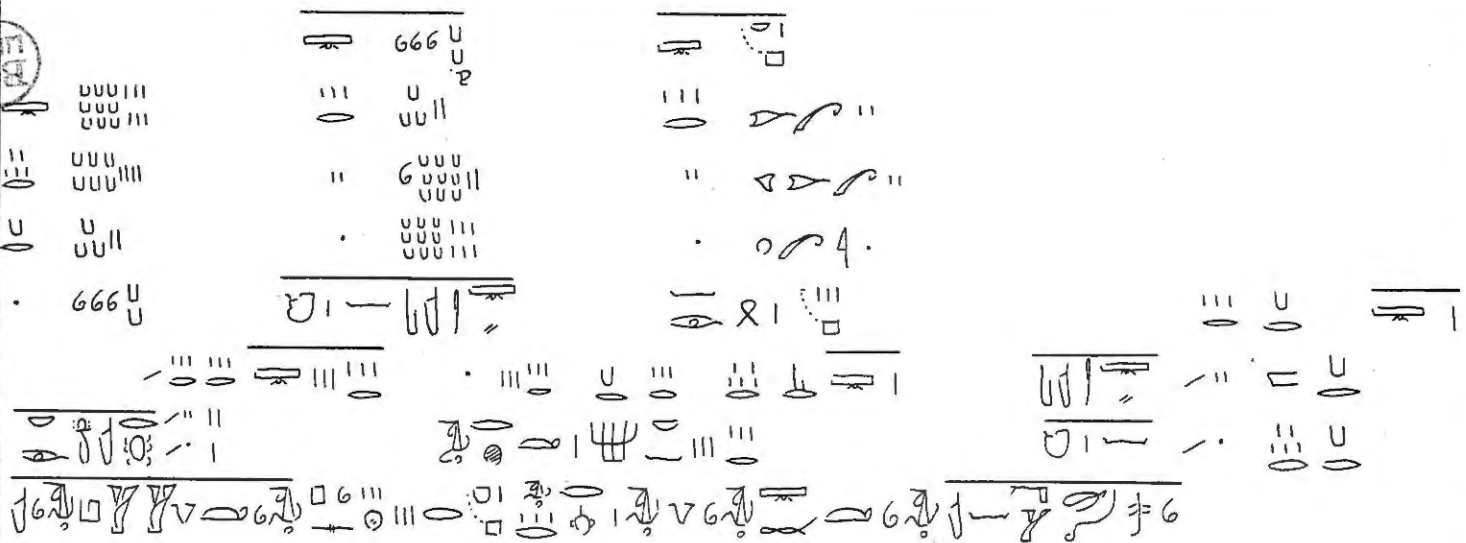


a. With erroneous fractional dot.	b. Dot placed over gir instead of before it.

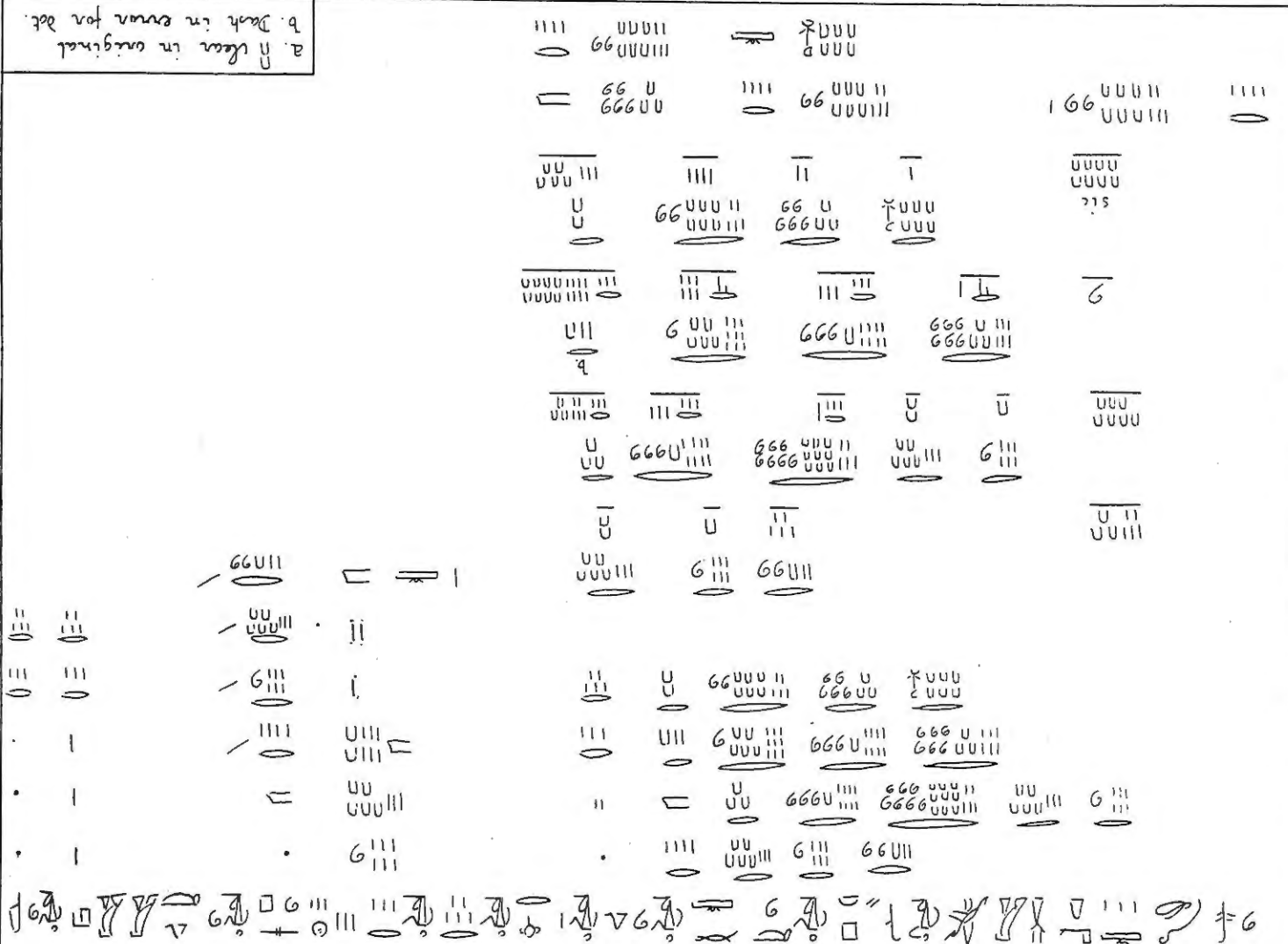
34



35



36



a. For group misplaced here see No. 51.
b. Added in black.

41		
40		
39		
38		

37

Handwritten musical notation on a staff, including various notes, rests, and bar lines. The notation is in a cursive script, likely Arabic or Persian. The staff is divided into measures by vertical bar lines. The notes are written in a stylized manner, with some characters resembling 'a', 'b', 'c', 'd', 'e', 'f', 'g', 'h', 'i', 'j', 'k', 'l', 'm', 'n', 'o', 'p', 'q', 'r', 's', 't', 'u', 'v', 'w', 'x', 'y', 'z'.

38

Handwritten musical notation on a staff, continuing the sequence from the previous page. The notation is in a cursive script, likely Arabic or Persian. The staff is divided into measures by vertical bar lines. The notes are written in a stylized manner, with some characters resembling 'a', 'b', 'c', 'd', 'e', 'f', 'g', 'h', 'i', 'j', 'k', 'l', 'm', 'n', 'o', 'p', 'q', 'r', 's', 't', 'u', 'v', 'w', 'x', 'y', 'z'.

39

Handwritten musical notation on a staff, continuing the sequence from the previous page. The notation is in a cursive script, likely Arabic or Persian. The staff is divided into measures by vertical bar lines. The notes are written in a stylized manner, with some characters resembling 'a', 'b', 'c', 'd', 'e', 'f', 'g', 'h', 'i', 'j', 'k', 'l', 'm', 'n', 'o', 'p', 'q', 'r', 's', 't', 'u', 'v', 'w', 'x', 'y', 'z'.

40

Handwritten musical notation on a staff, continuing the sequence from the previous page. The notation is in a cursive script, likely Arabic or Persian. The staff is divided into measures by vertical bar lines. The notes are written in a stylized manner, with some characters resembling 'a', 'b', 'c', 'd', 'e', 'f', 'g', 'h', 'i', 'j', 'k', 'l', 'm', 'n', 'o', 'p', 'q', 'r', 's', 't', 'u', 'v', 'w', 'x', 'y', 'z'.

41

Handwritten musical notation on a staff, continuing the sequence from the previous page. The notation is in a cursive script, likely Arabic or Persian. The staff is divided into measures by vertical bar lines. The notes are written in a stylized manner, with some characters resembling 'a', 'b', 'c', 'd', 'e', 'f', 'g', 'h', 'i', 'j', 'k', 'l', 'm', 'n', 'o', 'p', 'q', 'r', 's', 't', 'u', 'v', 'w', 'x', 'y', 'z'.

43

Handwritten musical notation on a single staff, featuring various rhythmic values (e.g., minims, crotchets, quavers) and bar lines. The notation is written in a cursive, historical style.

43

44

Handwritten mathematical notes and diagrams, including various symbols, fractions, and geometric shapes, likely related to the study of the circle and its properties.

[illegible]

46

a. Ligature: not like

46

Handwritten text in a cursive script, likely a historical form of English or a related language. The text is written in a single line across the top of the page, with some words appearing to be ligatures or abbreviations. The script is dense and difficult to decipher without a key.

45

Handwritten text in a cursive script, likely a historical form of English or a related language. The text is written in a single line across the top of the page, with some words appearing to be ligatures or abbreviations. The script is dense and difficult to decipher without a key.

44

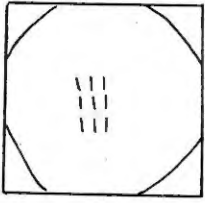
Handwritten text in a cursive script, likely a historical form of English or a related language. The text is written in a single line across the top of the page, with some words appearing to be ligatures or abbreviations. The script is dense and difficult to decipher without a key.

43

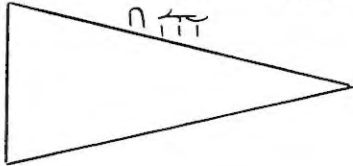
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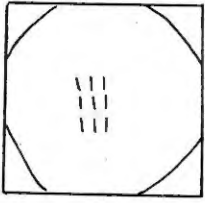
Handwritten text in a cursive script, likely a historical form of English or a related language. The text is written in a single line across the top of the page, with some words appearing to be ligatures or abbreviations. The script is dense and difficult to decipher without a key.

47
48
49
50
51

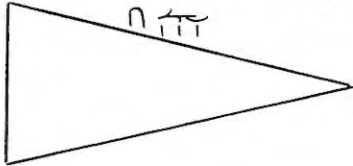


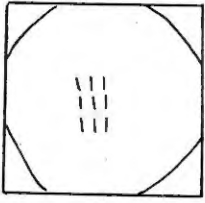




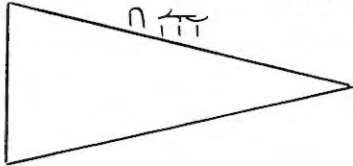


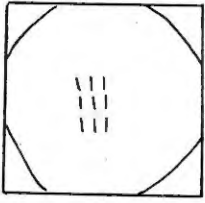




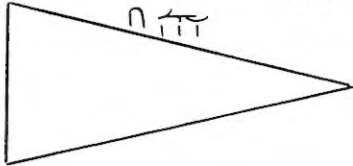


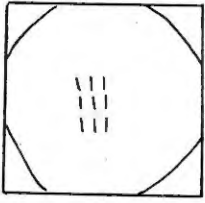




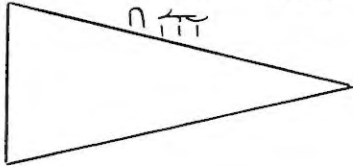


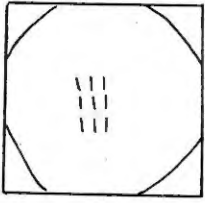




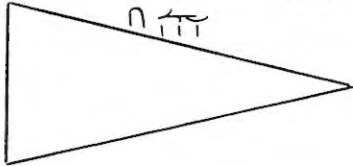


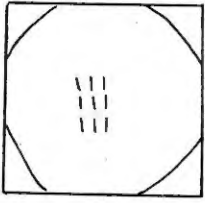




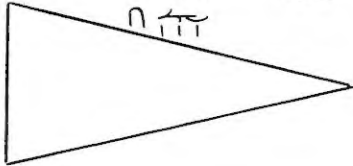


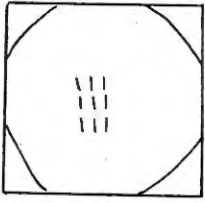




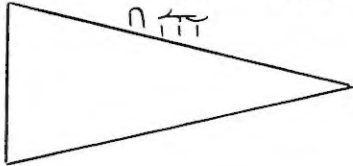


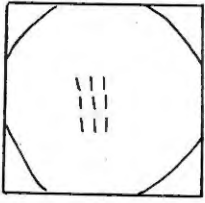




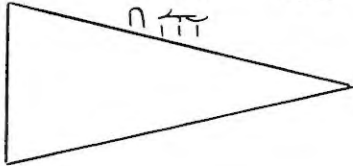


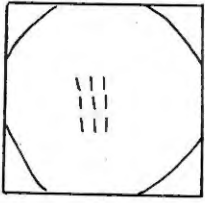




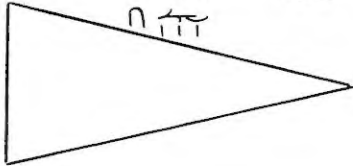


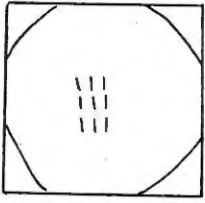




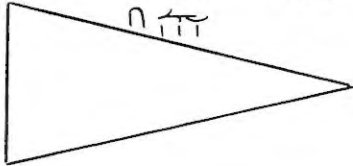


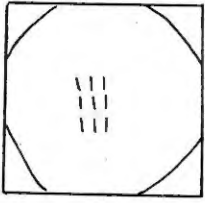




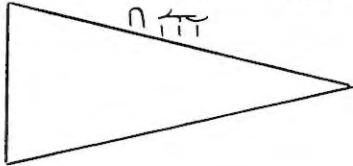


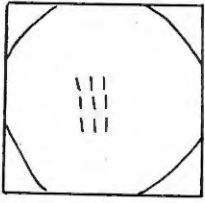




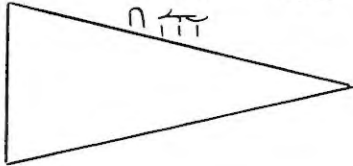


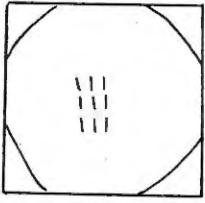




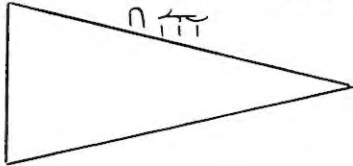


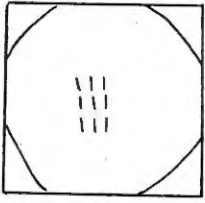




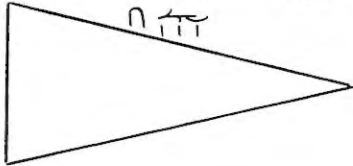


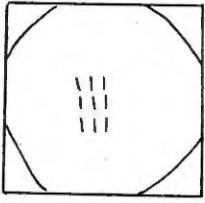




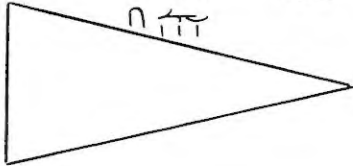


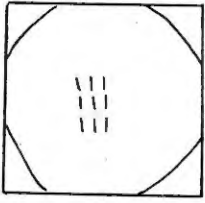




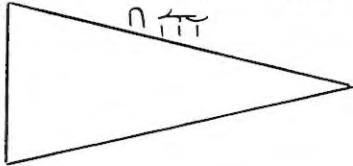


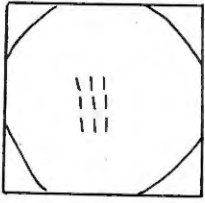




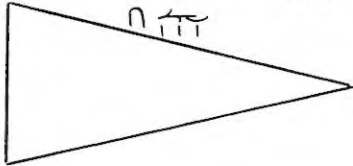


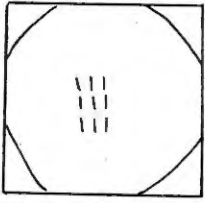




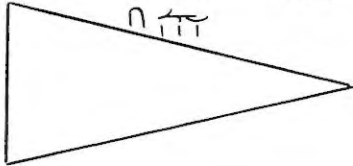


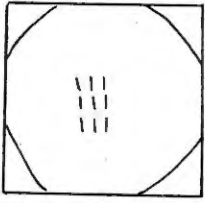




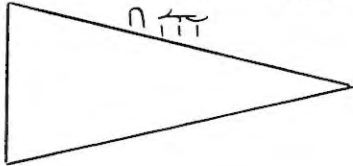


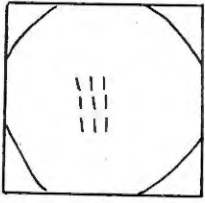




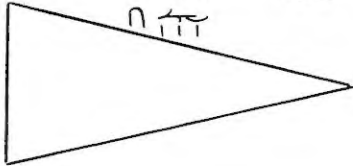


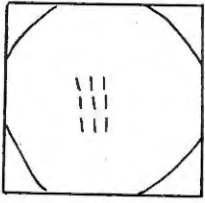




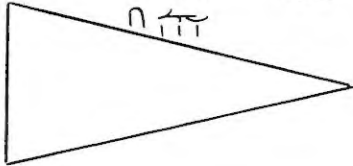


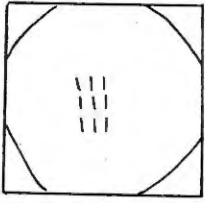




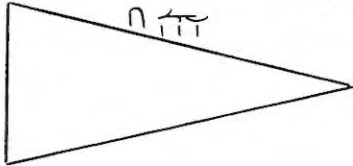


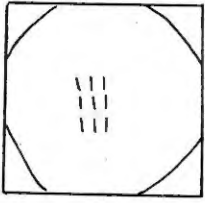




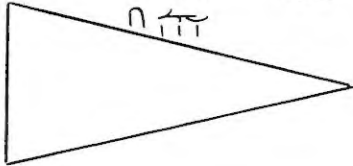


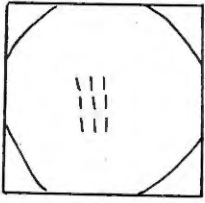




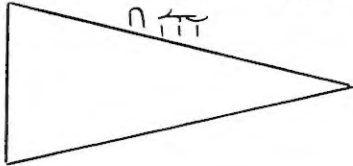


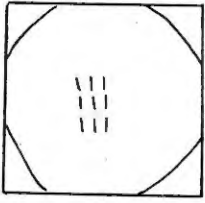




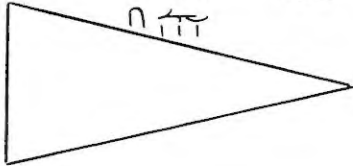


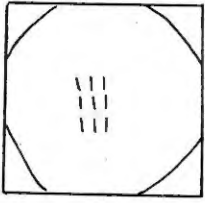




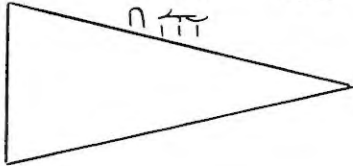


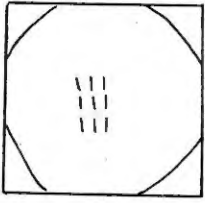




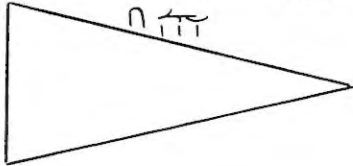


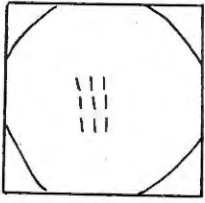




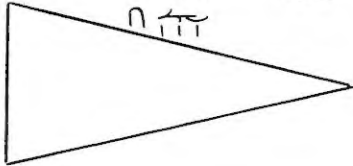


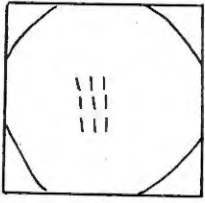




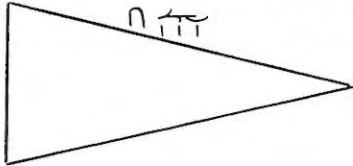


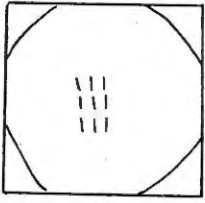




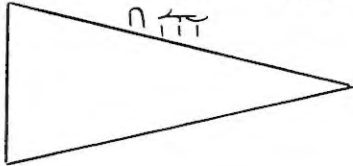


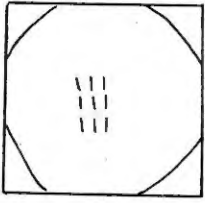




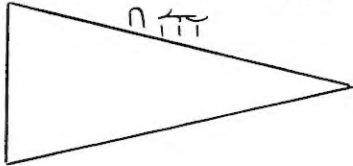


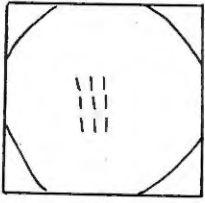




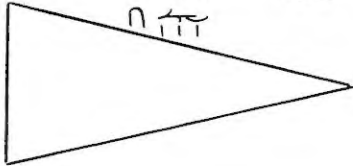


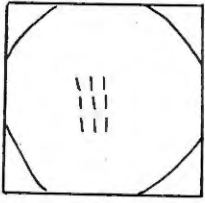




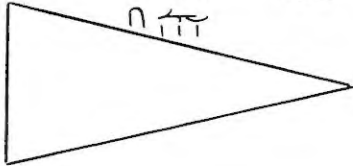


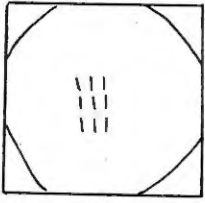




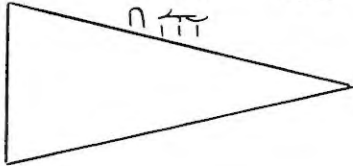


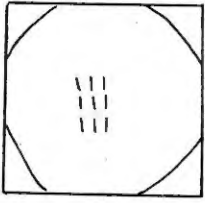




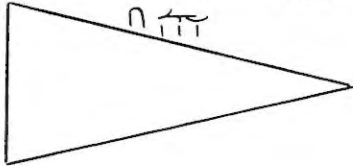


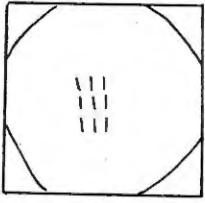




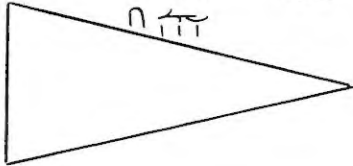


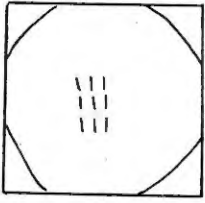




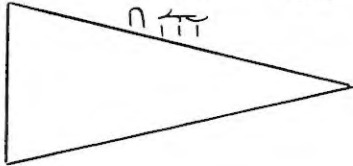


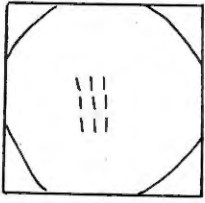




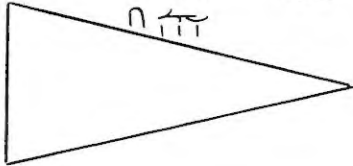


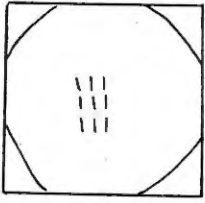




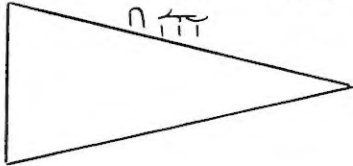


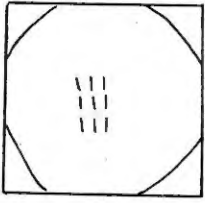




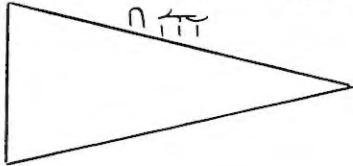


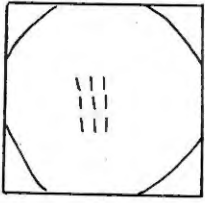




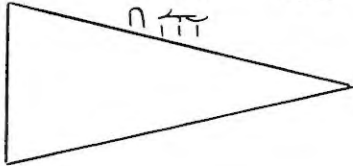


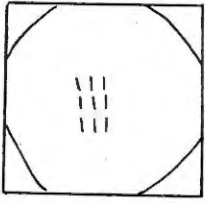




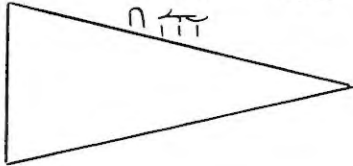


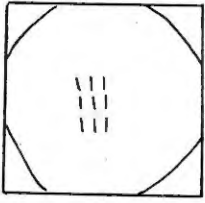




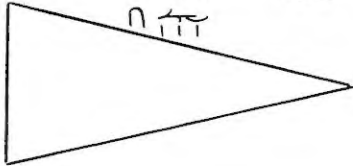


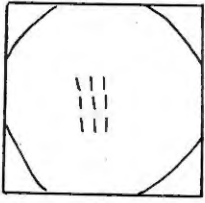




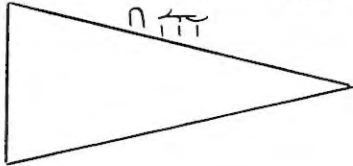


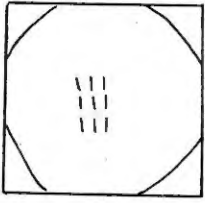




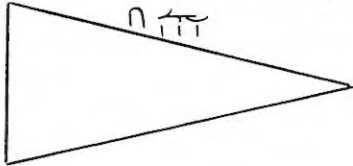


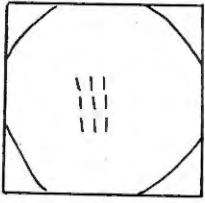




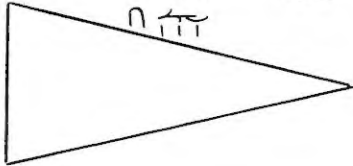


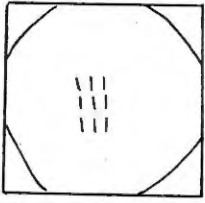




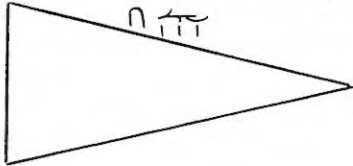


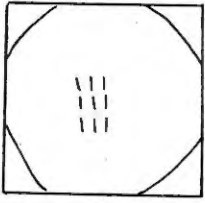




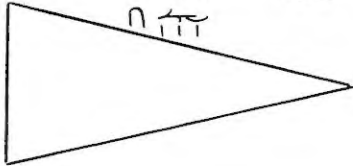


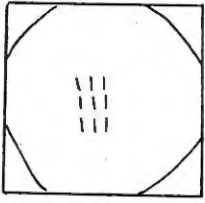




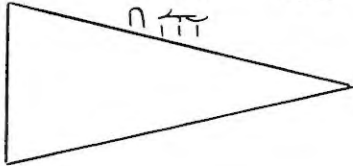


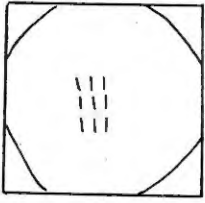




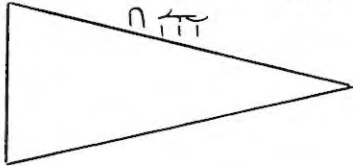


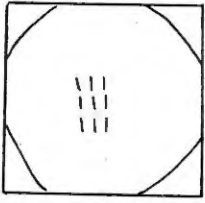




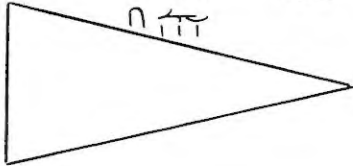


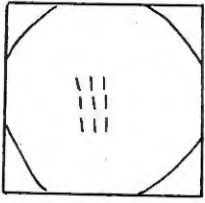




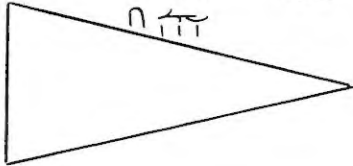


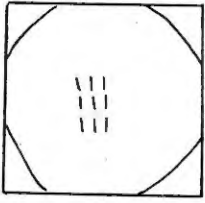




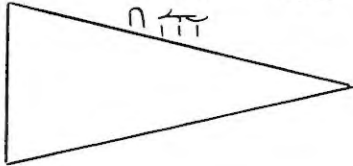


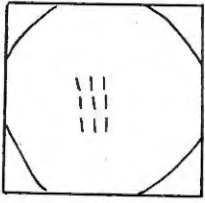




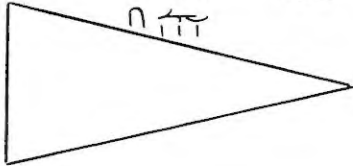


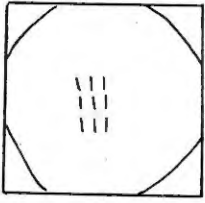




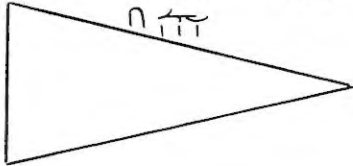


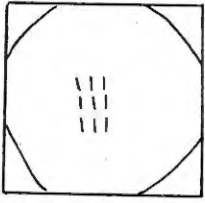




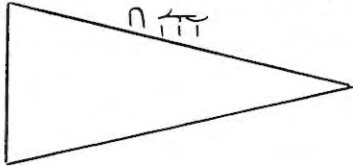


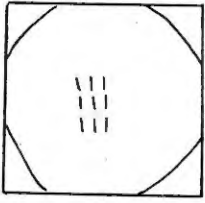




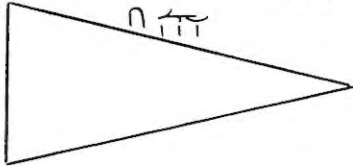


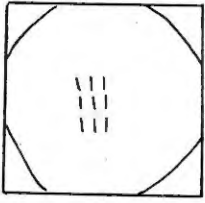




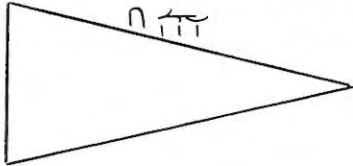


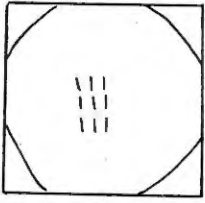




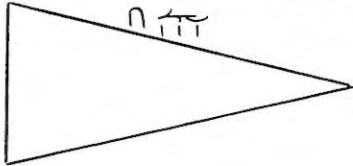


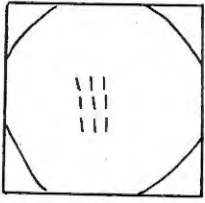




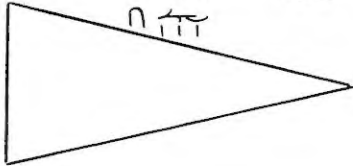


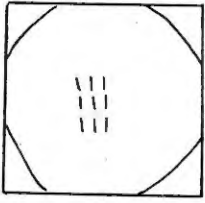




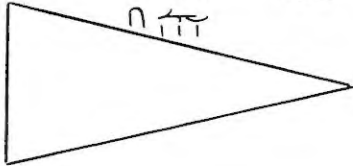












52

53

54

55

a. Or $\frac{500}{0}$

b.. Hieratic 3

c. All figures in red except the $\frac{111}{11}$

d. Read $\begin{matrix} \text{|||} \\ \text{||} \end{matrix}$

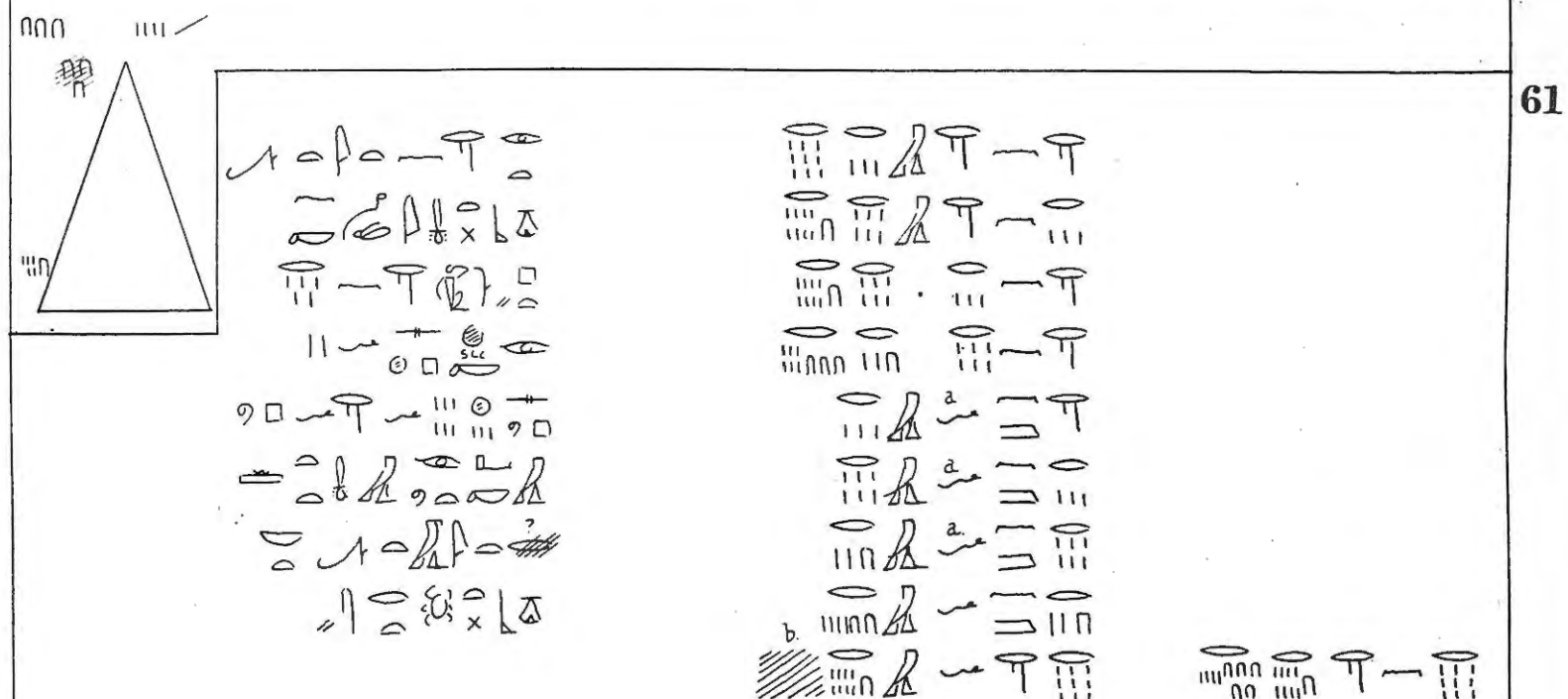
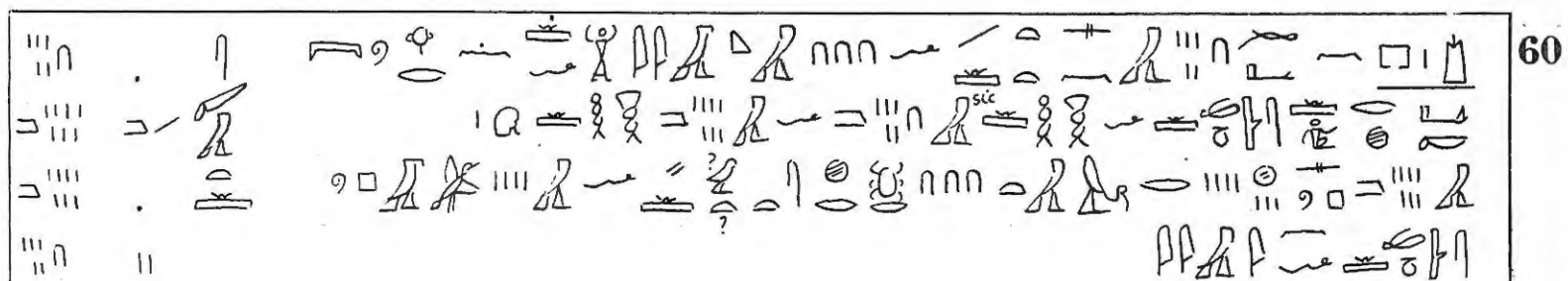
598

69

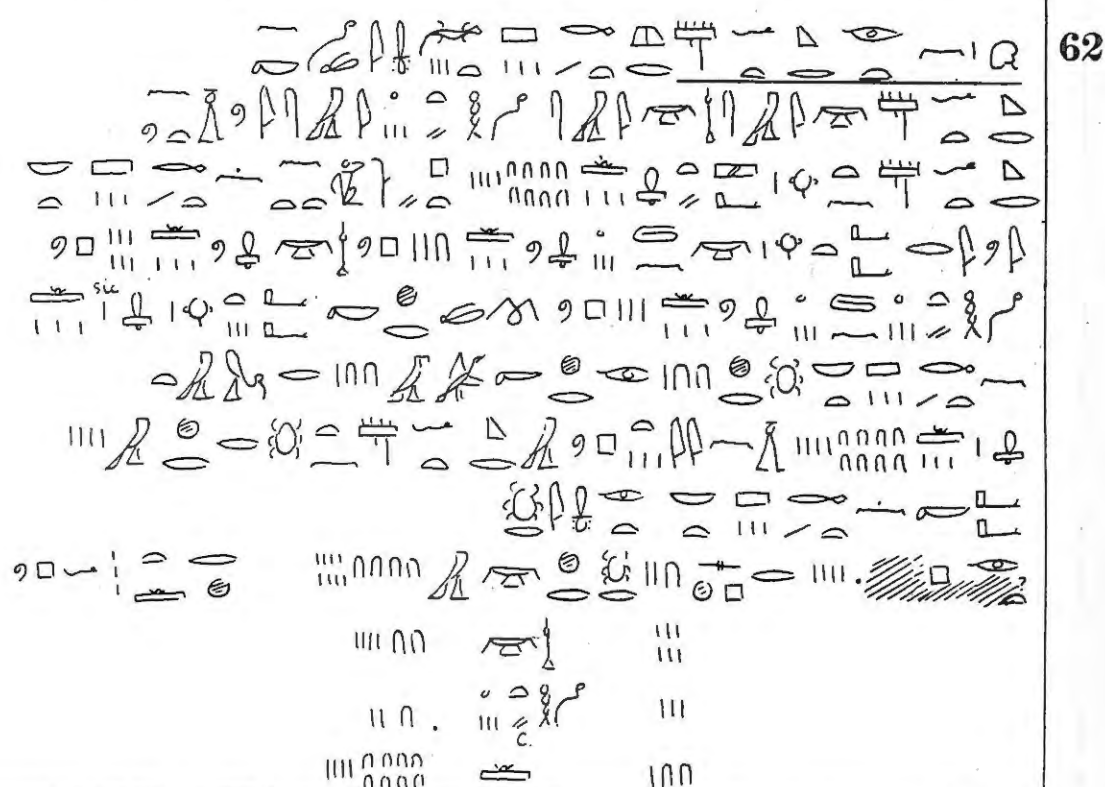
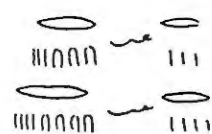
89

57

99

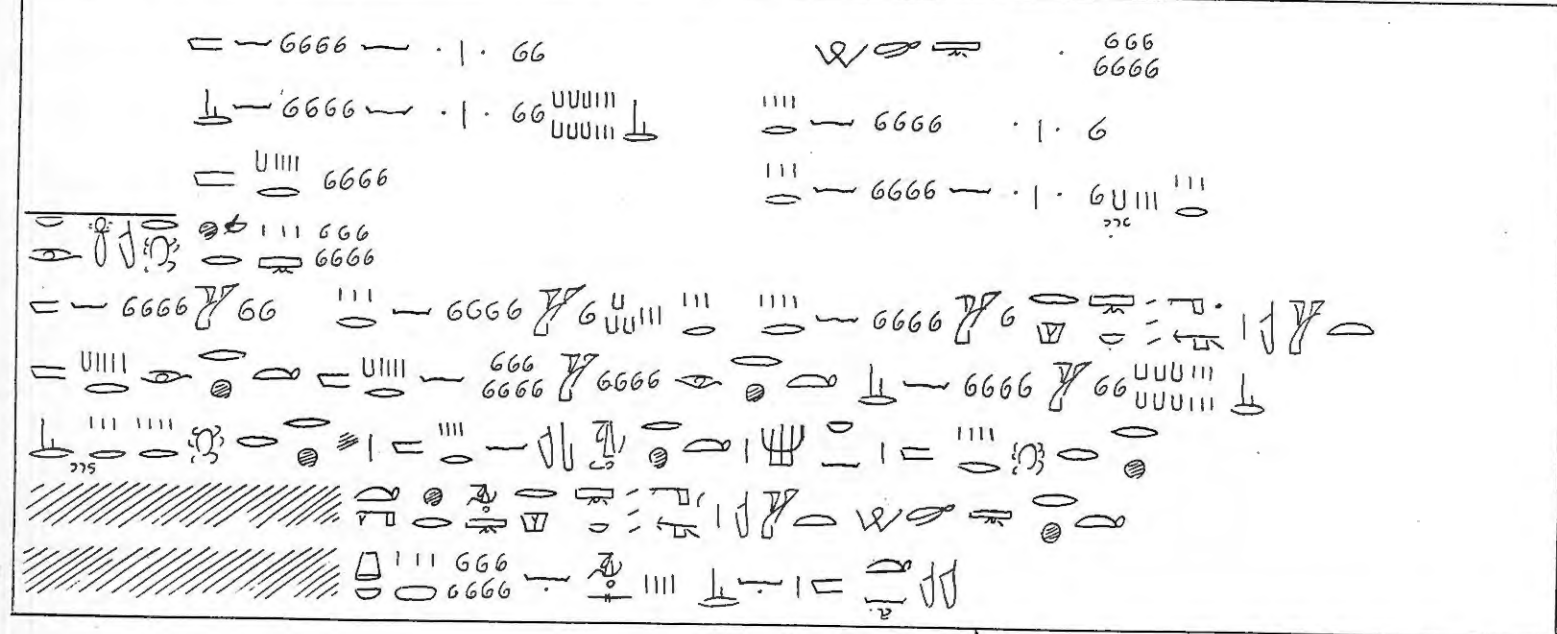


About 5 Lines Lost

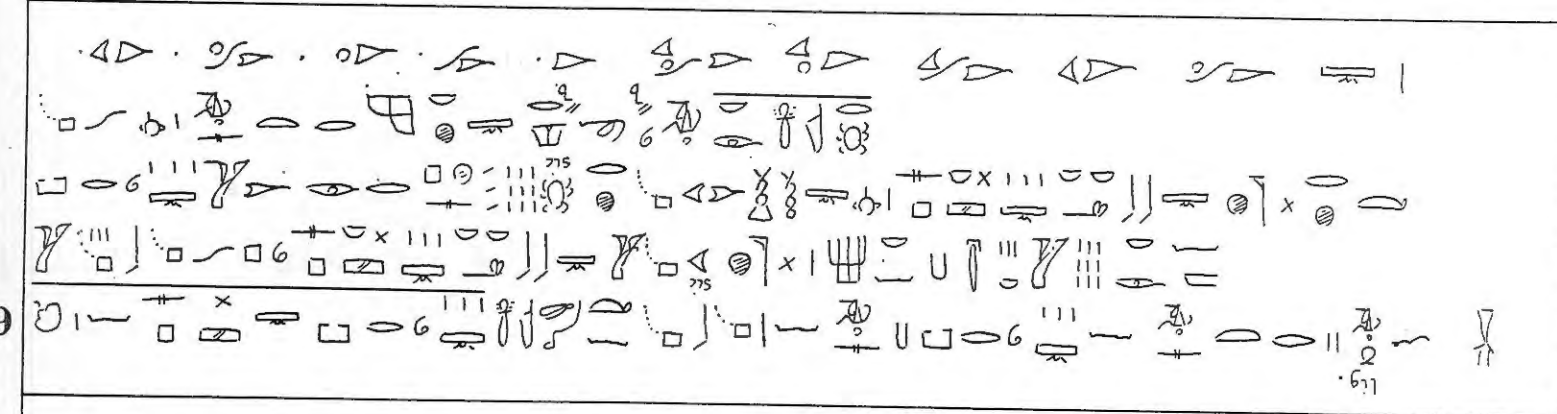


- a. Half erased.
b. Traces do not suit so.
c. over a vertical stroke.

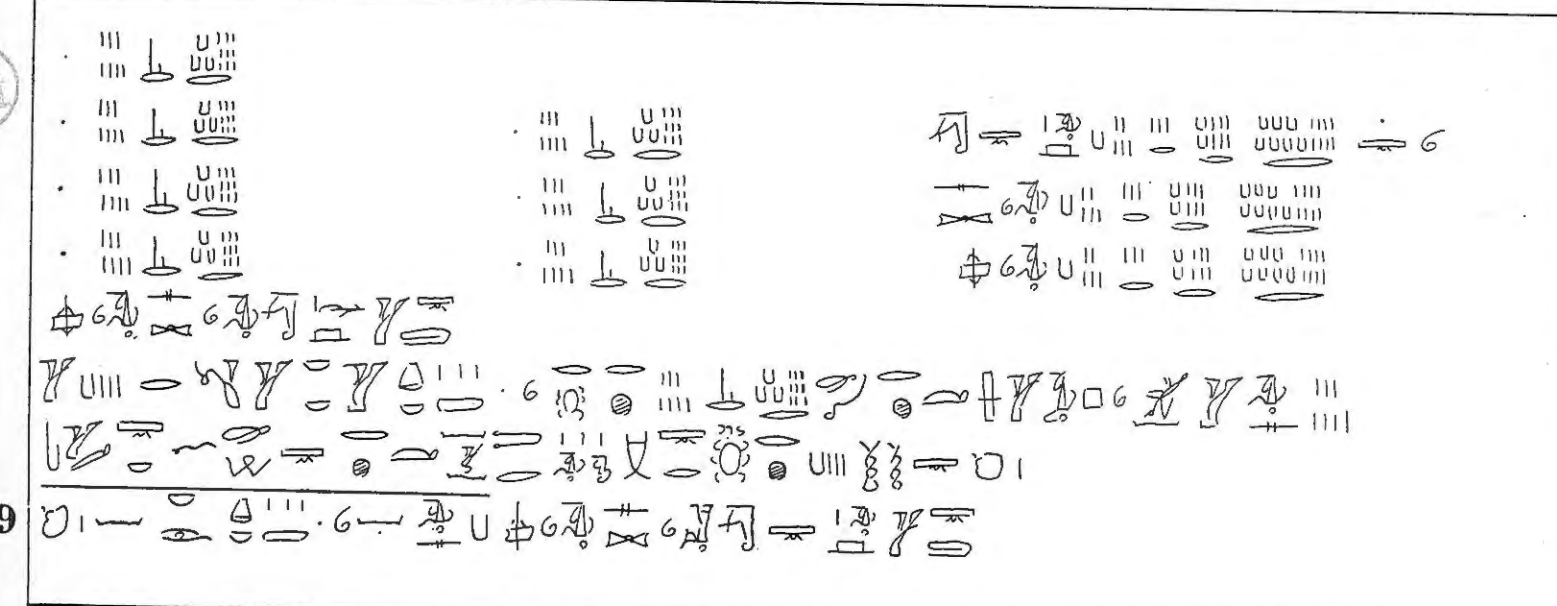
63



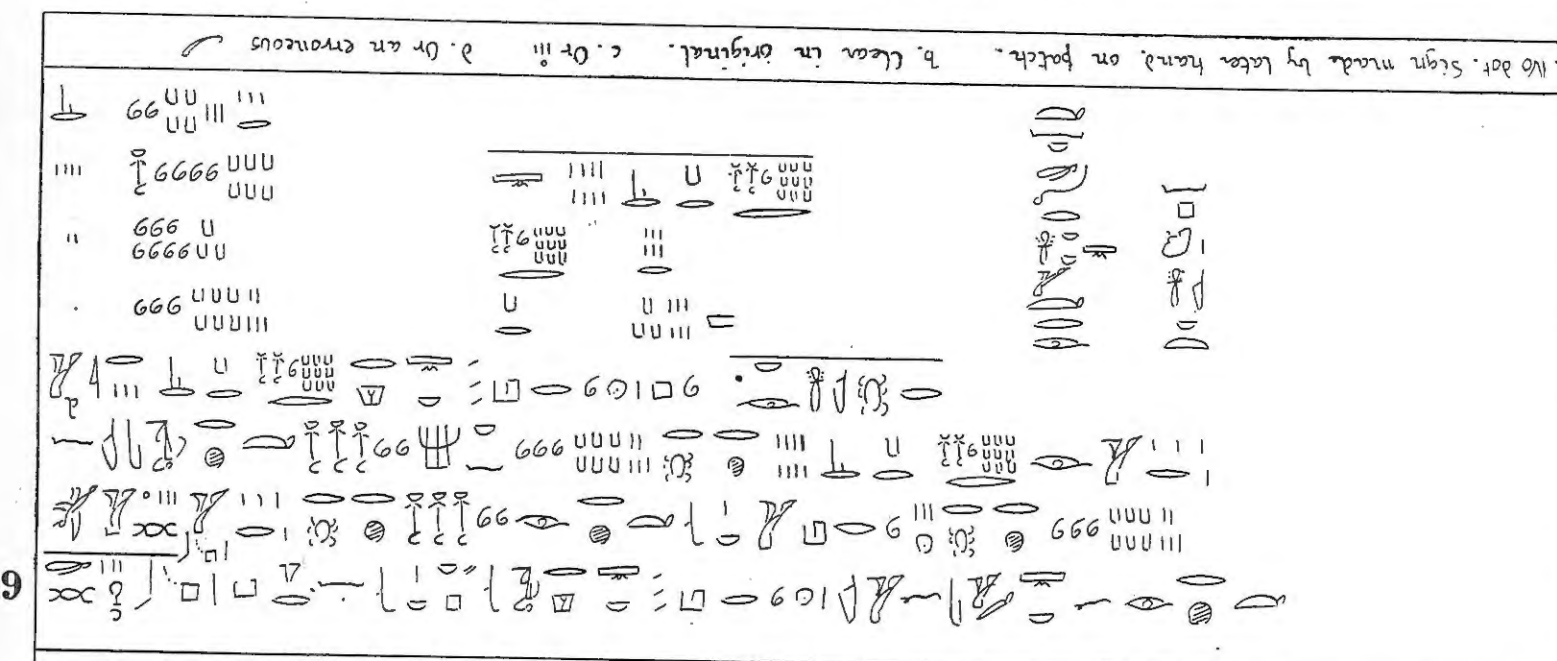
64

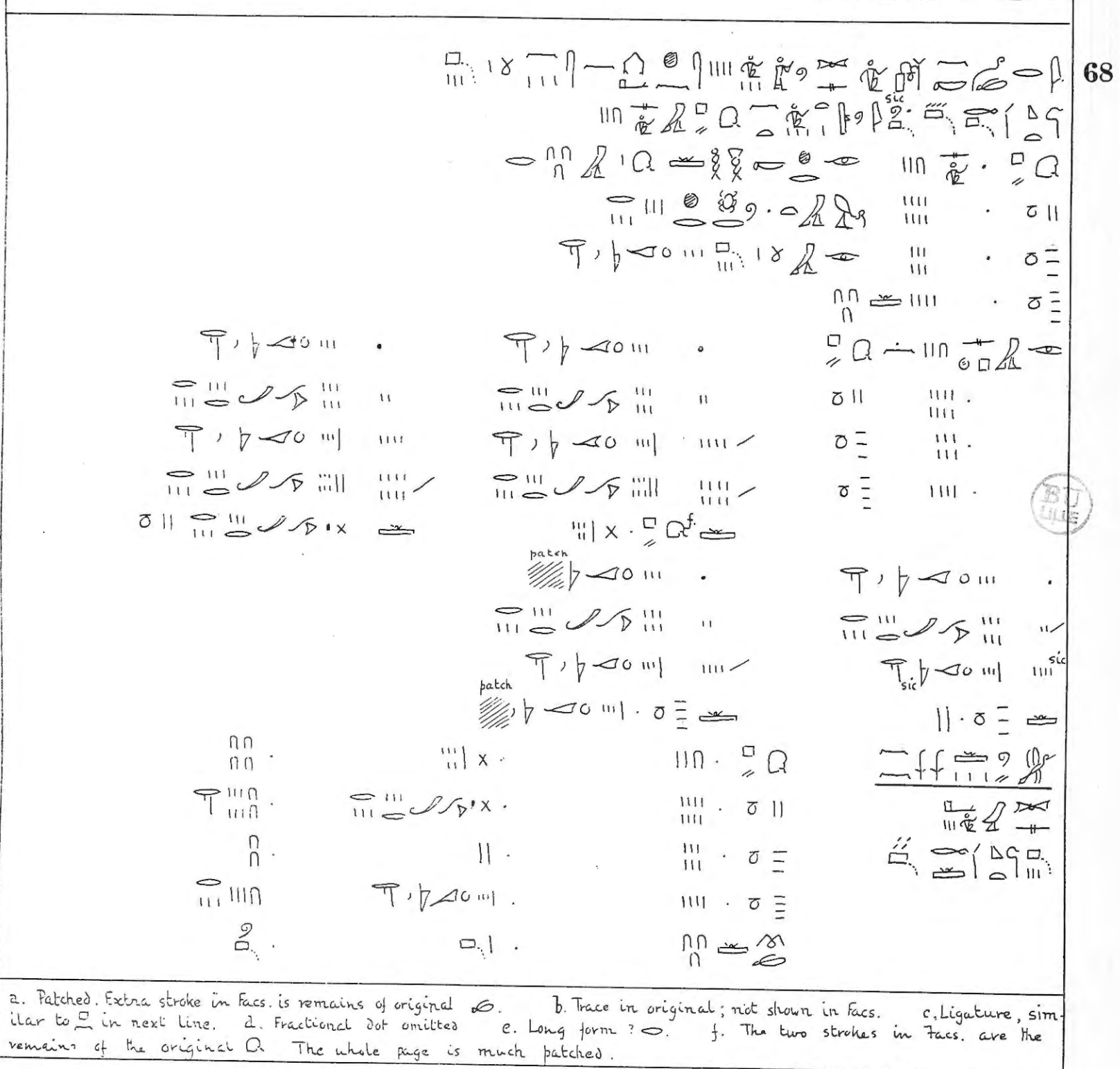
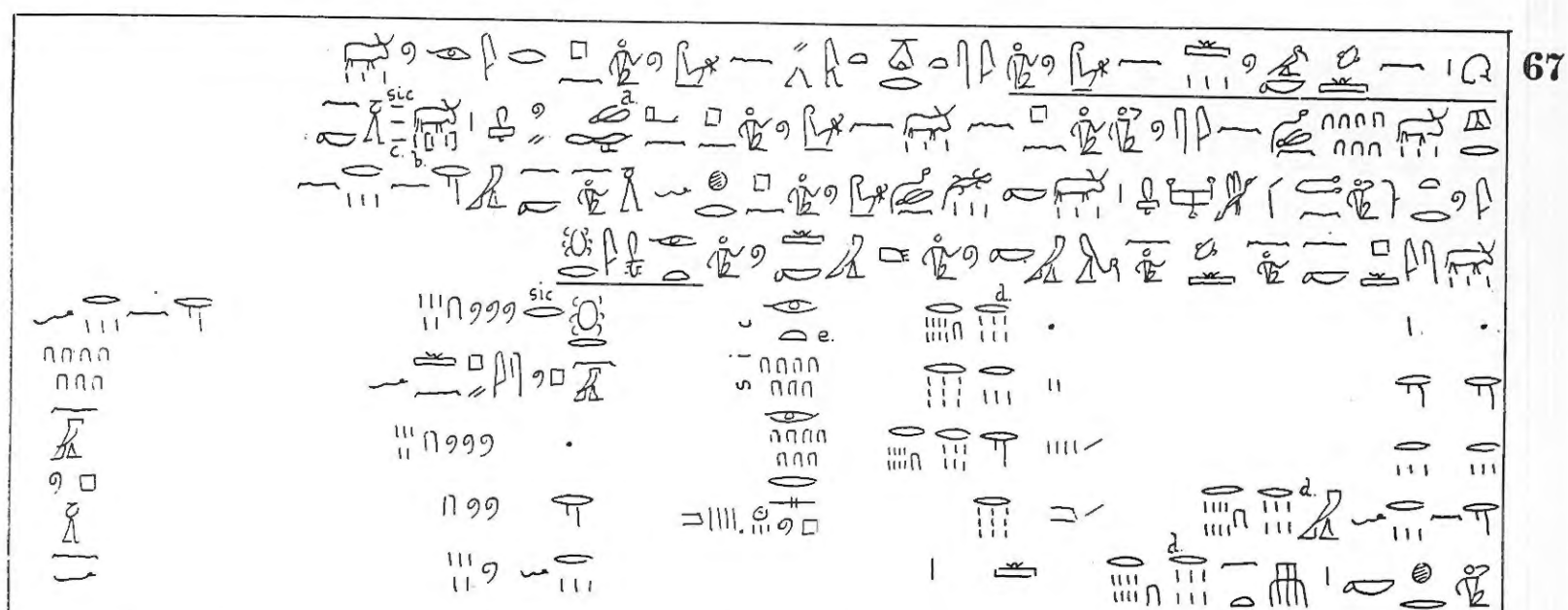


65



66





69

70

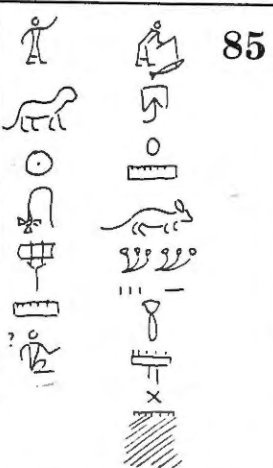
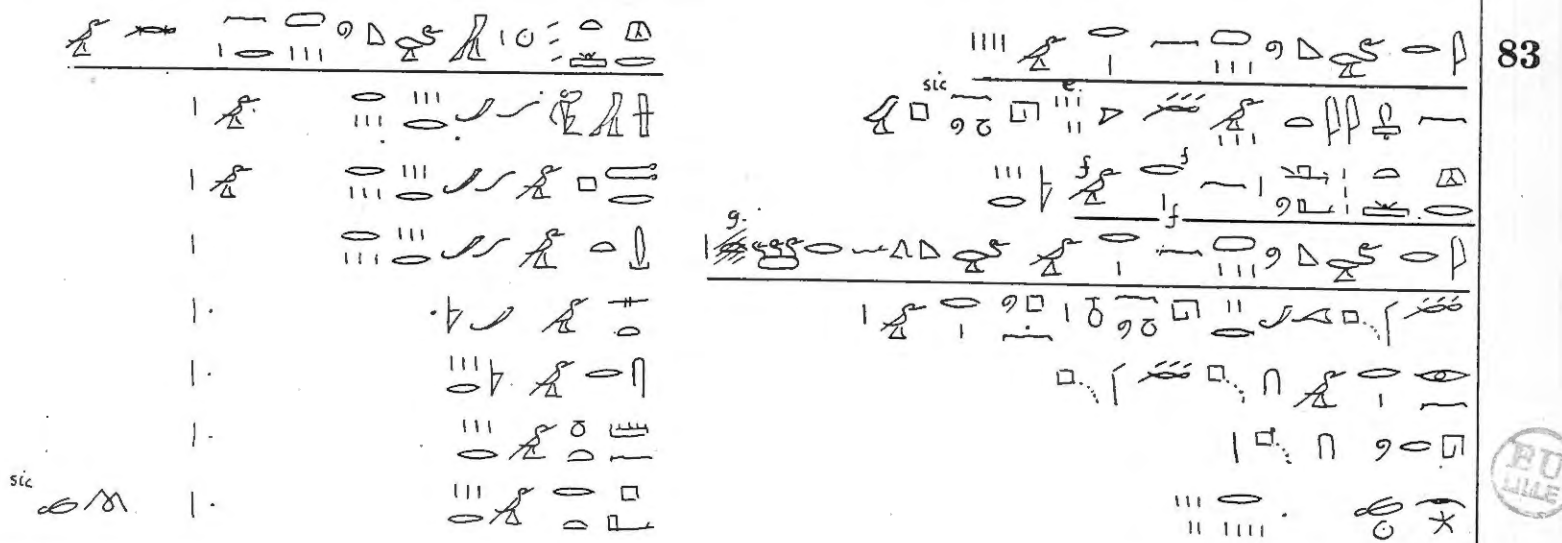
71

a.  on patch, but no trace of  on original. b. Hieratic 3. c. A trace. d. Clear in original.

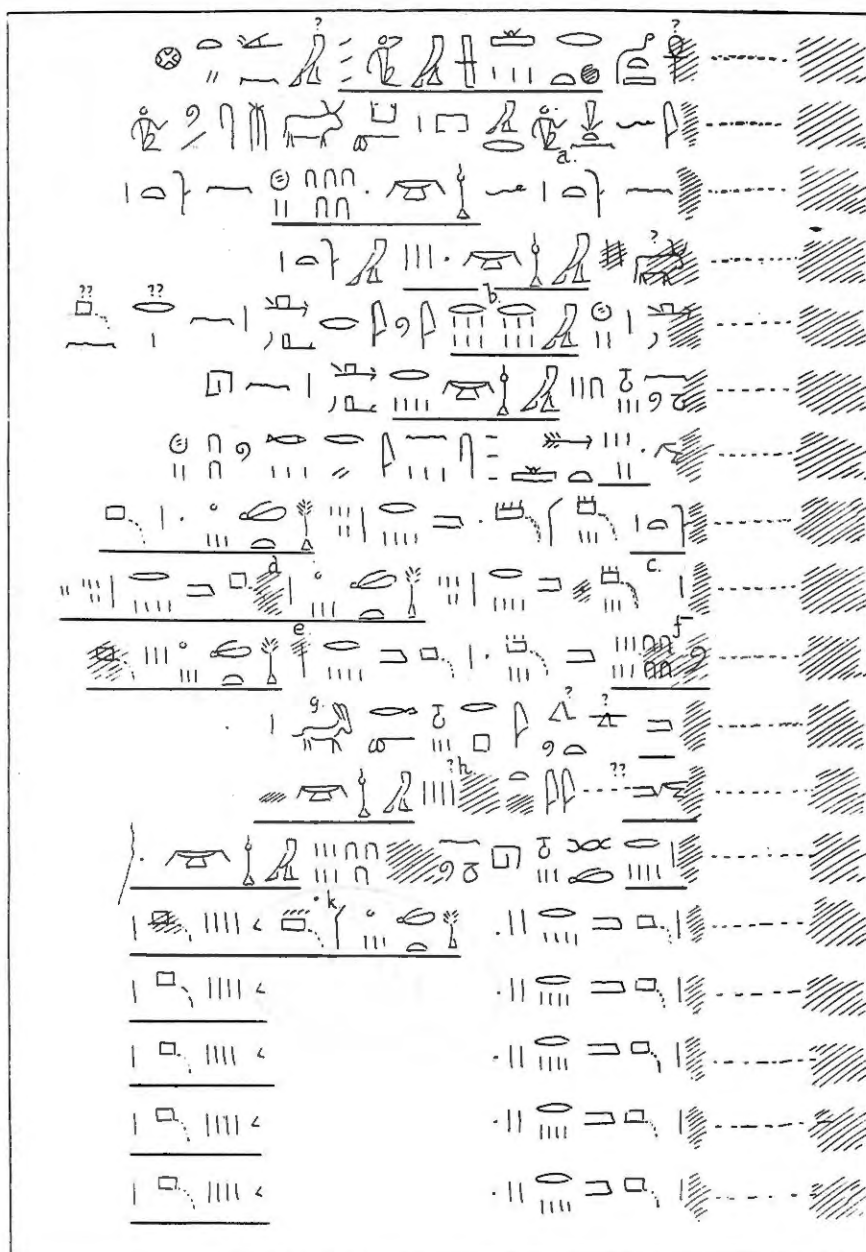
72	72
73	73
74	74
75	75
76	76
77	77

a. Perhaps a mere separator rather than an b. Probably an erased vertical stroke. c. Hieratic

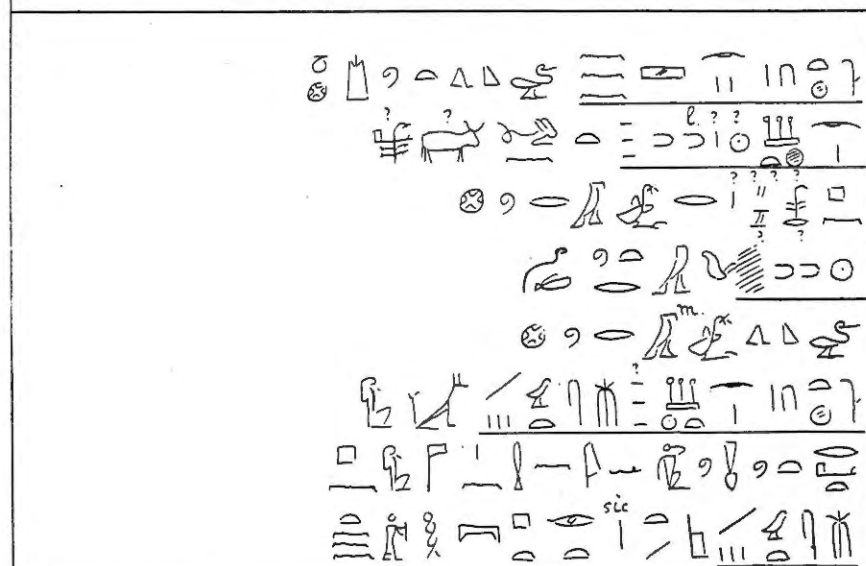
a. Cross stroke is part of . Read an erroneous ? b. Over a sign like c. Uncertain. Form very unusual. d. Abnormal form. e. Numeral made like 5 hkt. f. Red, not black as in Facs. g. Uncertain: fold in papyrus. h. Made like 60. k. Hieratic l. Not convincing; but what else possible?



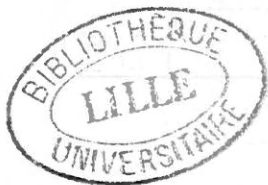
86



87



- a. The — +  both clear in original. b. Or simply 
 c. Probably blank in original d. Perhaps no loss. e. Probably
 10 not 5 as in facsimile f. Traces g. Clear in original.
 h. First stroke doubtful. k. Certain, despite facsimile.
 l. Not convincing on original which is here obscured by dis-
 coloured gelatine. m. Stroke shown by facsimile across
 this sign is not there.



T. Eric Peet — THE RHIND MATHEMATICAL PAPYRUS —

1923

7A
40522
AZ